

Optimization When You Don't Know the Future

Roie Levin

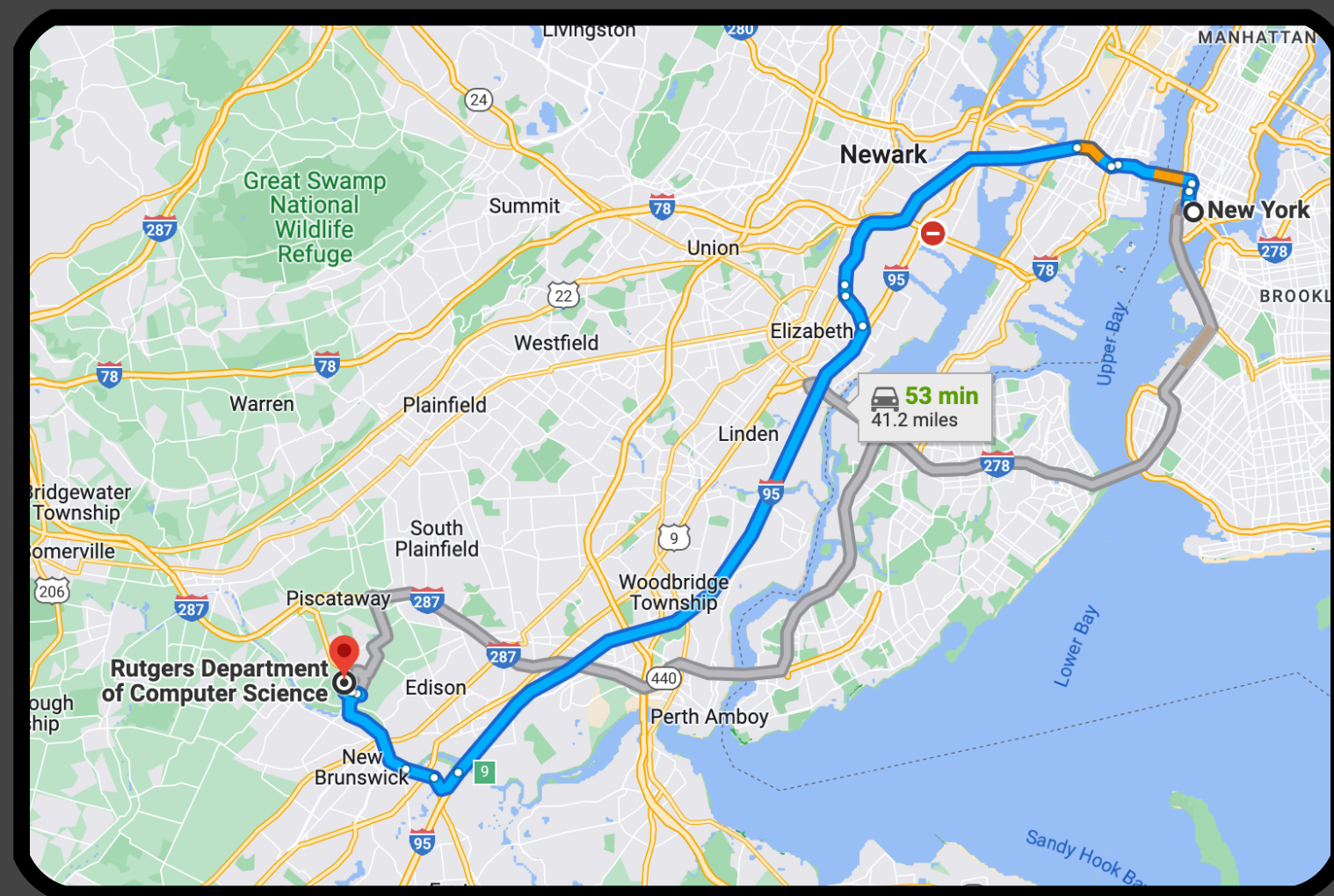
Introduction

My Research

I research algorithms
for optimization
in the face of uncertainty.

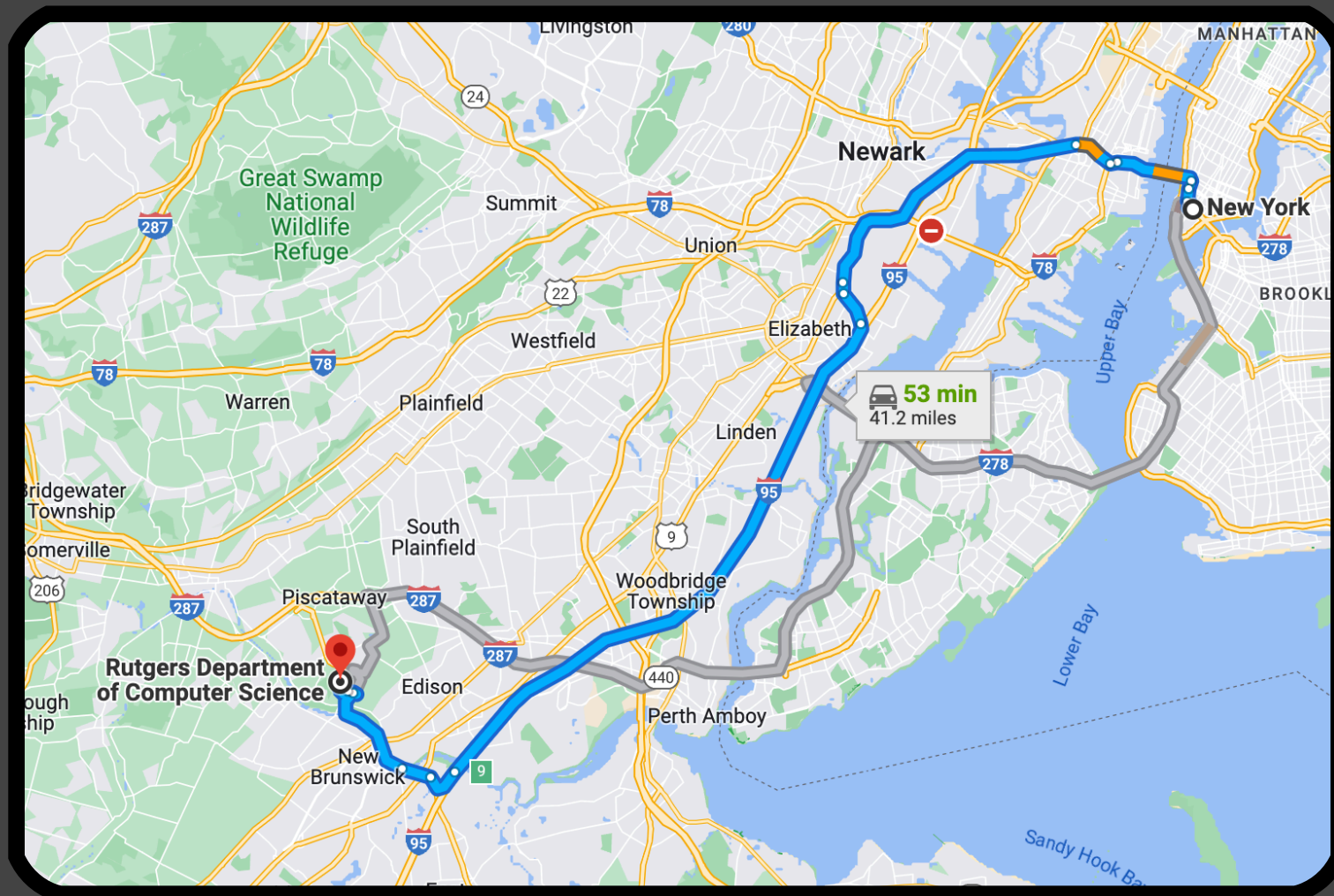
Classical CS is about Computational Challenges

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ShortestPath

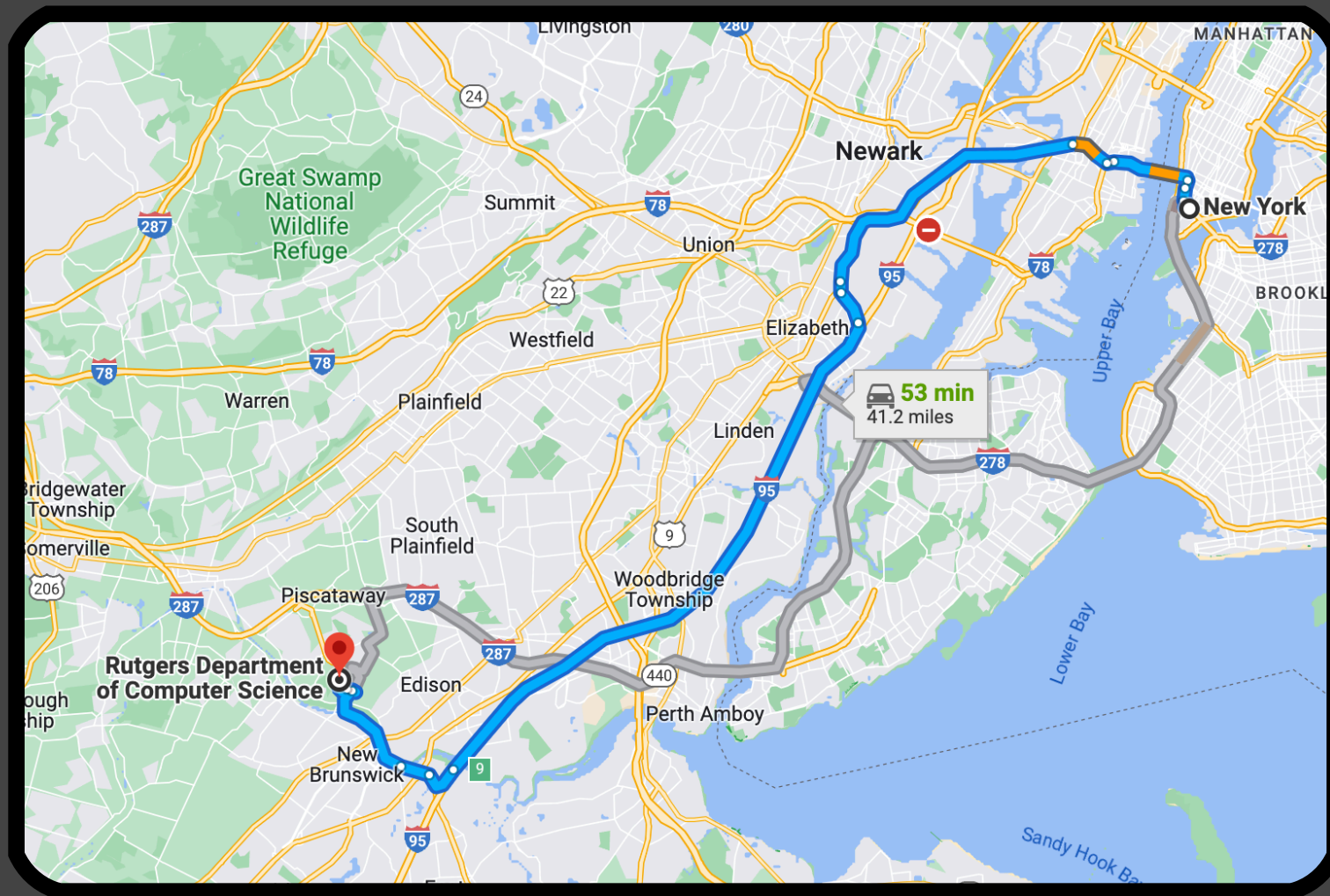


Knapsack



Classical CS is about Computational Challenges

ShortestPath



Knapsack

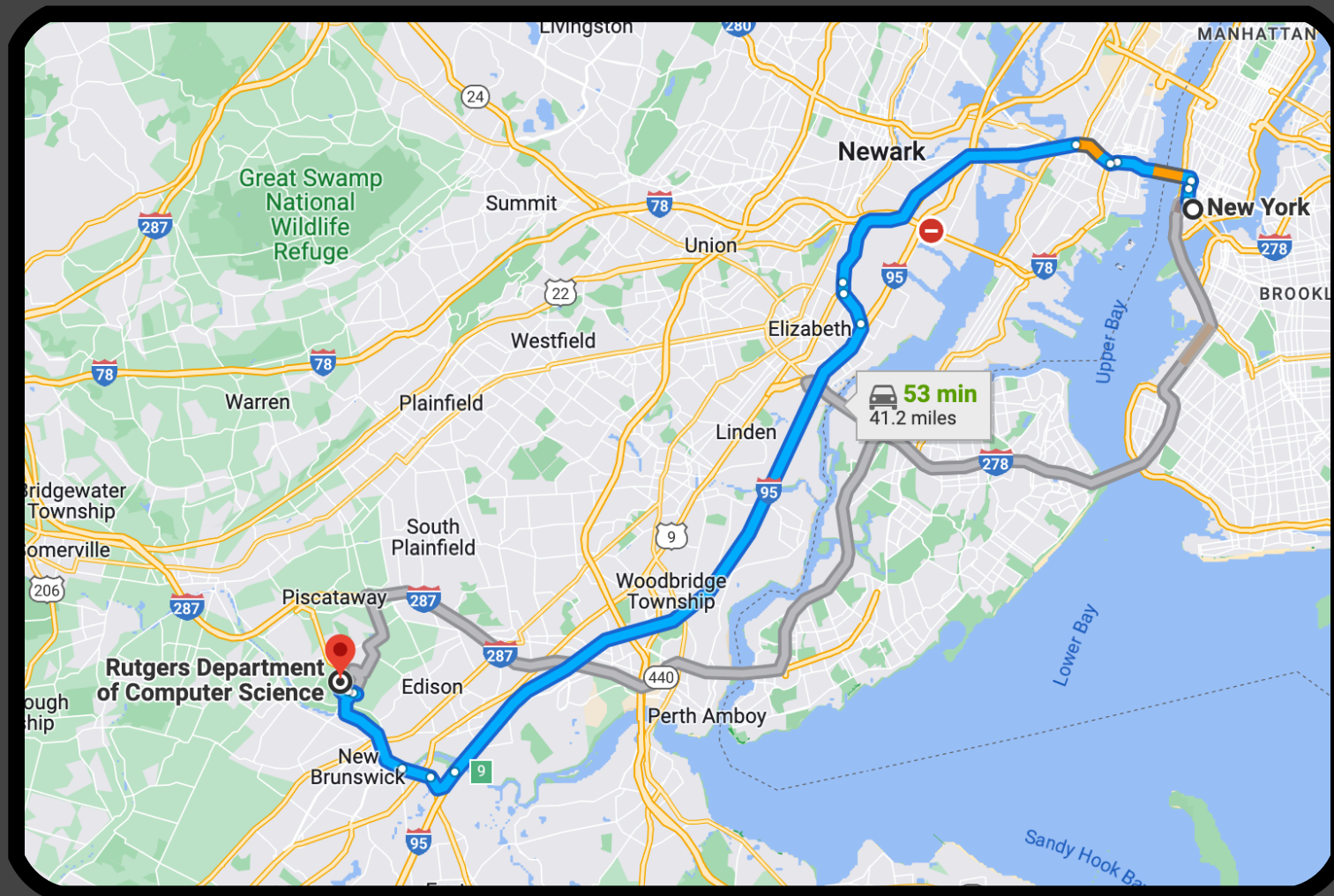


Computationally Easy

Computationally Hard

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ShortestPath



Knapsack



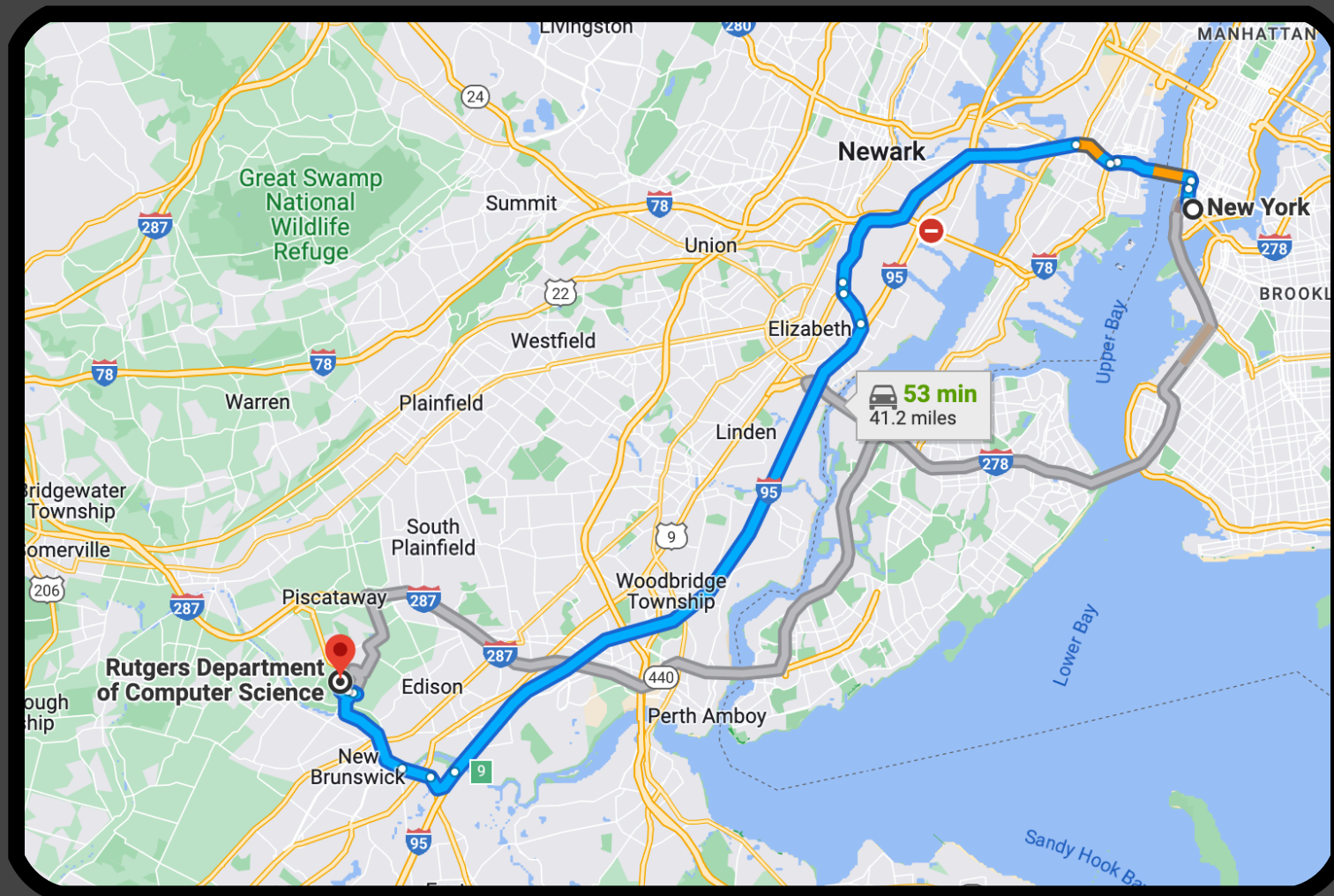
NP-hard

Computationally Easy

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ShortestPath



Approximate
Knapsack



Knapsack



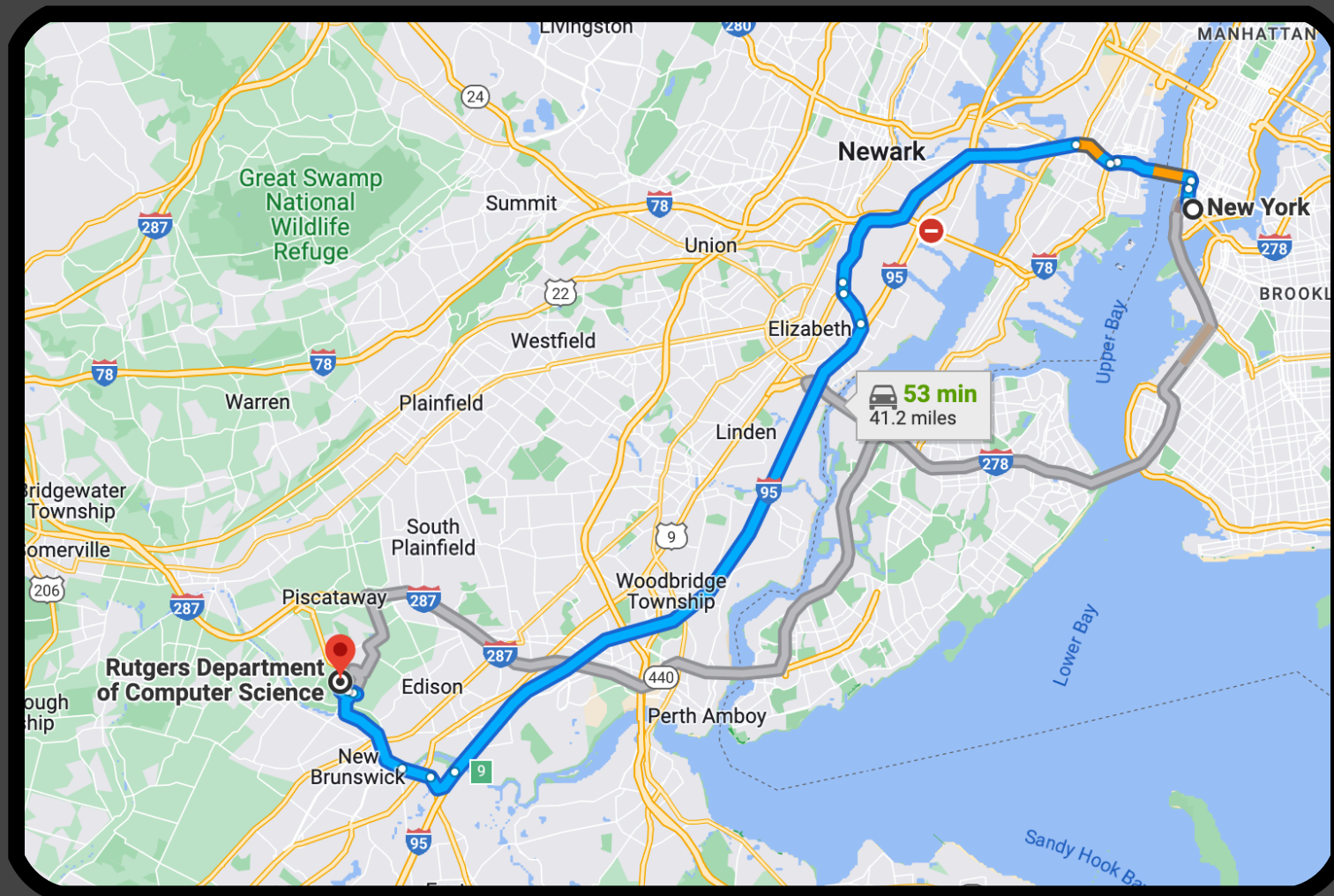
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Approximate
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NP-hard

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Beautiful theory of Approximation Algorithms!

A Different Source of Hardness: Uncertainty

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FindMax

4	1	10	-2	22	7
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A Different Source of Hardness: Uncertainty

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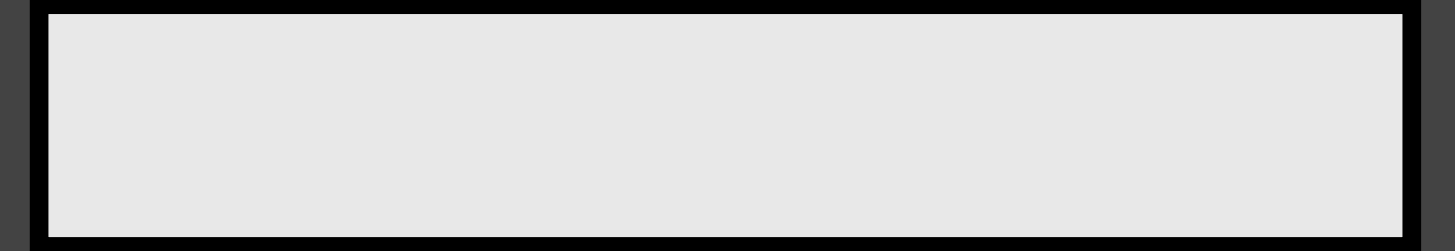
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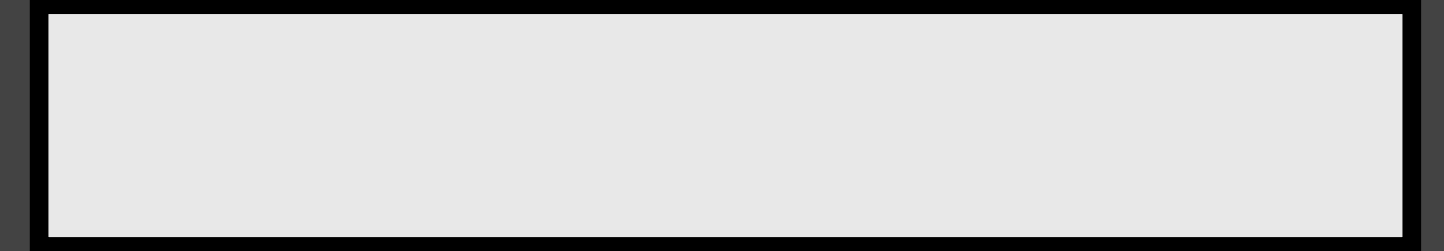
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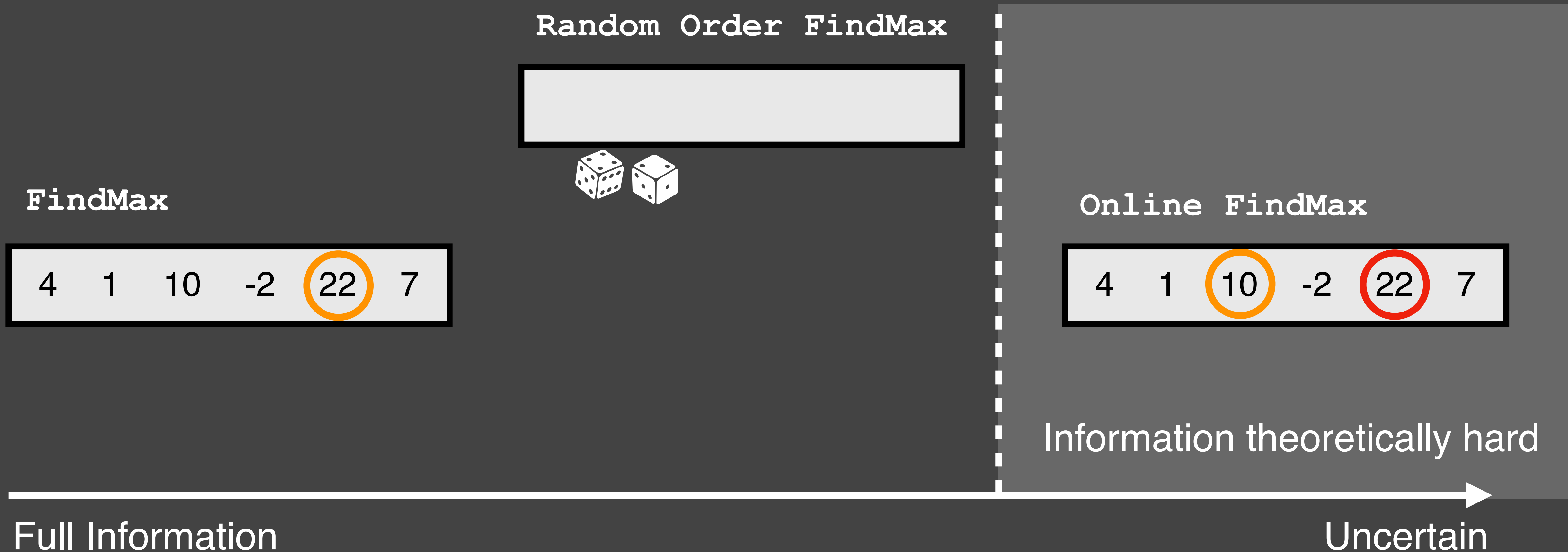
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Information theoretically hard

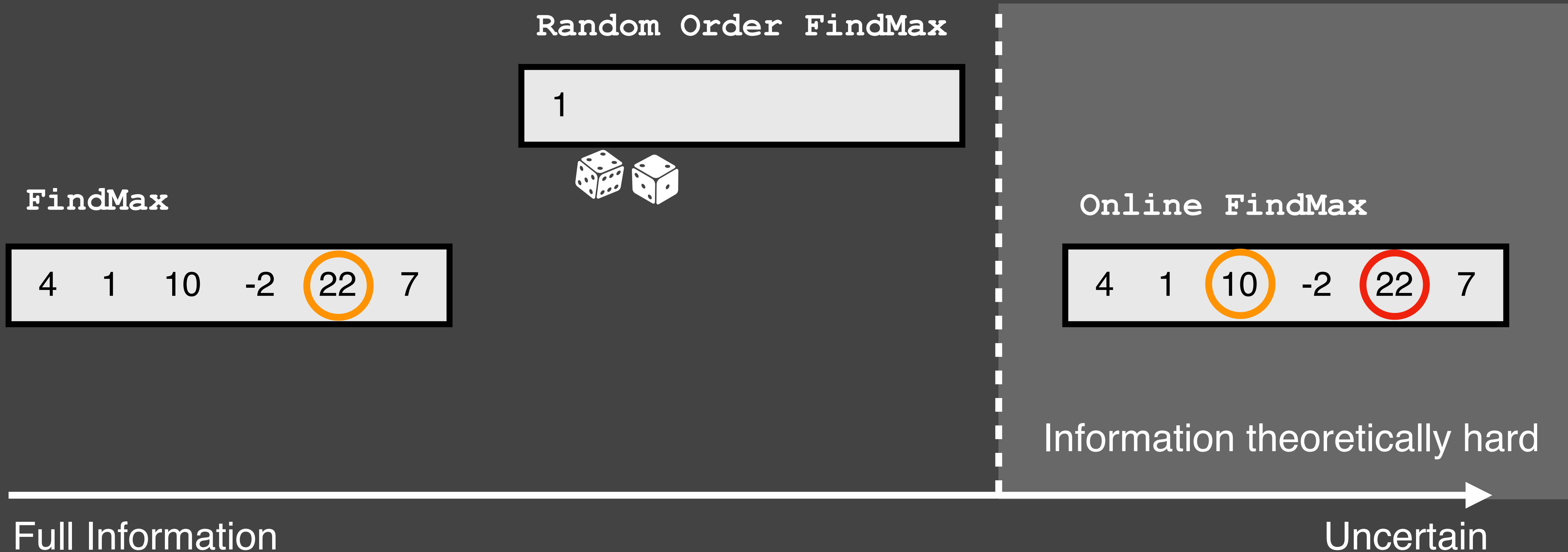
Full Information

Uncertain

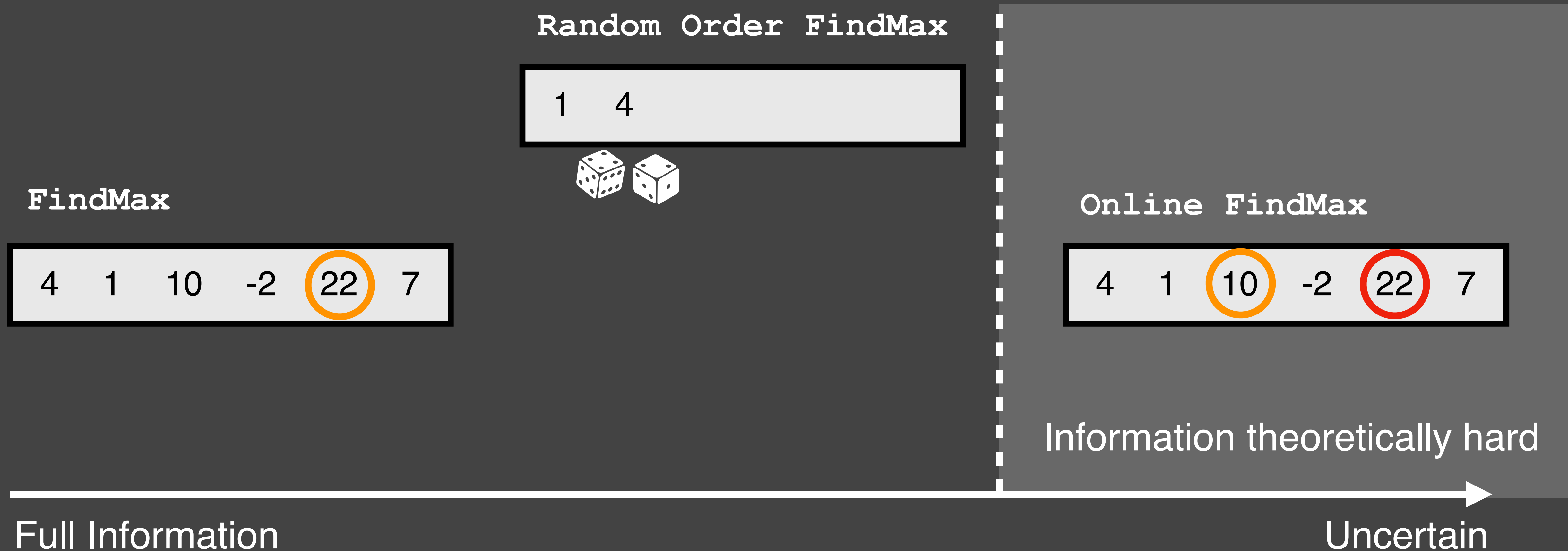
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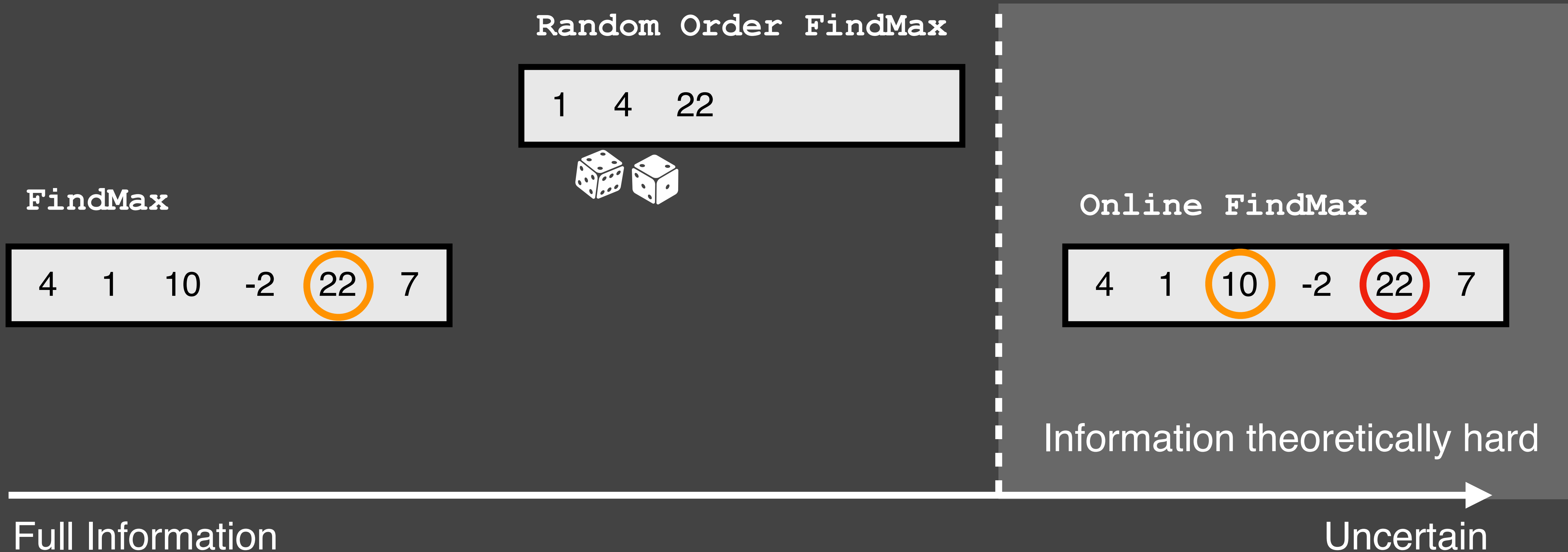
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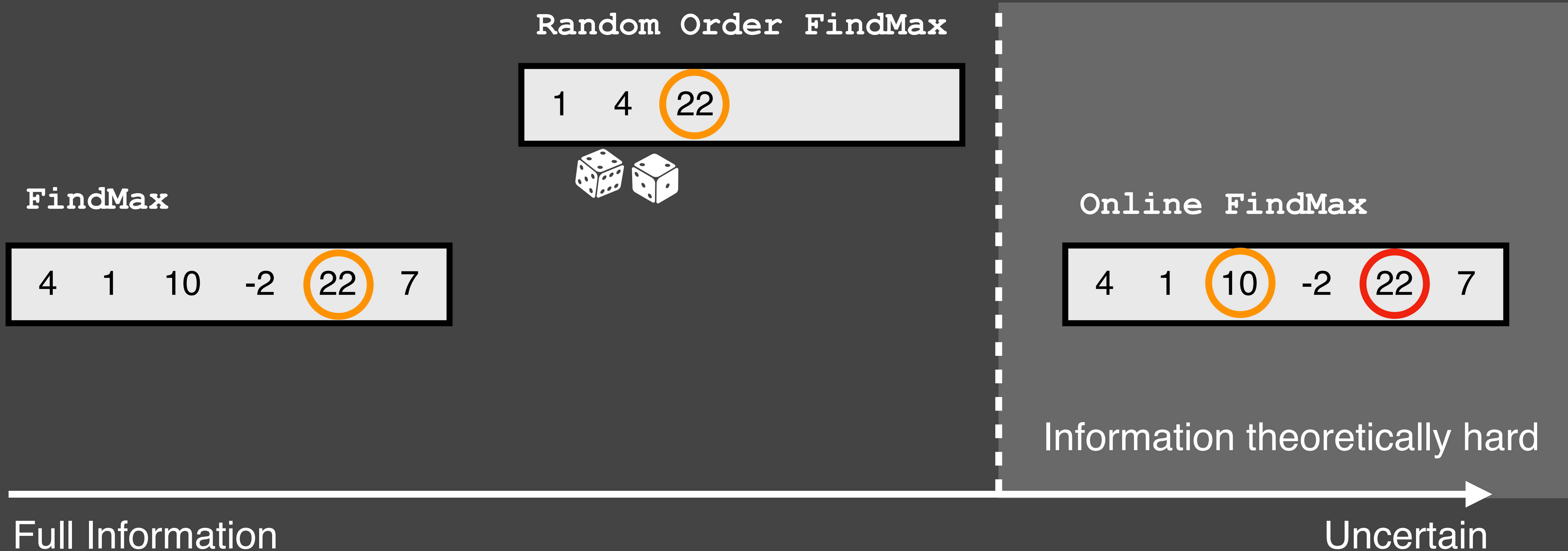
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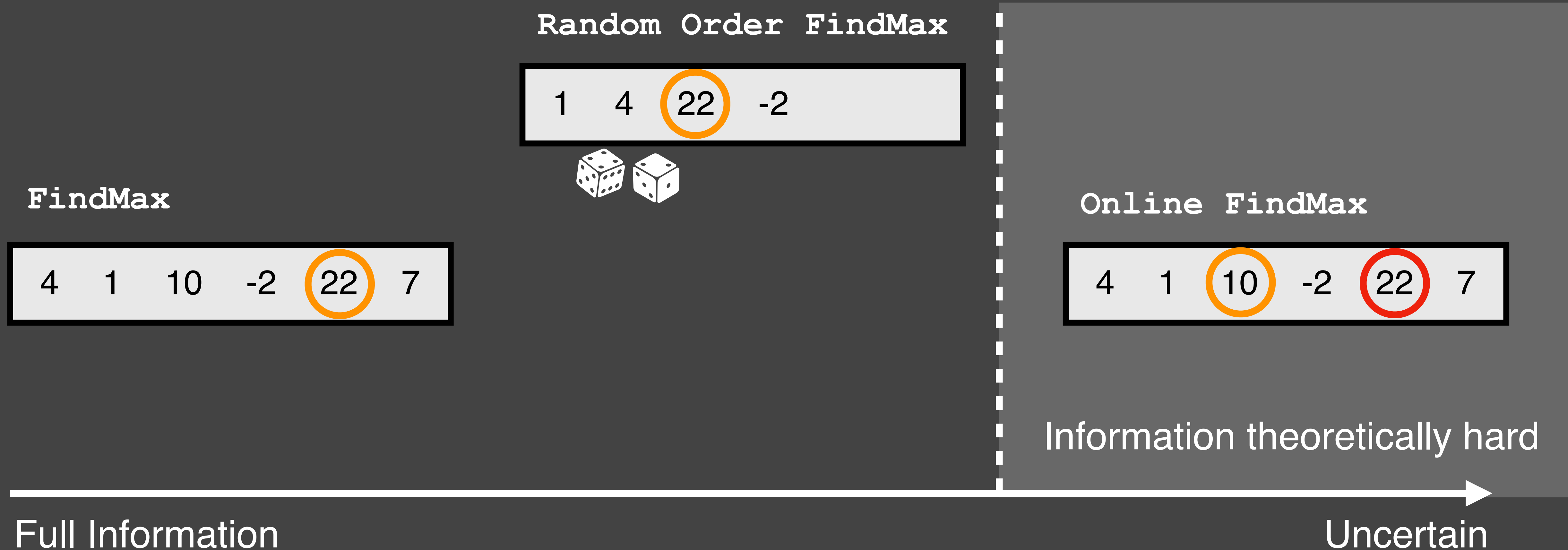
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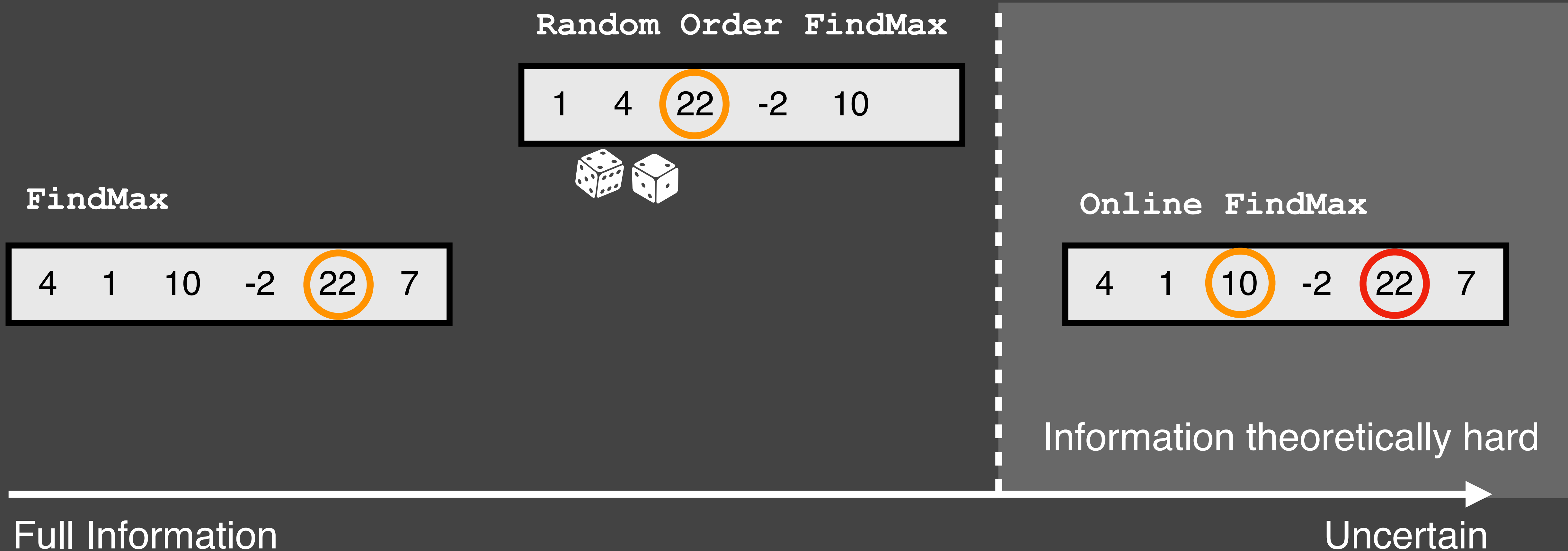
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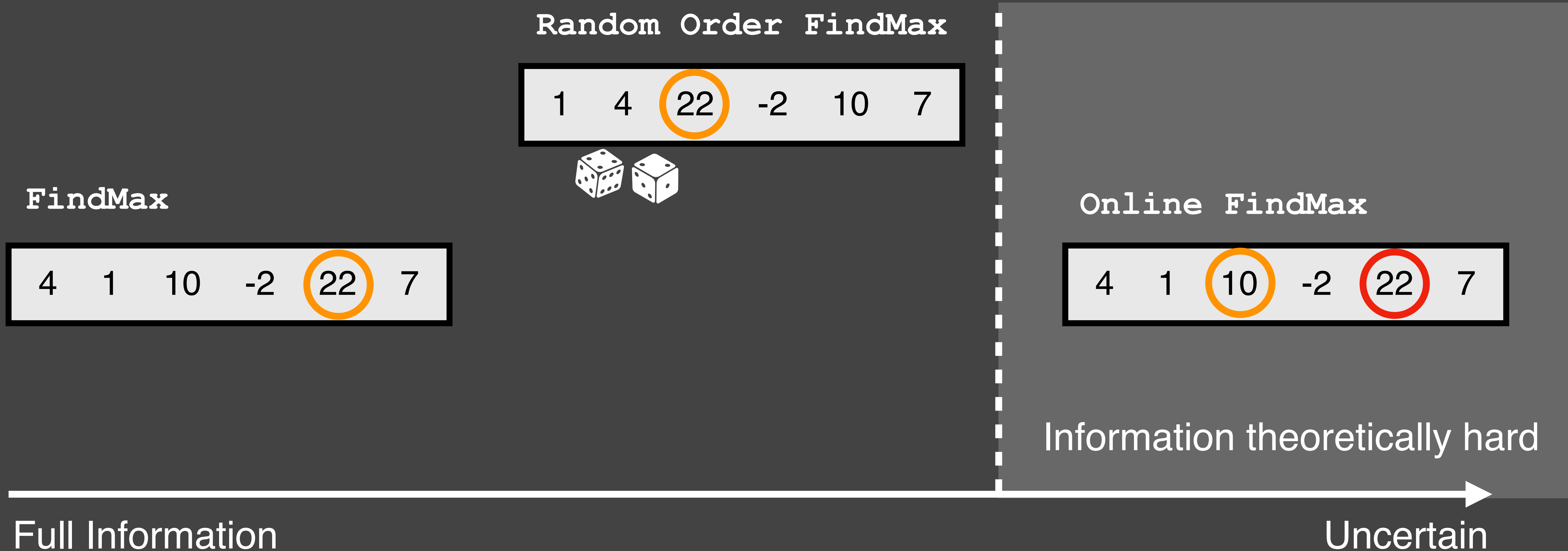
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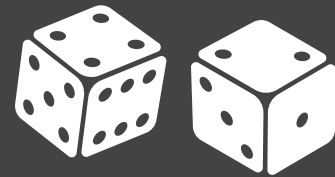
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a.k.a. Secretary Problem

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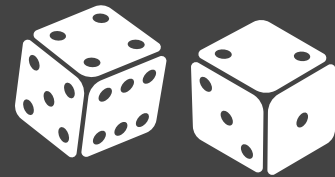
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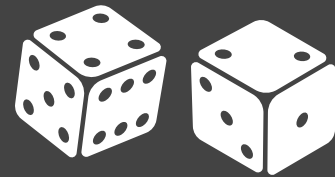
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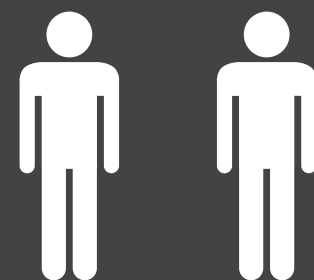
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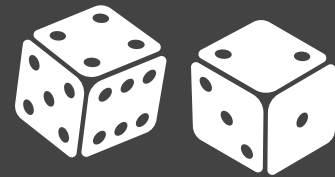
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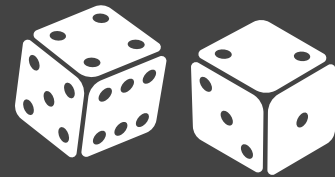
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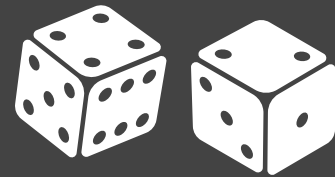
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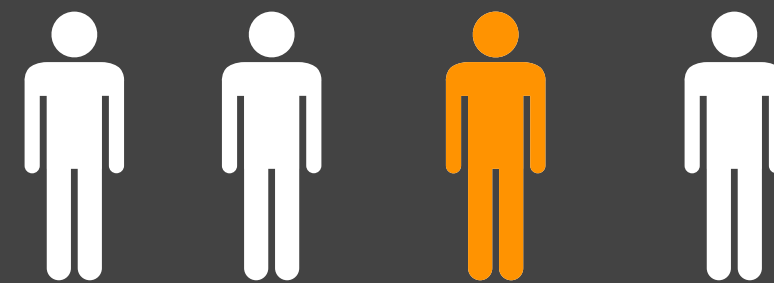
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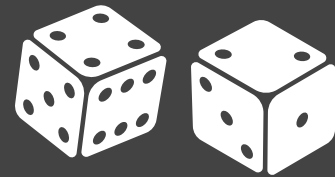
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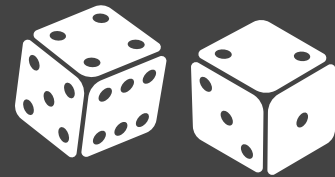
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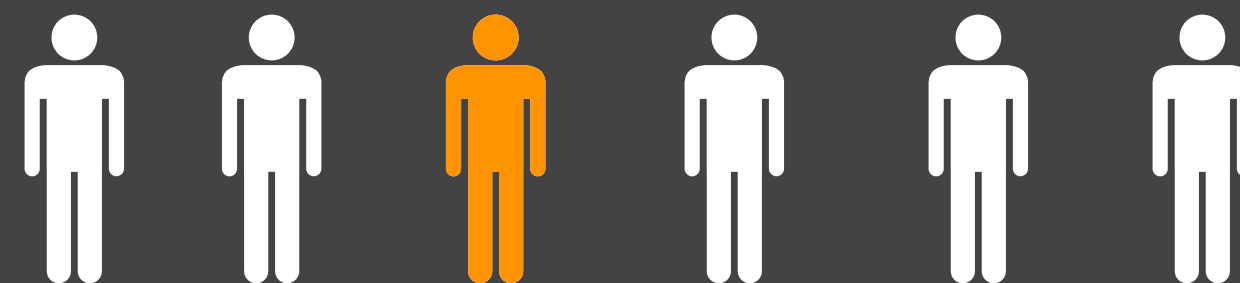
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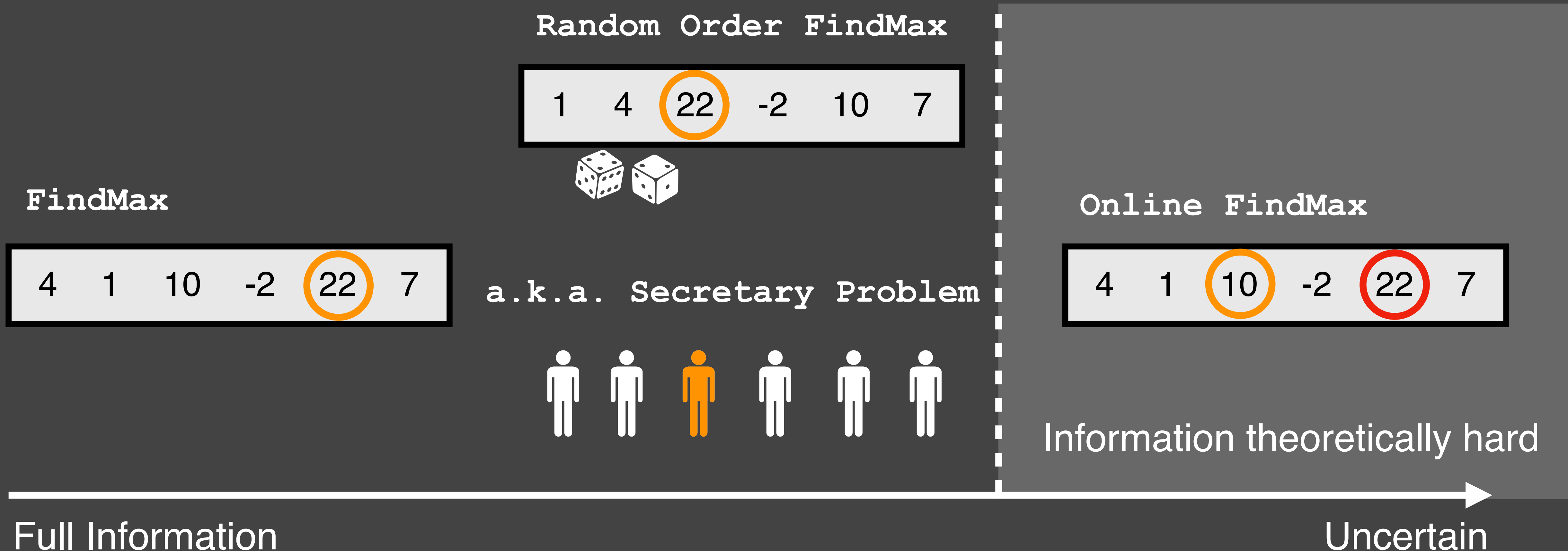
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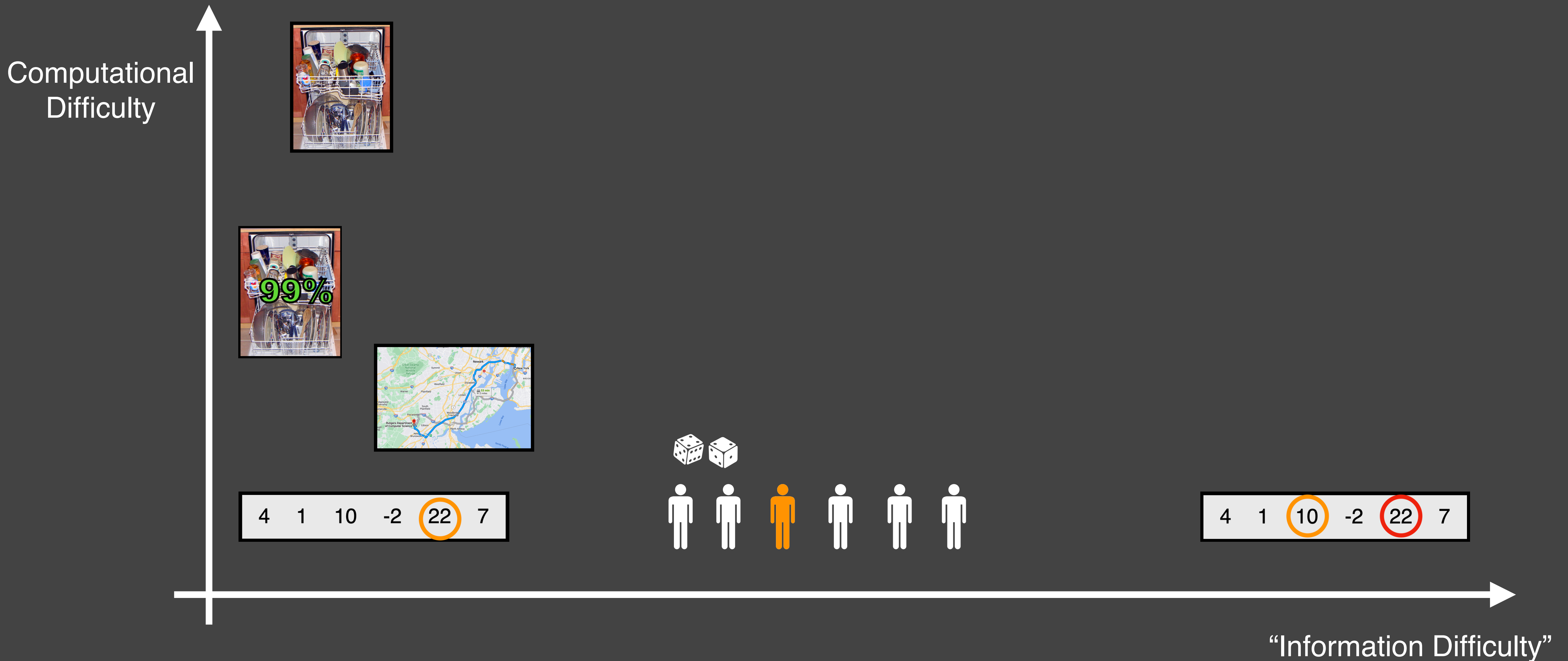


Beautiful theory of **Decision Making Under Uncertainty!**

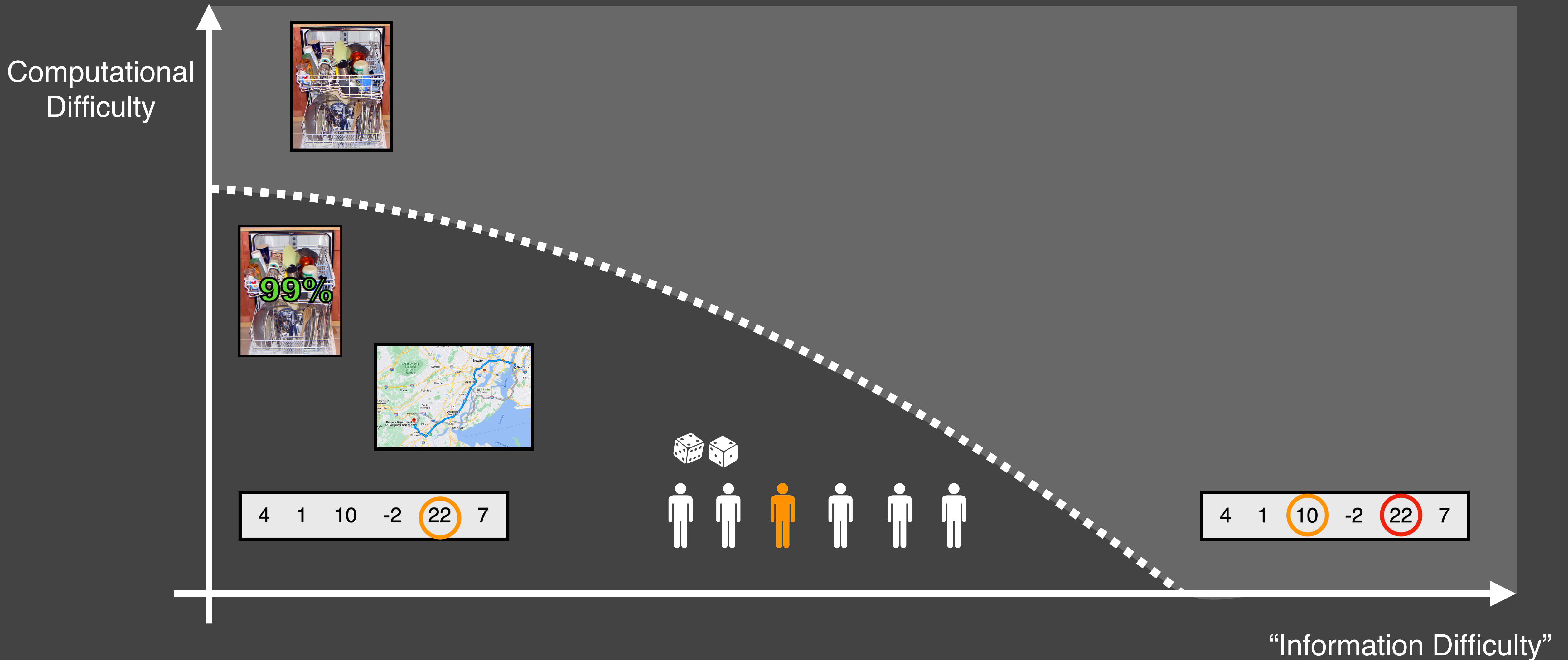
The Computation/Information Landscape



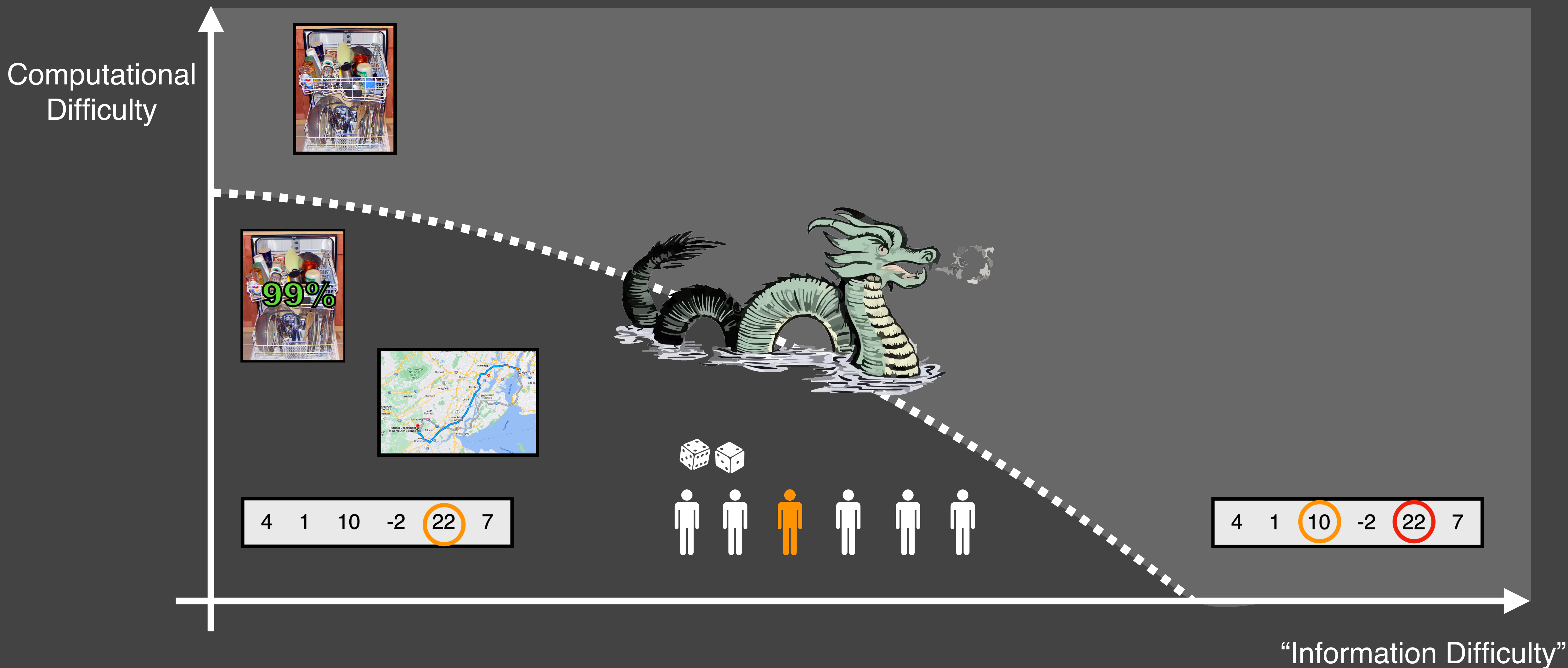
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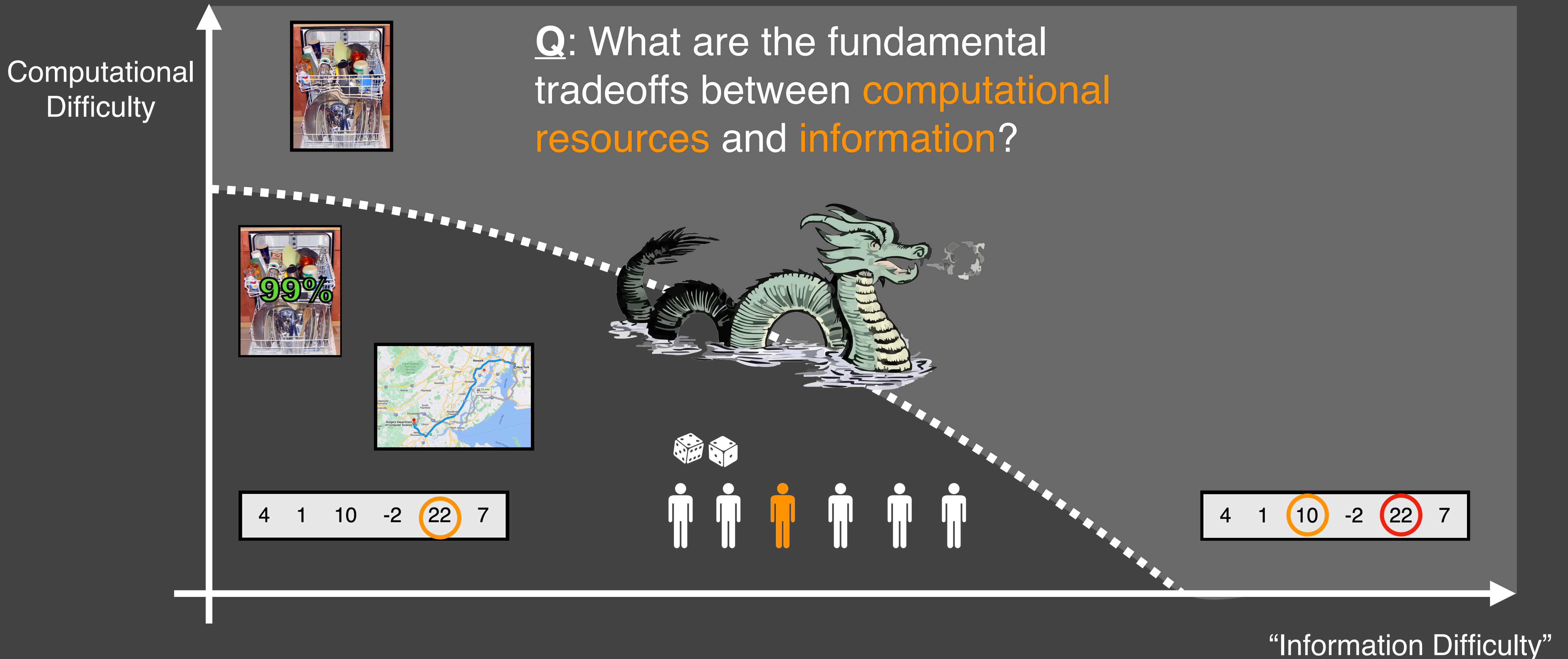
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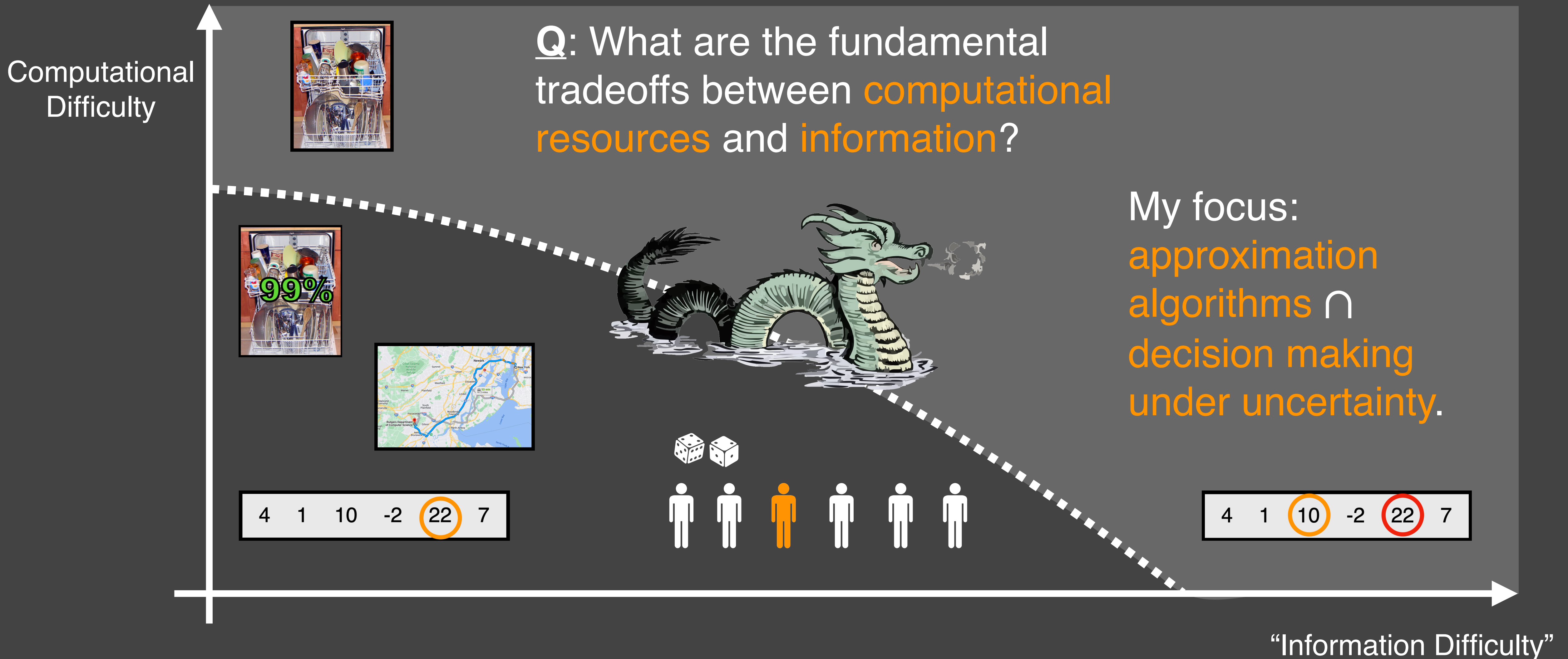
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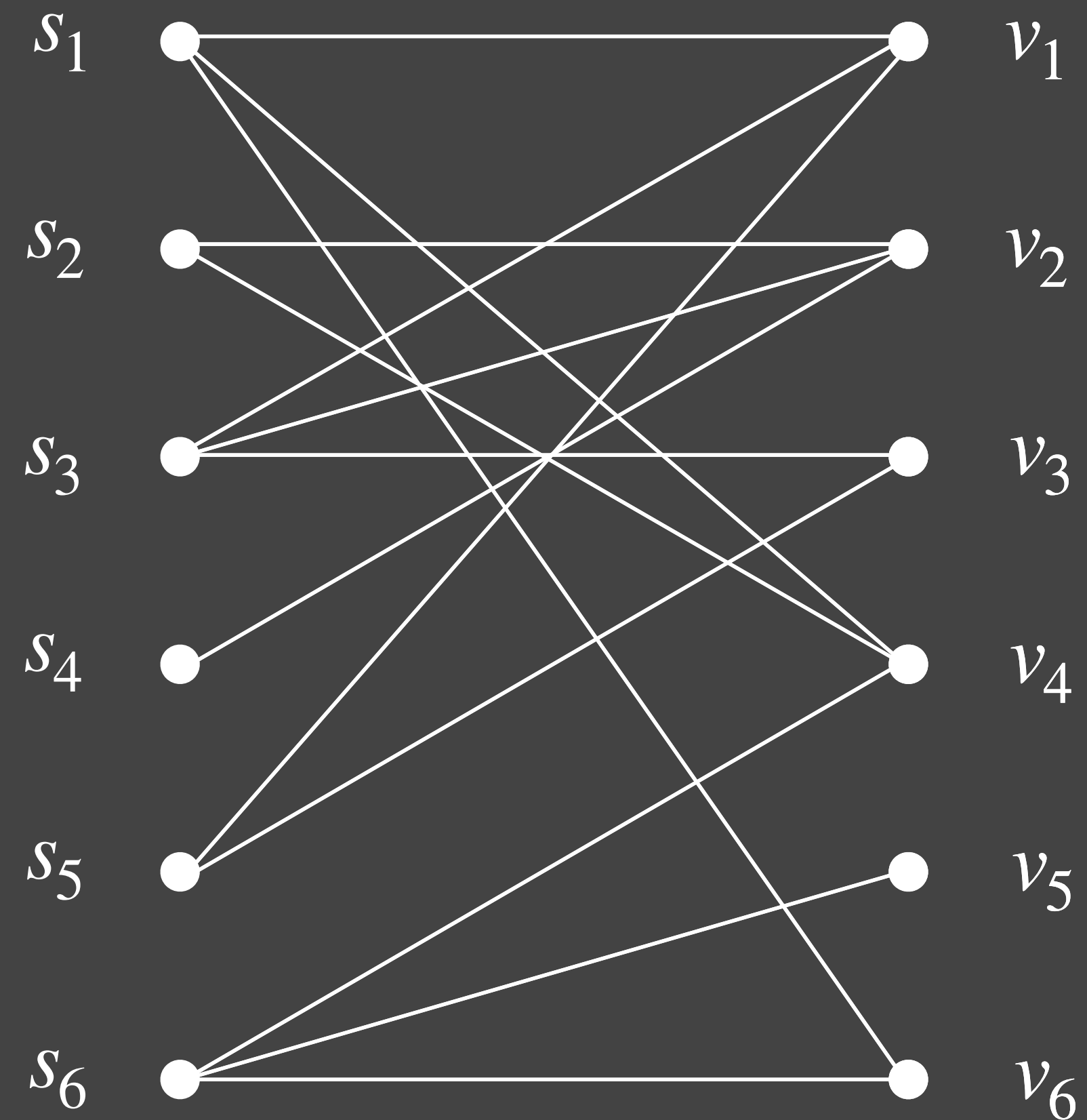
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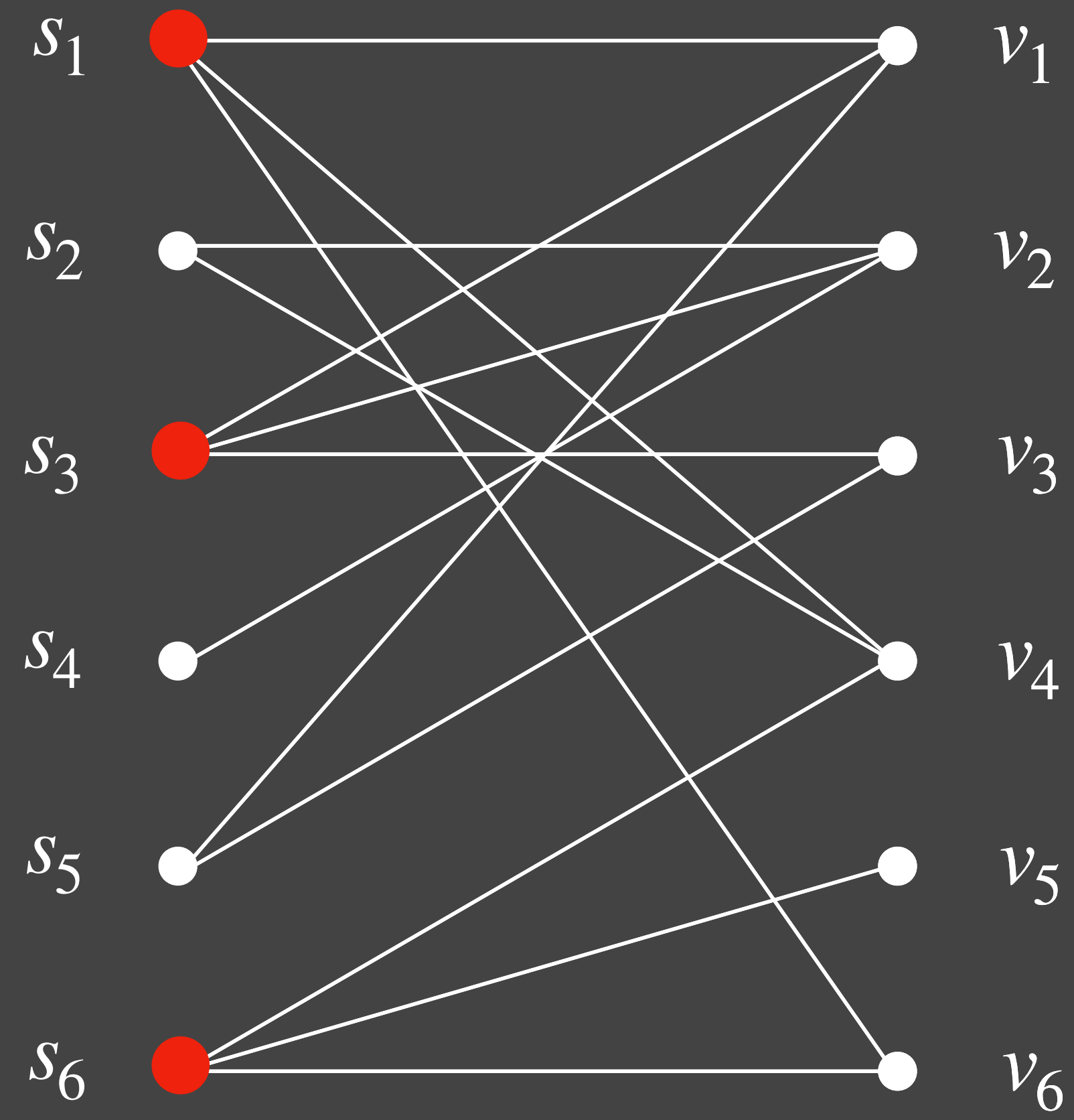
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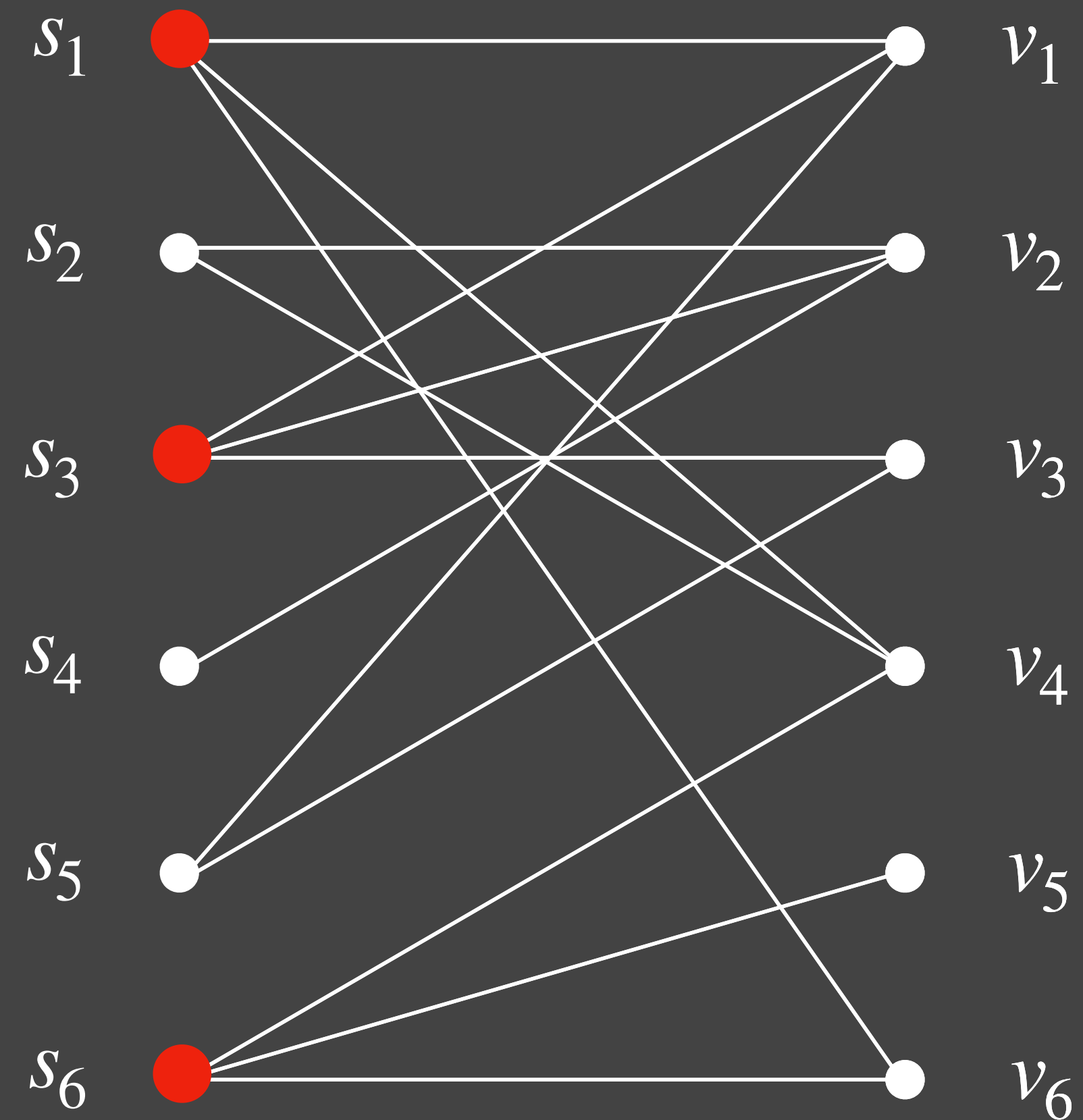
Running Example: Set Cover



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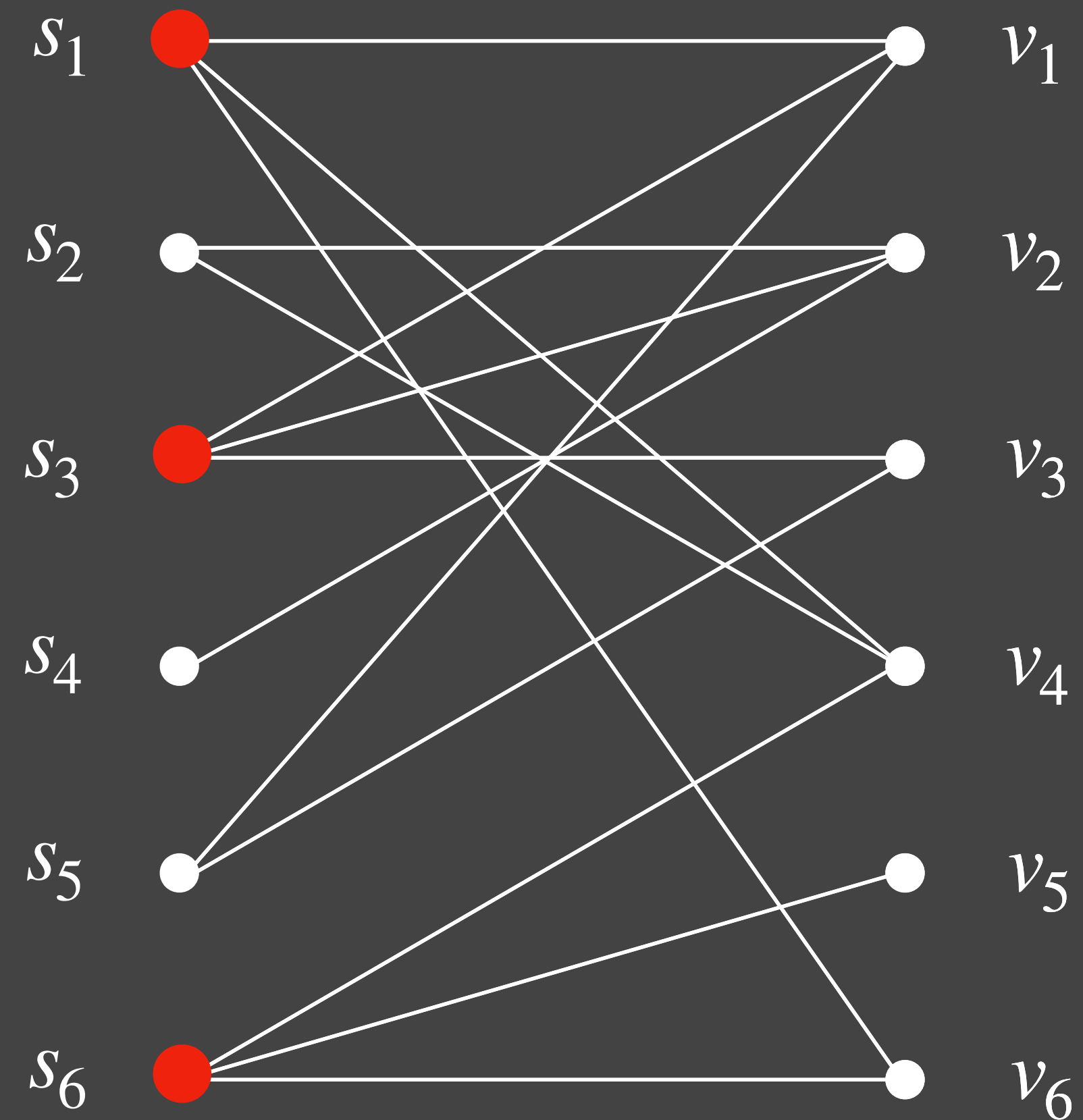


Running Example: Set Cover



Why should we care?

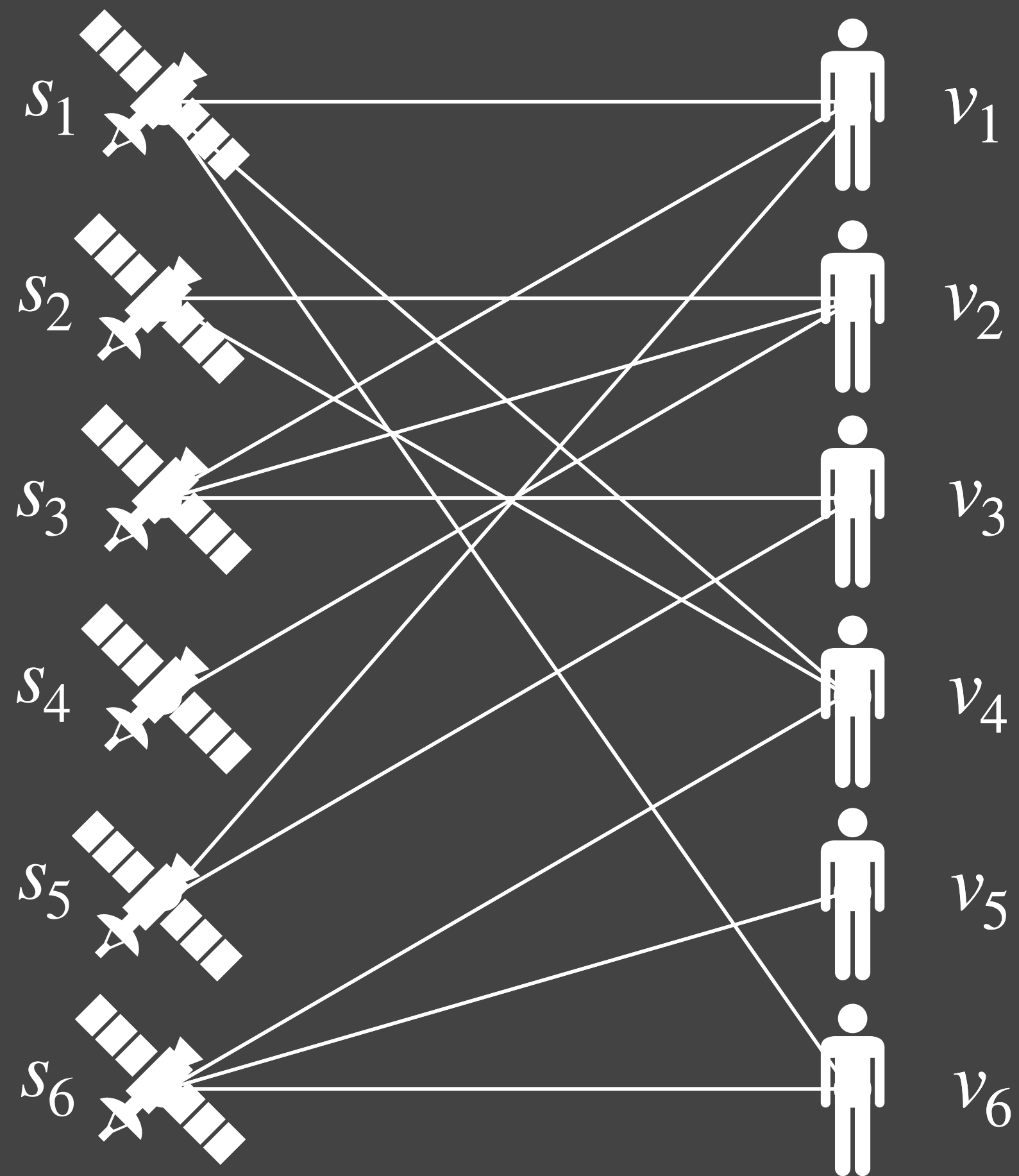
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Why should we care?

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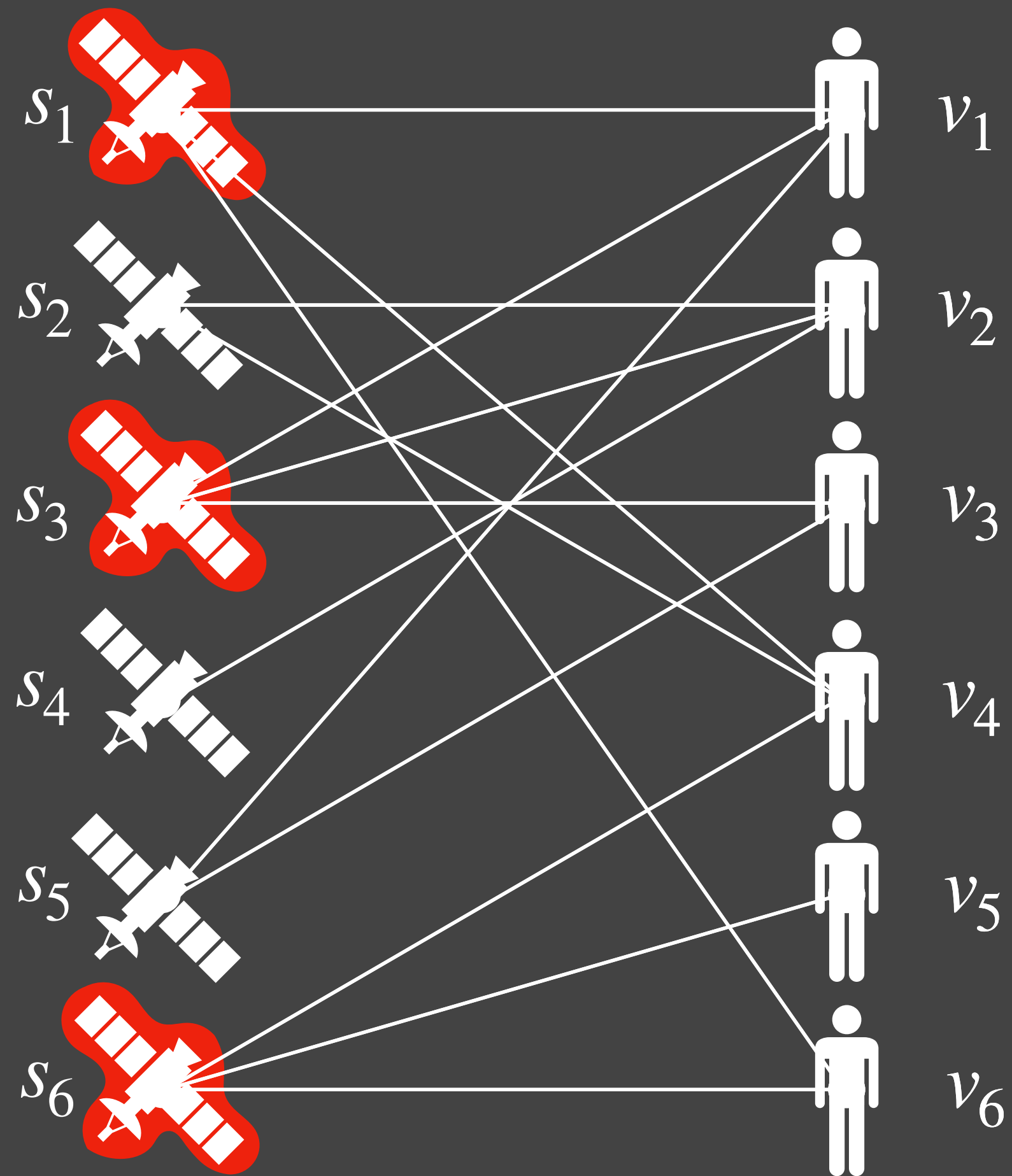
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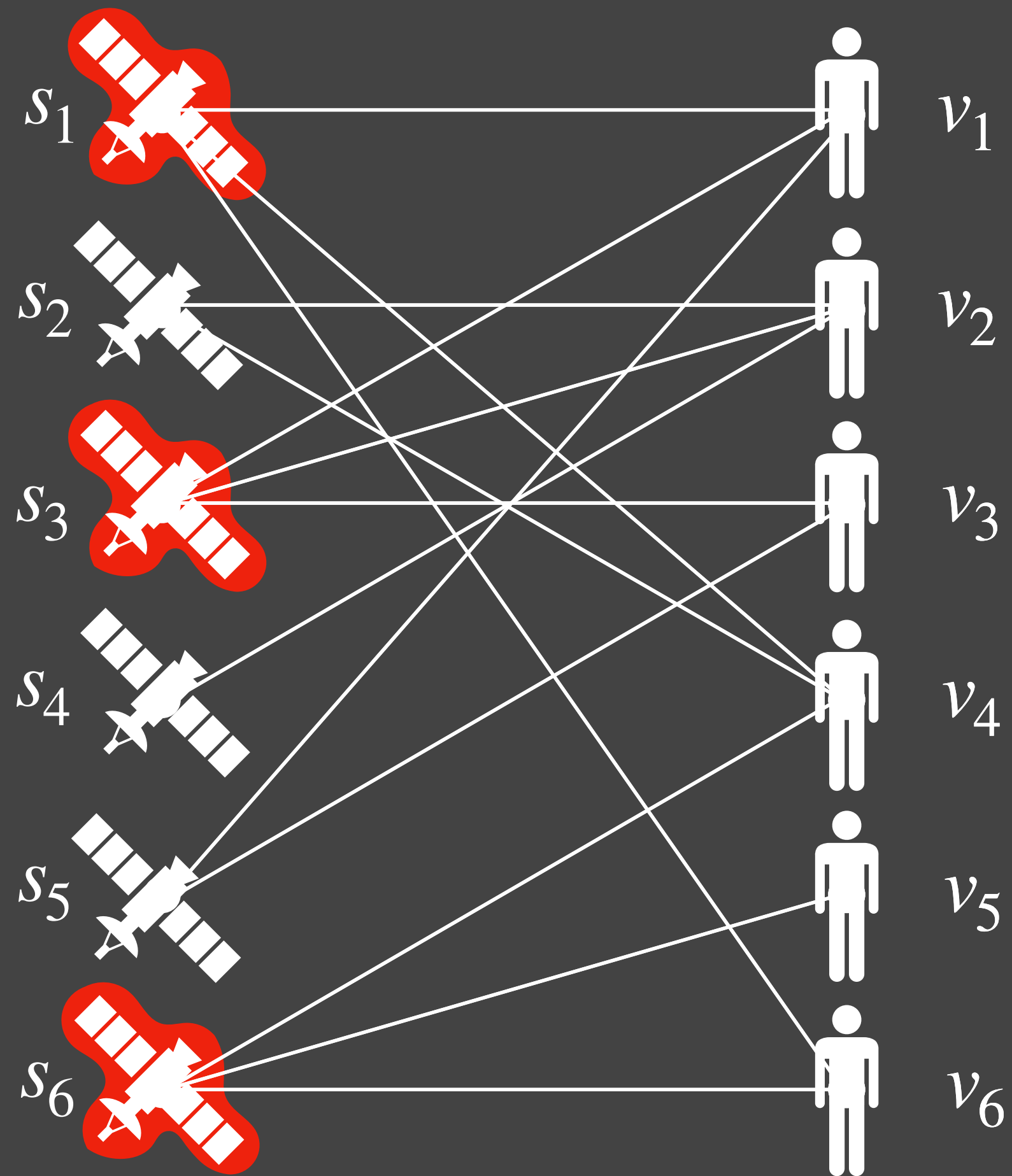
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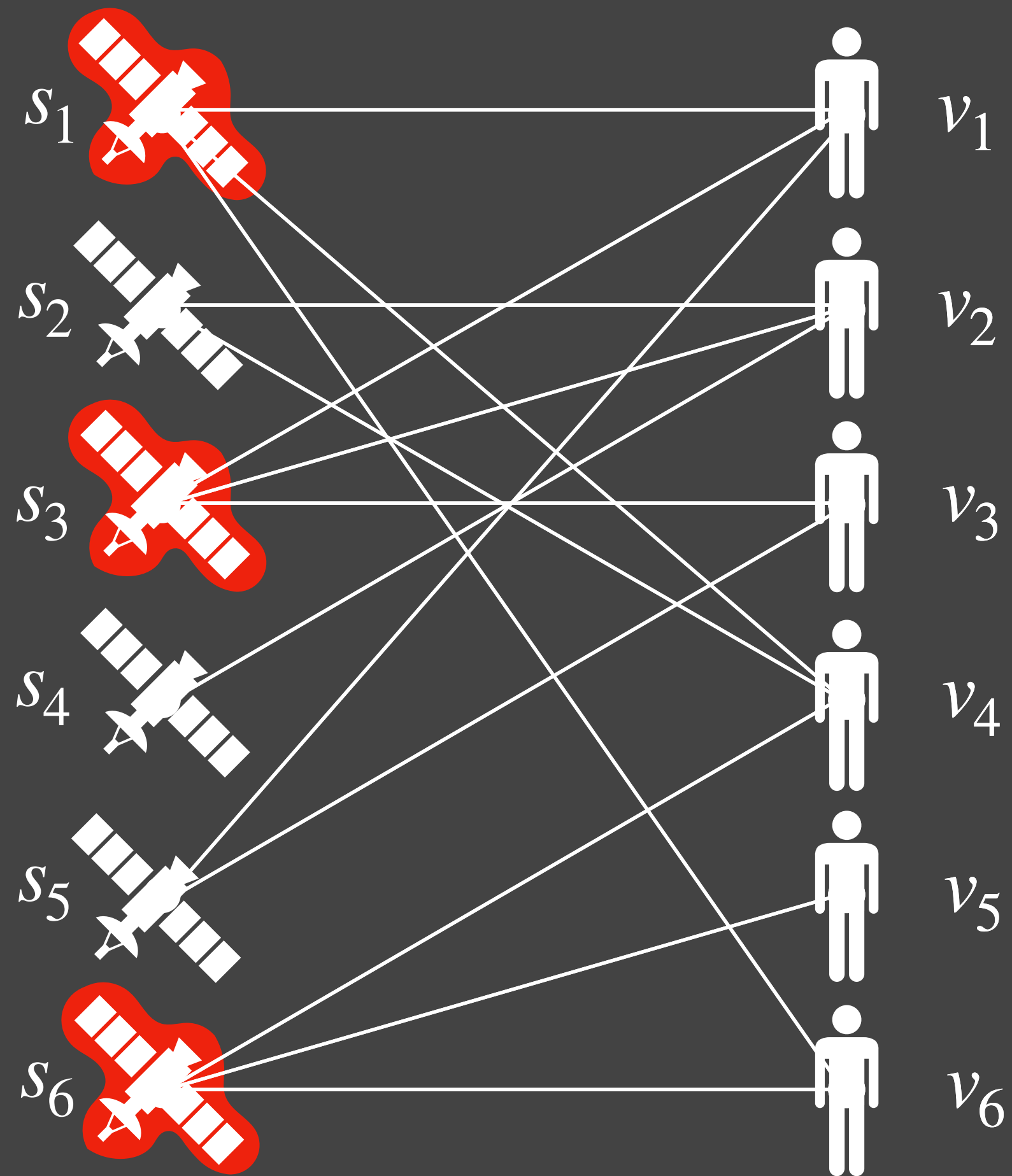
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2. **Sandbox** for fundamental algorithmic ideas.

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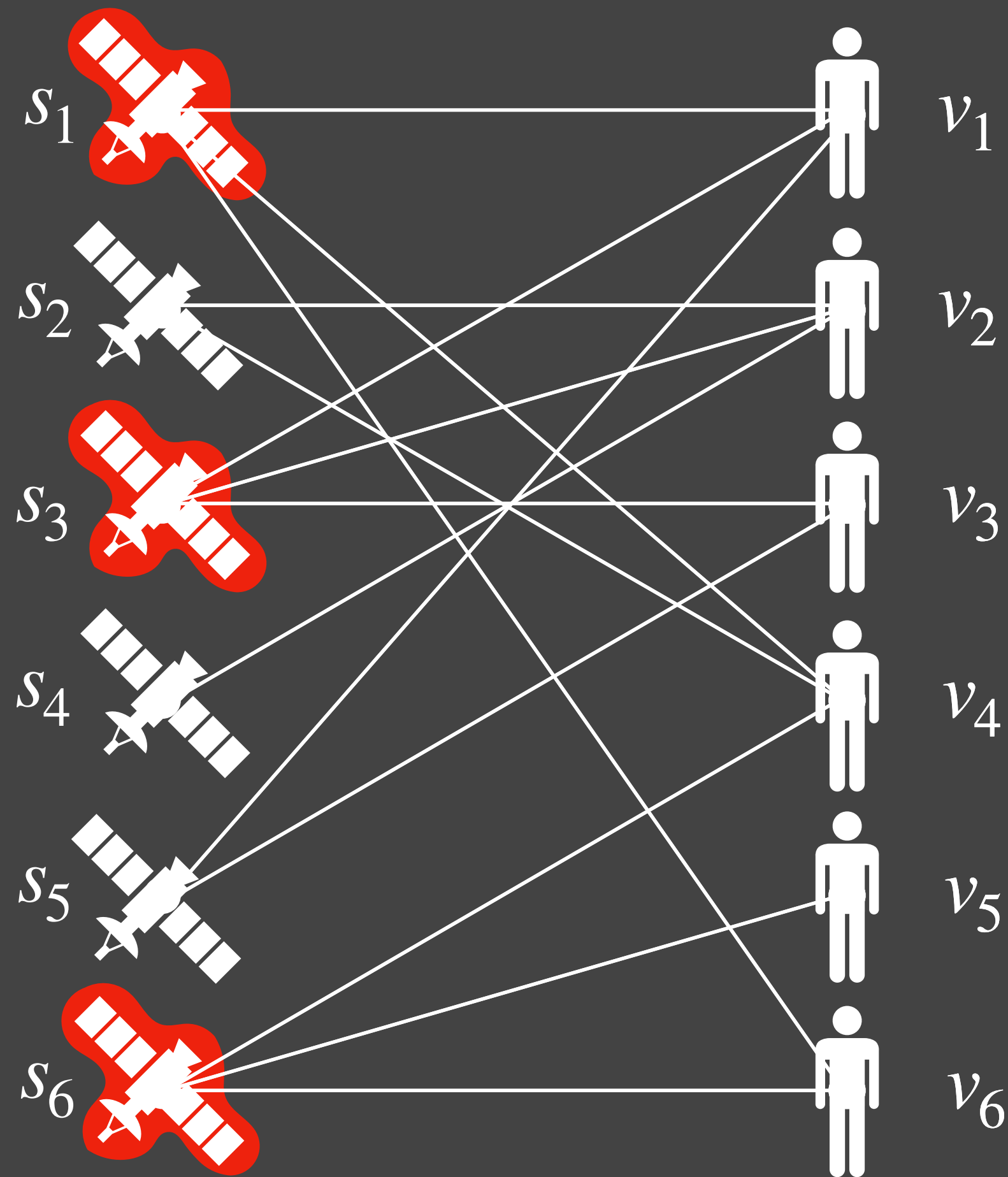
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$$\min c^T x$$

$$Ax \geq 1$$

$$x \in \mathbb{Z}_{\geq 0}^n$$

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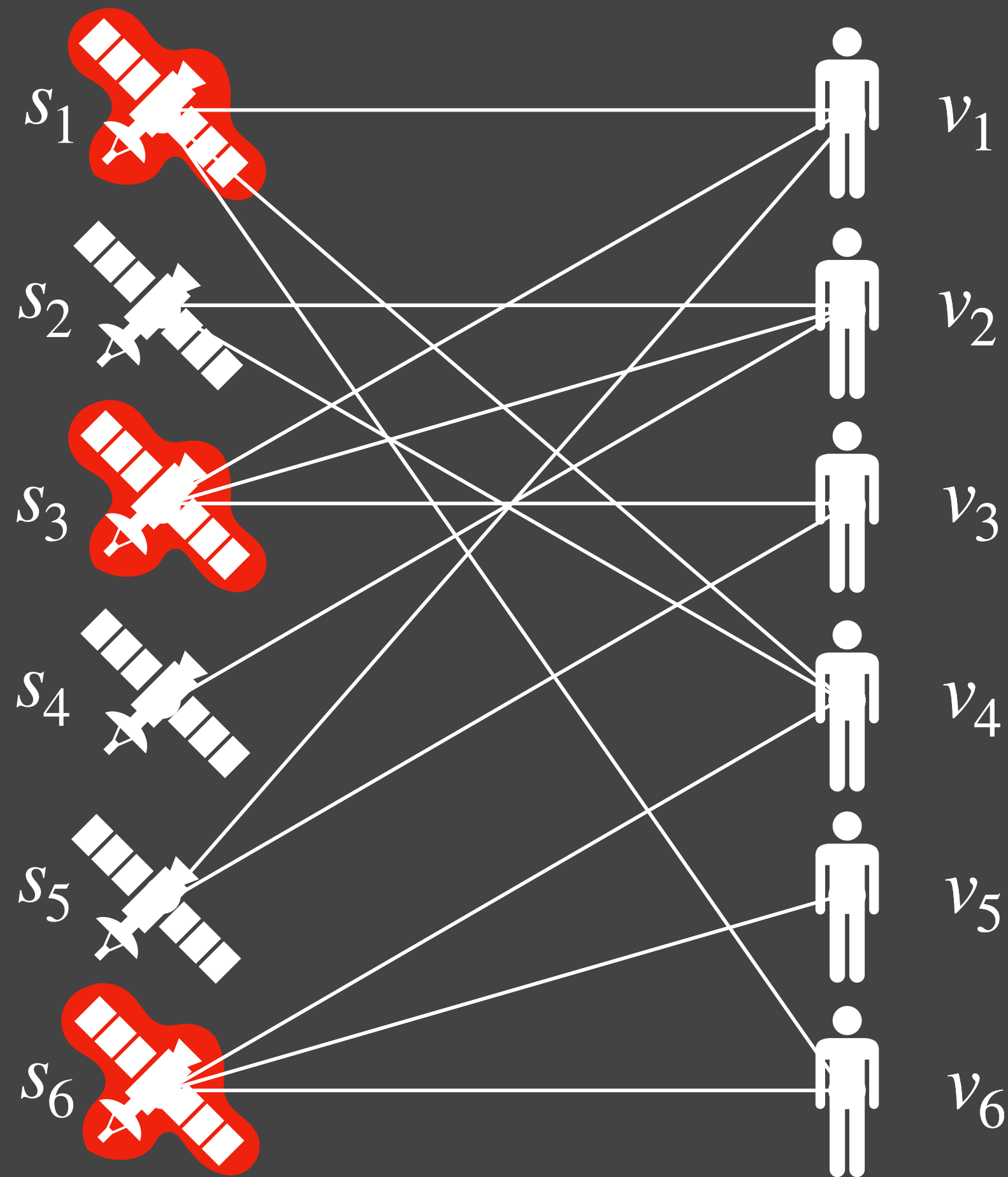
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Special case of
Integer Programming
where A is 0/1.

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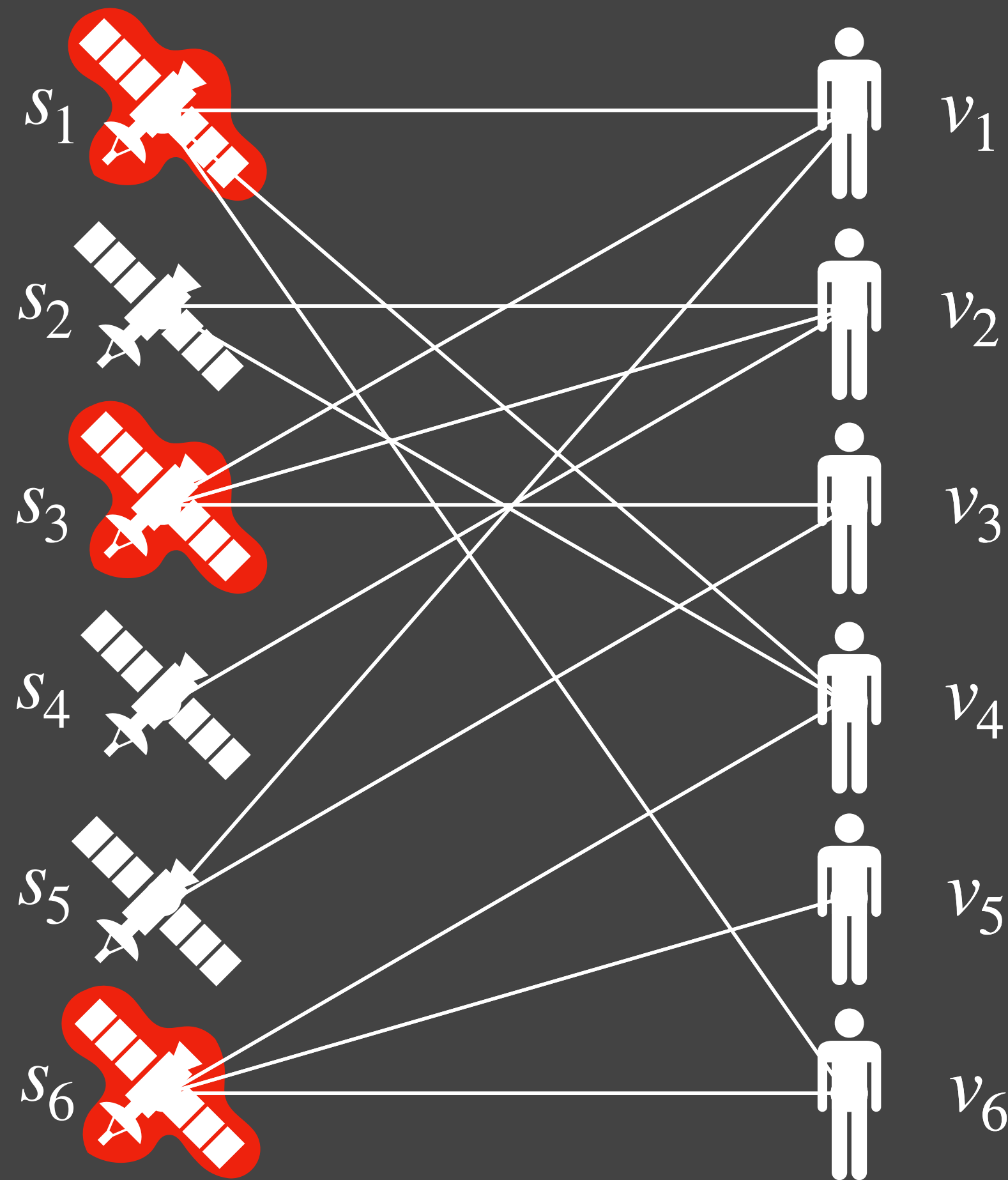
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Version 0 of EVERY discrete optimization problem!

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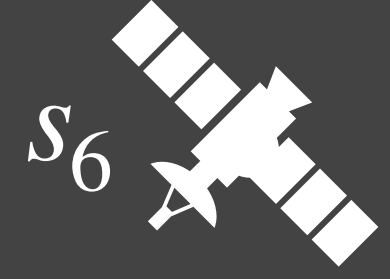
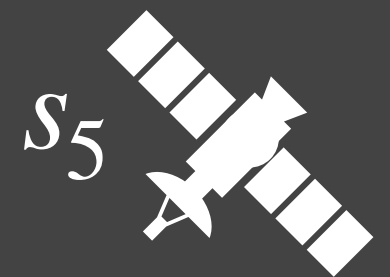
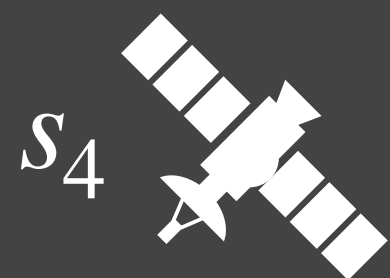
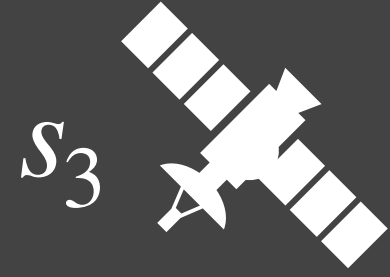
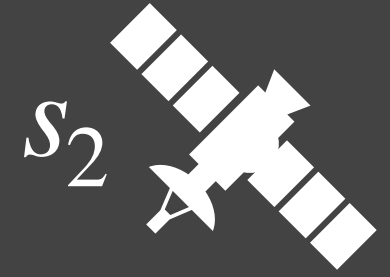
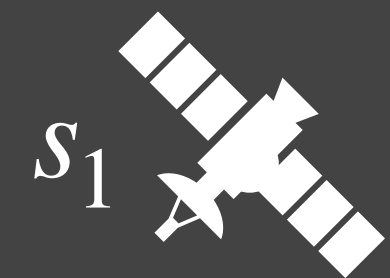
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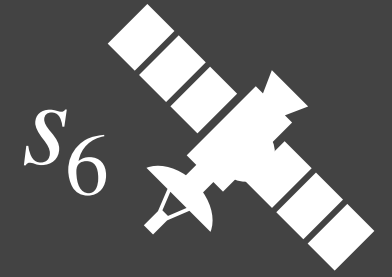
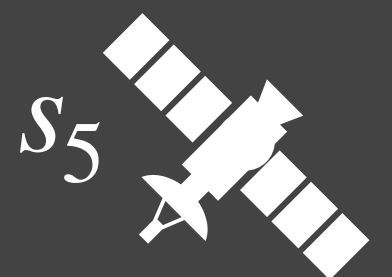
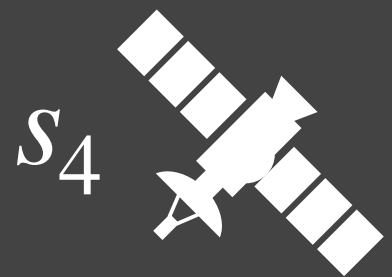
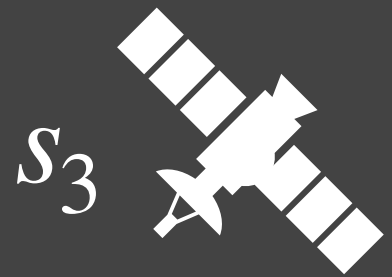
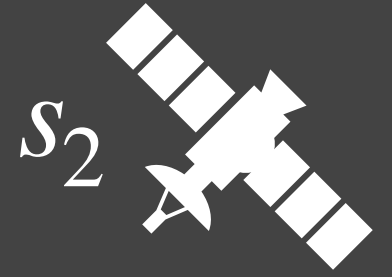
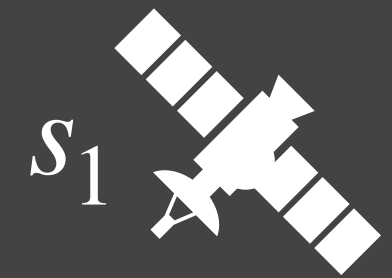
3. **Fast** algos get good approximation: $O(\log n)$
[Johnson 74], [Lovasz 75], [Chvatal 79]

Running Example: Set Cover



What if we **don't know** user demand a-priori?

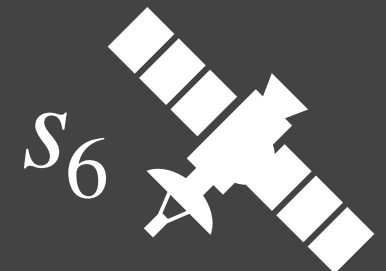
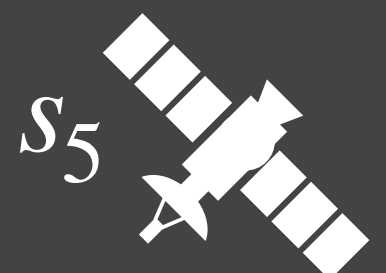
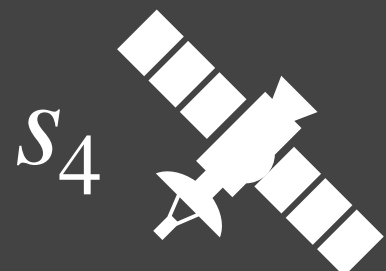
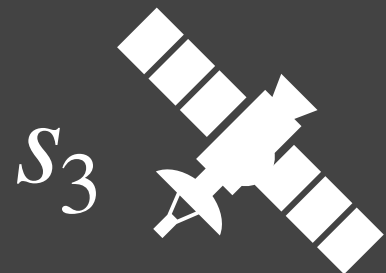
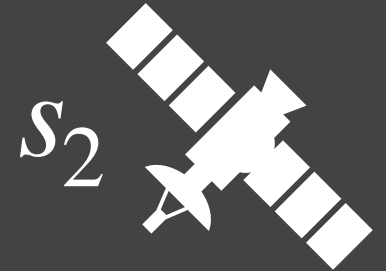
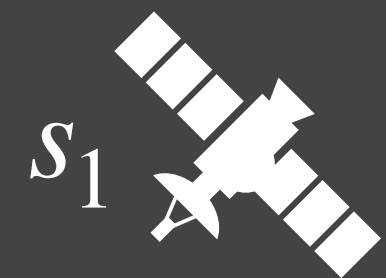
Running Example: Set Cover



What if we **don't know** user demand a-priori?

Requests arrive over time, need to satisfy immediately.

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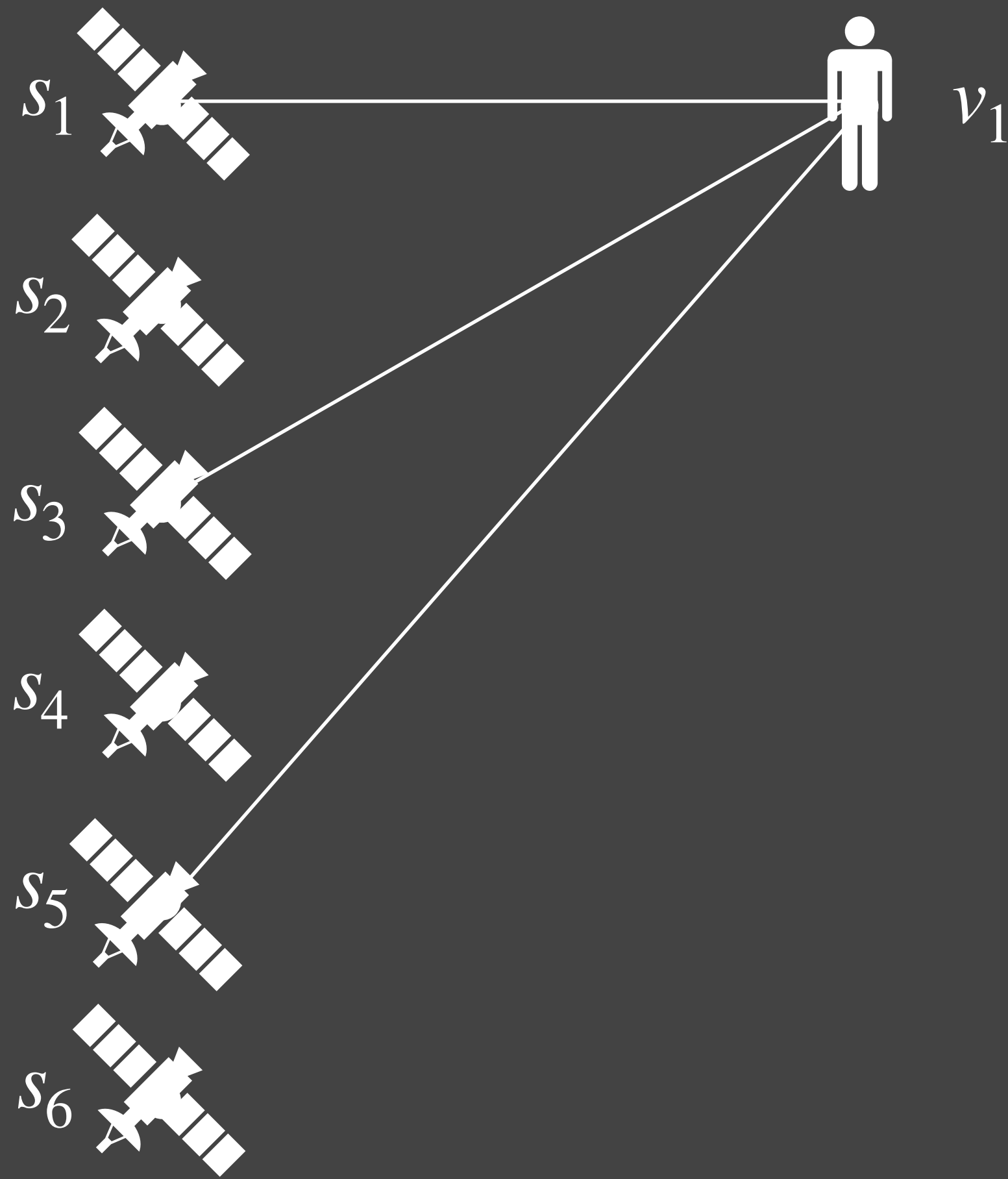


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Expensive to open satellites!
Model decisions as **irrevocable**.

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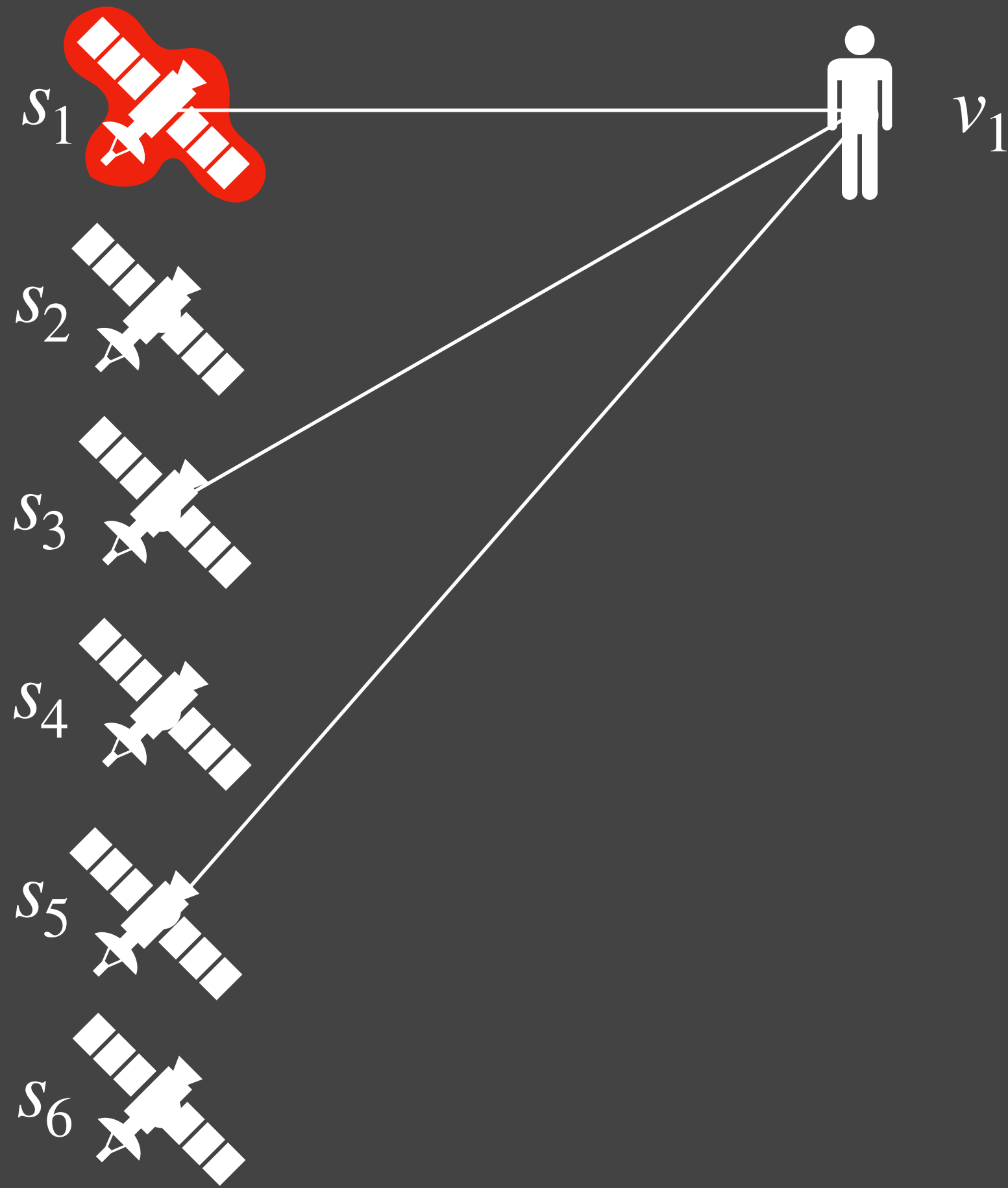


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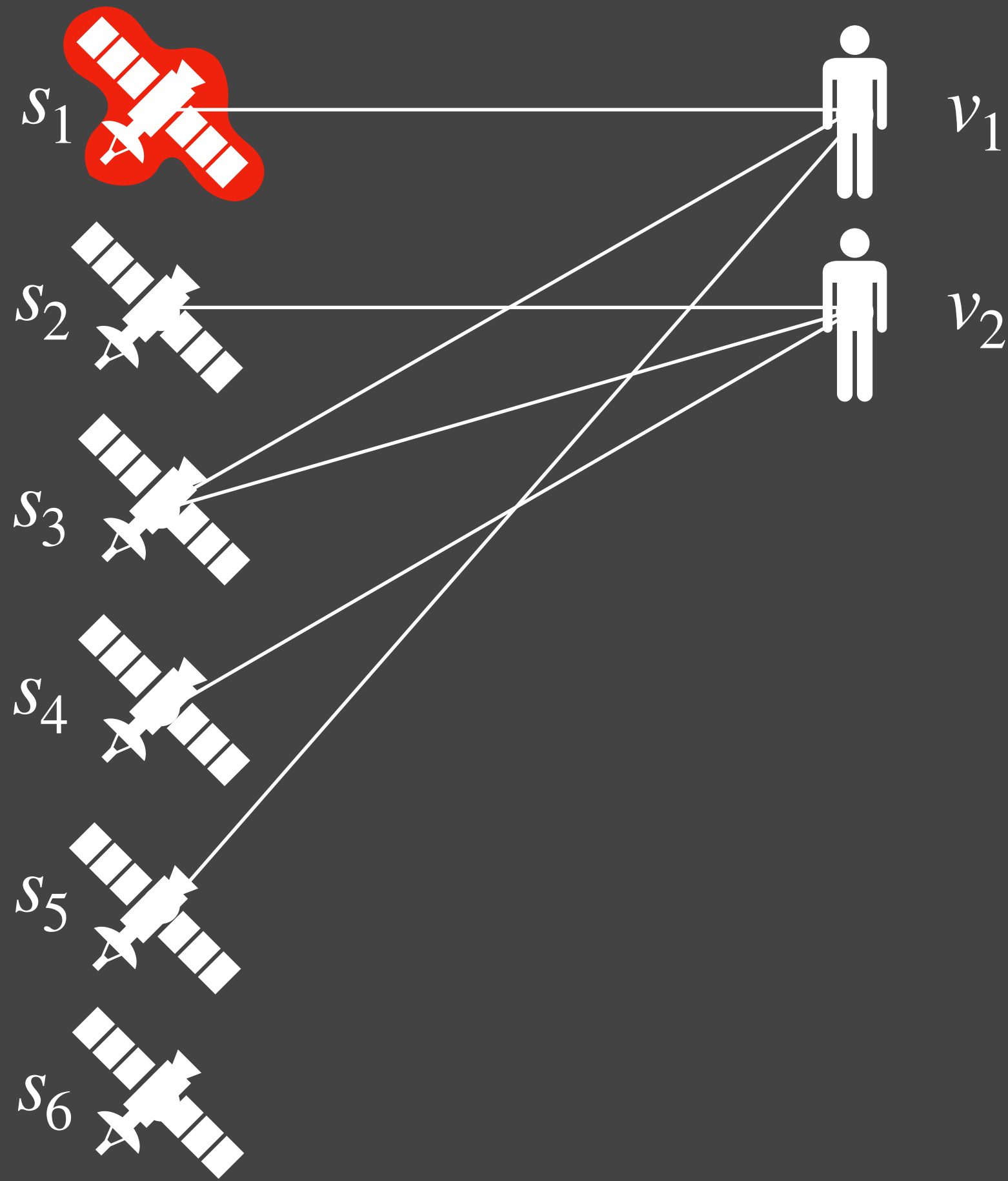


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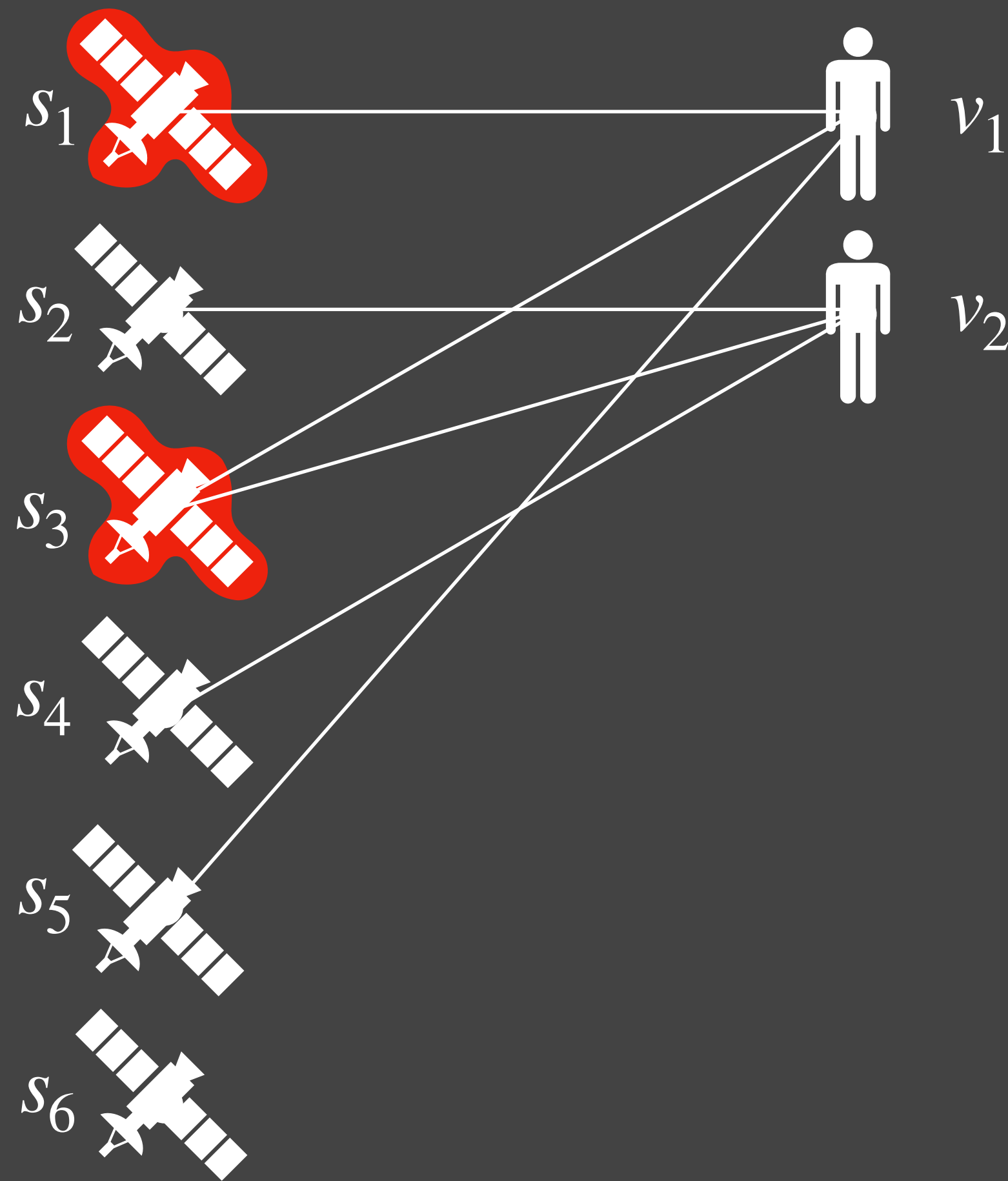


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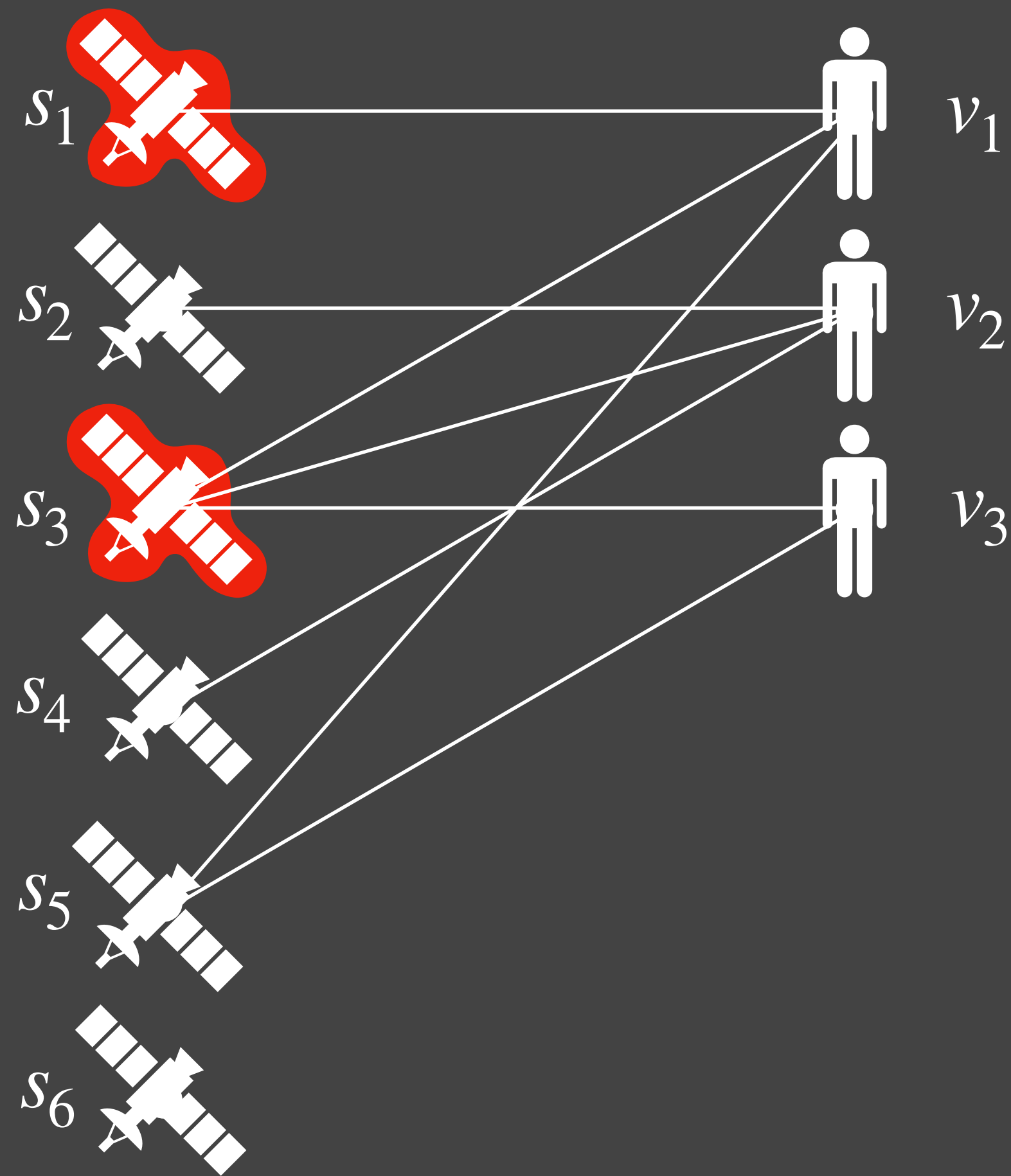


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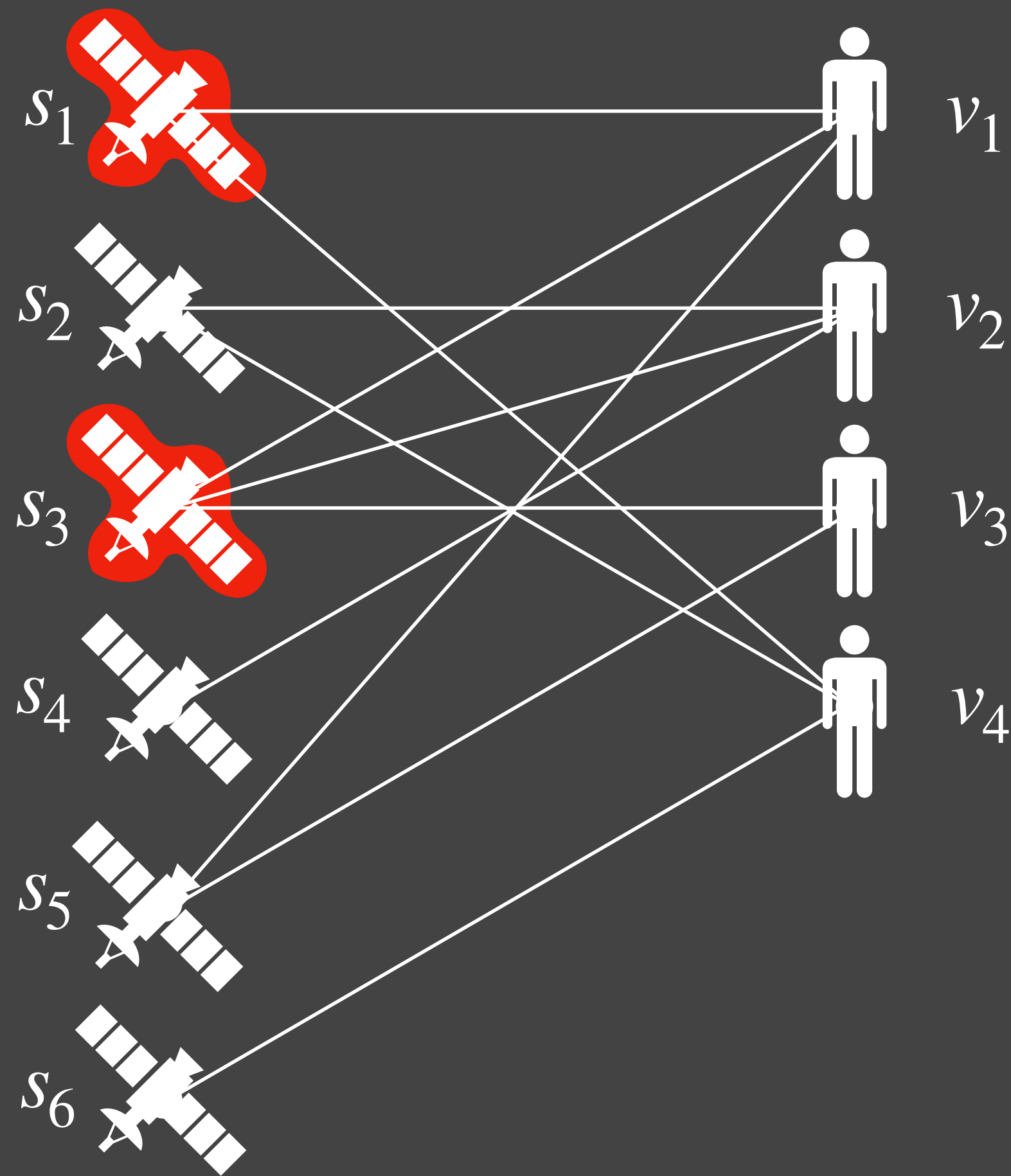


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Expensive to open satellites!
Model decisions as **irrevocable**.

Running Example: Set Cover

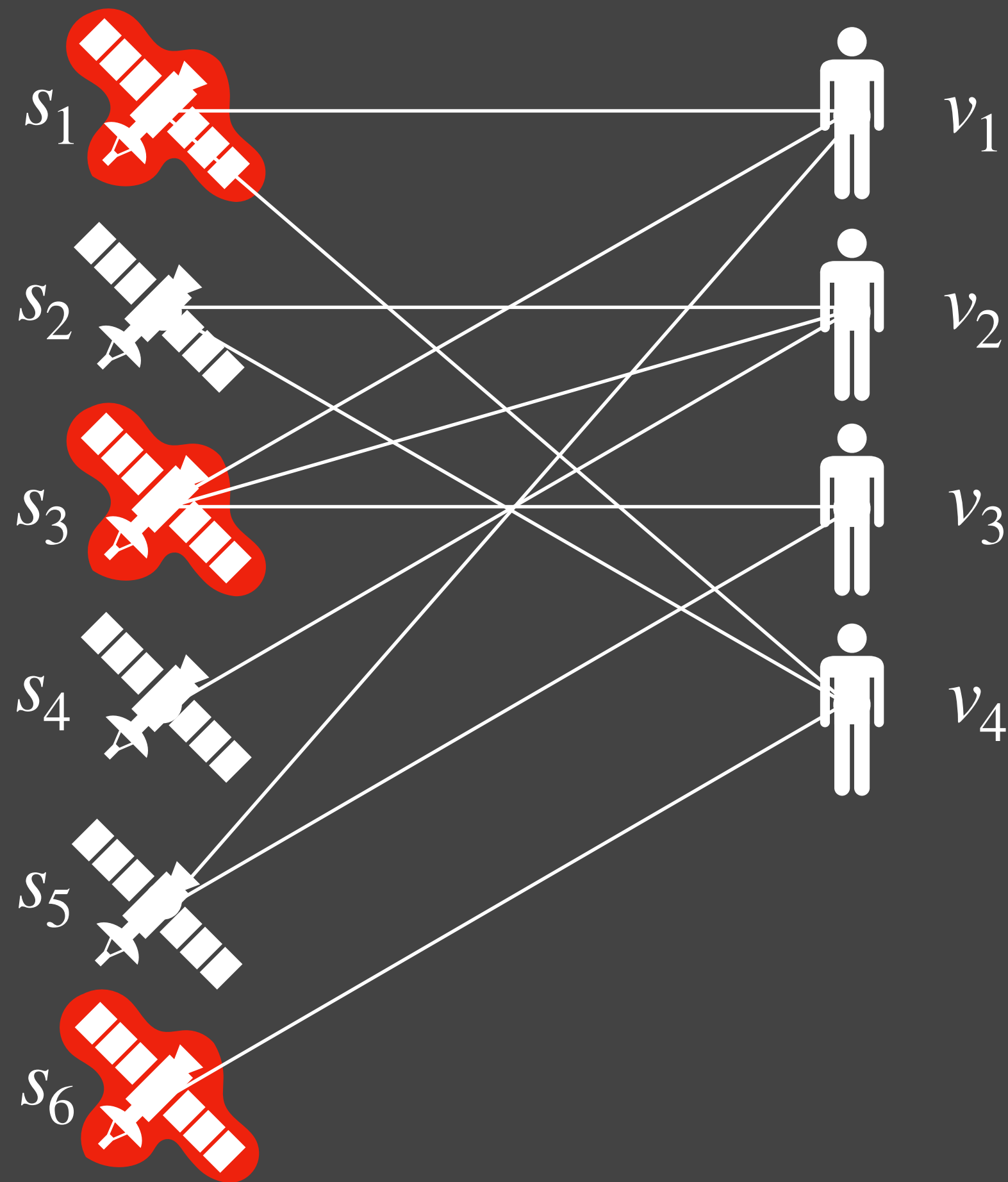


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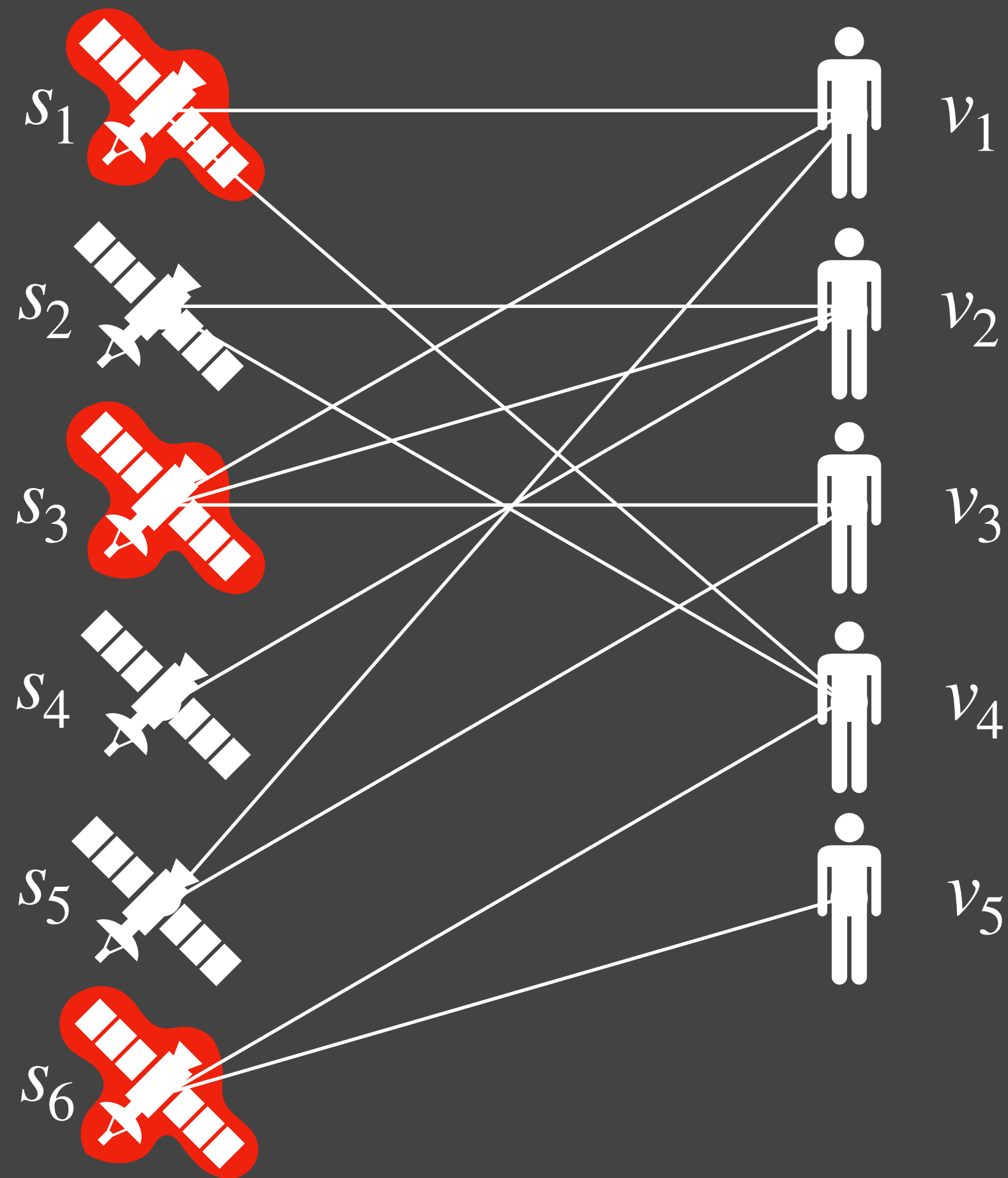


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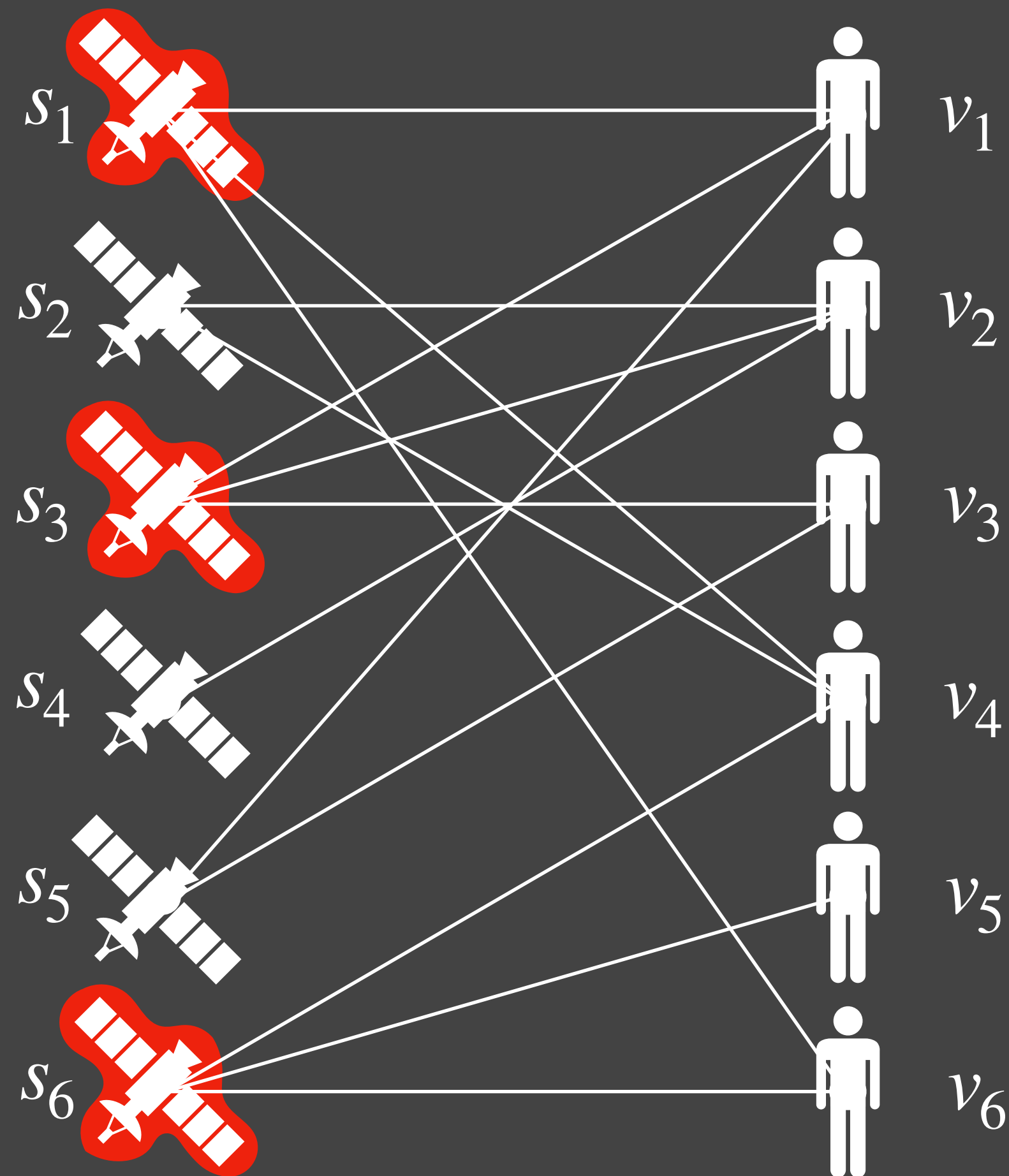


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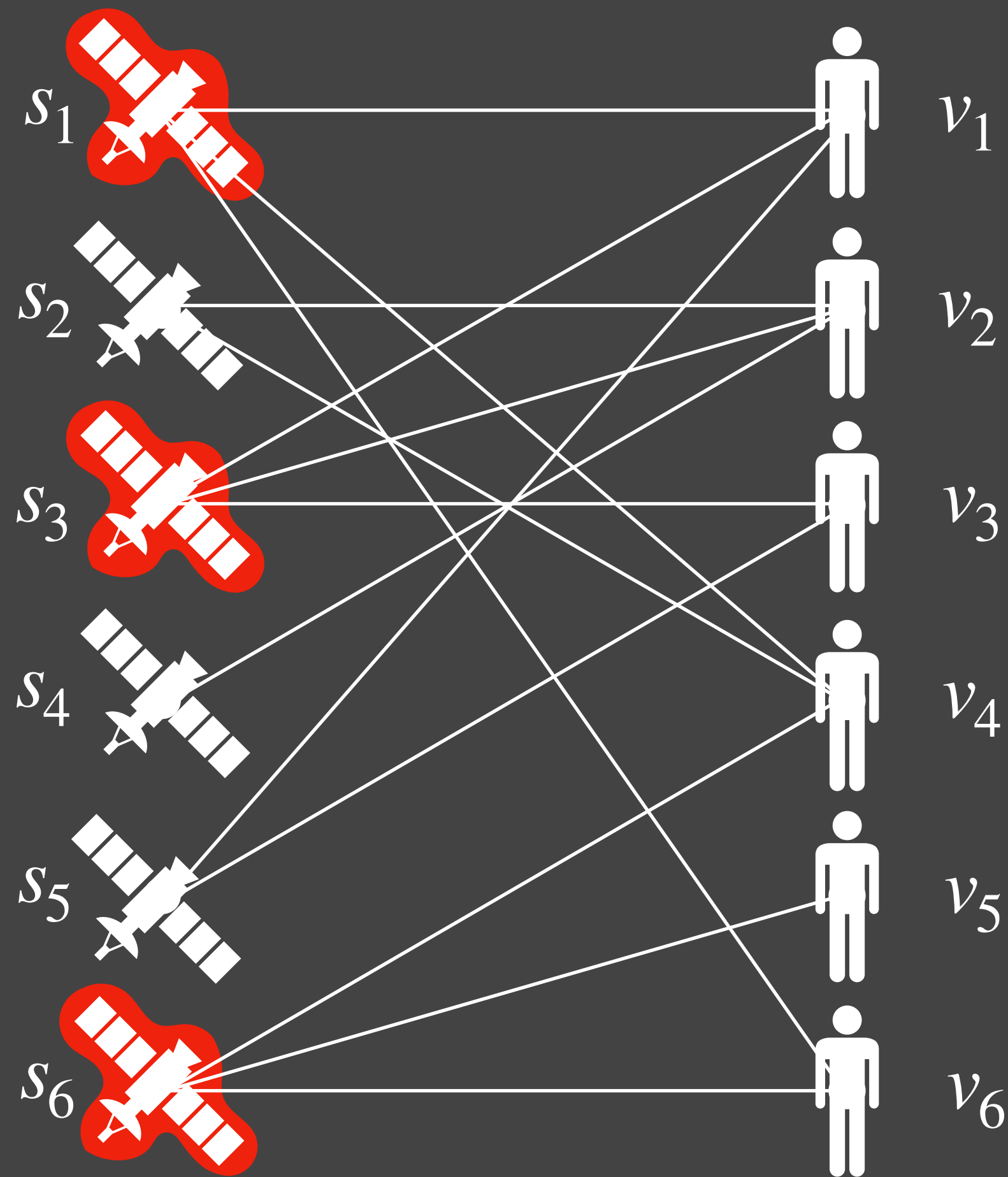


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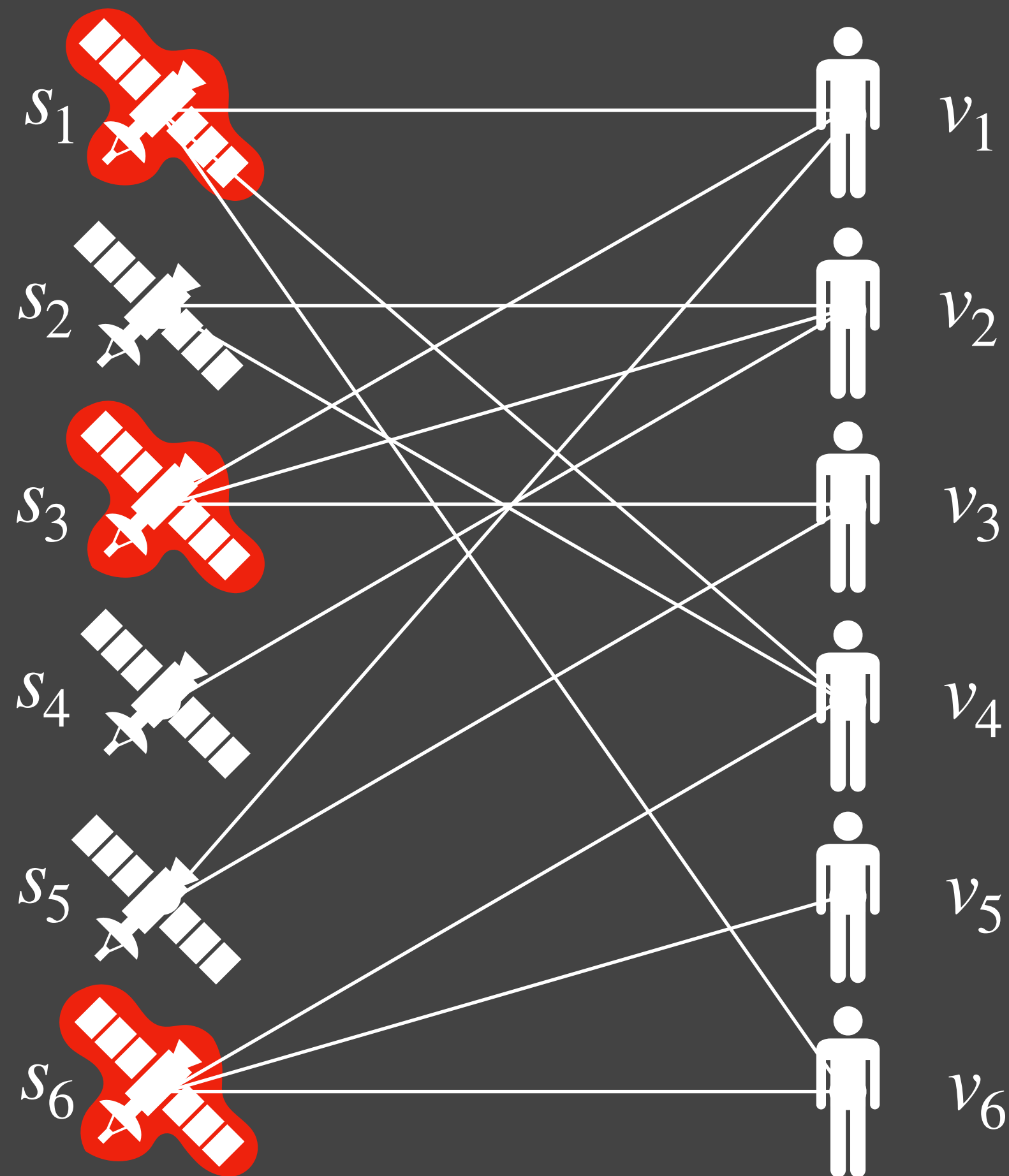
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Running Example: Set Cover



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Requests arrive over time, need to satisfy immediately.

Expensive to open satellites!
Model decisions as **irrevocable**.

Q: Can we get good approximation, **efficiently**, despite not knowing the future?

A: Yes! Approximation: $O(\log^2 n)$
[Alon Awerbuch Azar Buchbinder Naor 03]
[Buchbinder Naor 09], this is **optimal** for polynomial time algorithms.

My Work

Online

Dynamic

Streaming

My Work

Online

Dynamic

Streaming

No take-backs

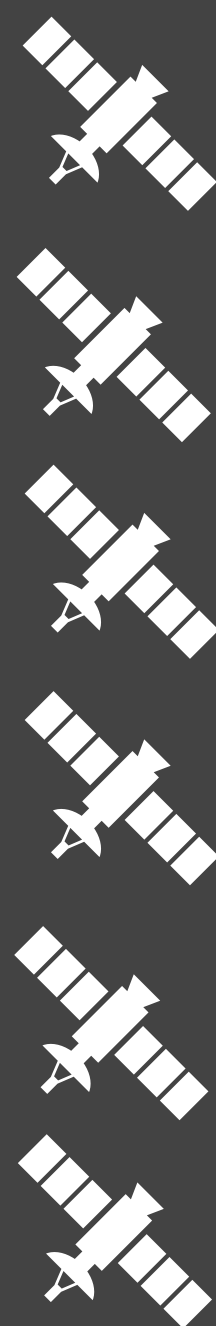
My Work

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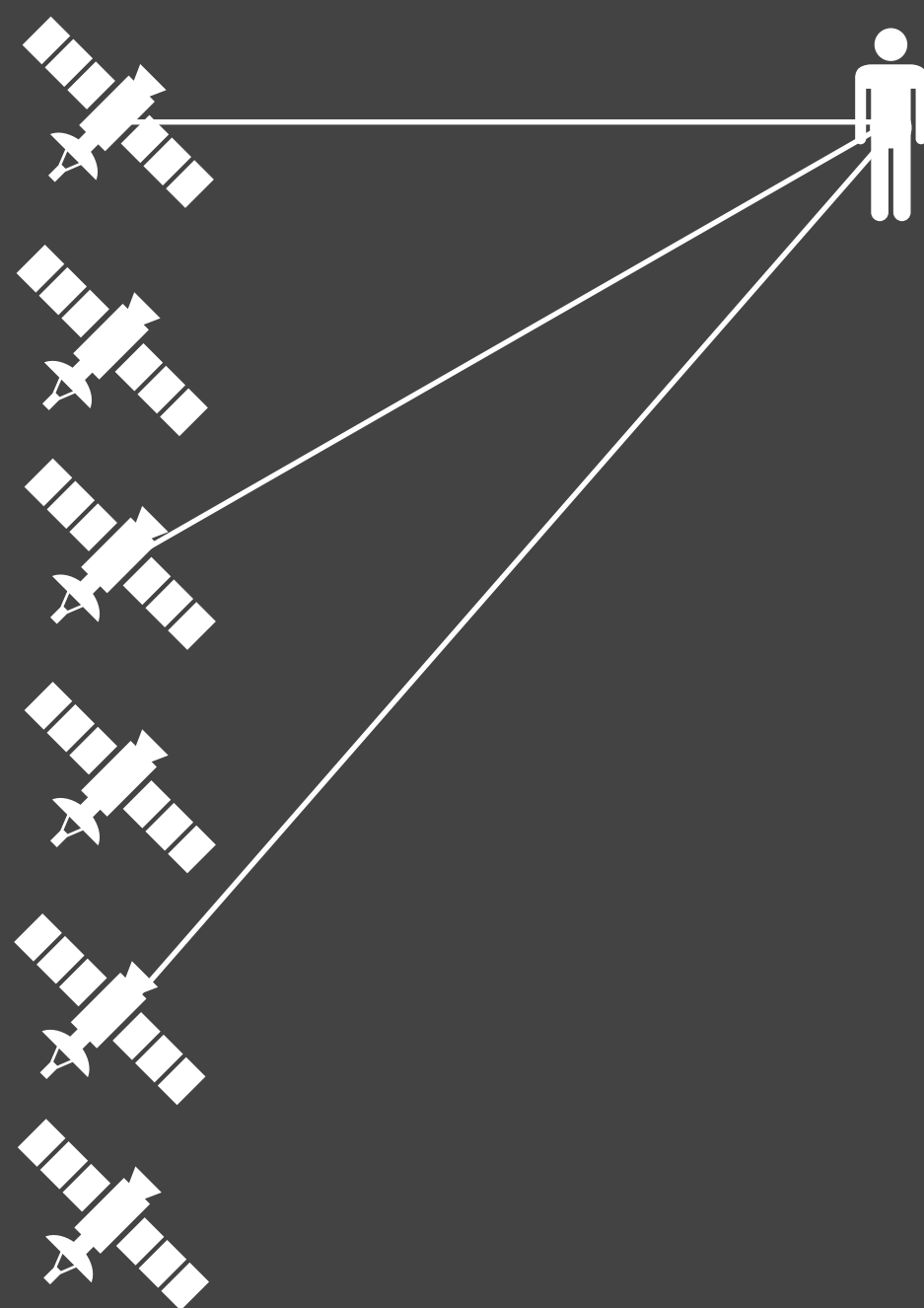
My Work

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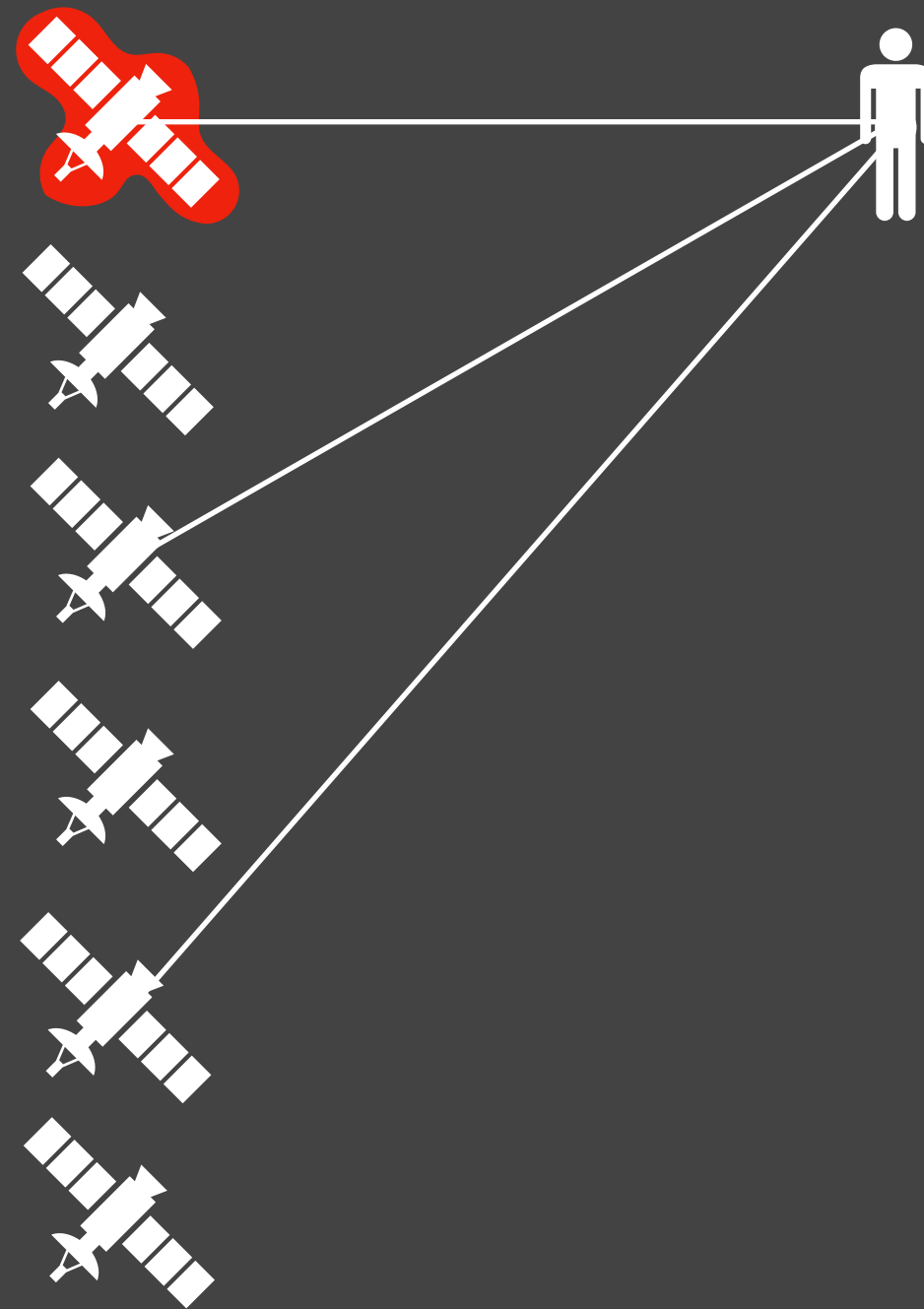
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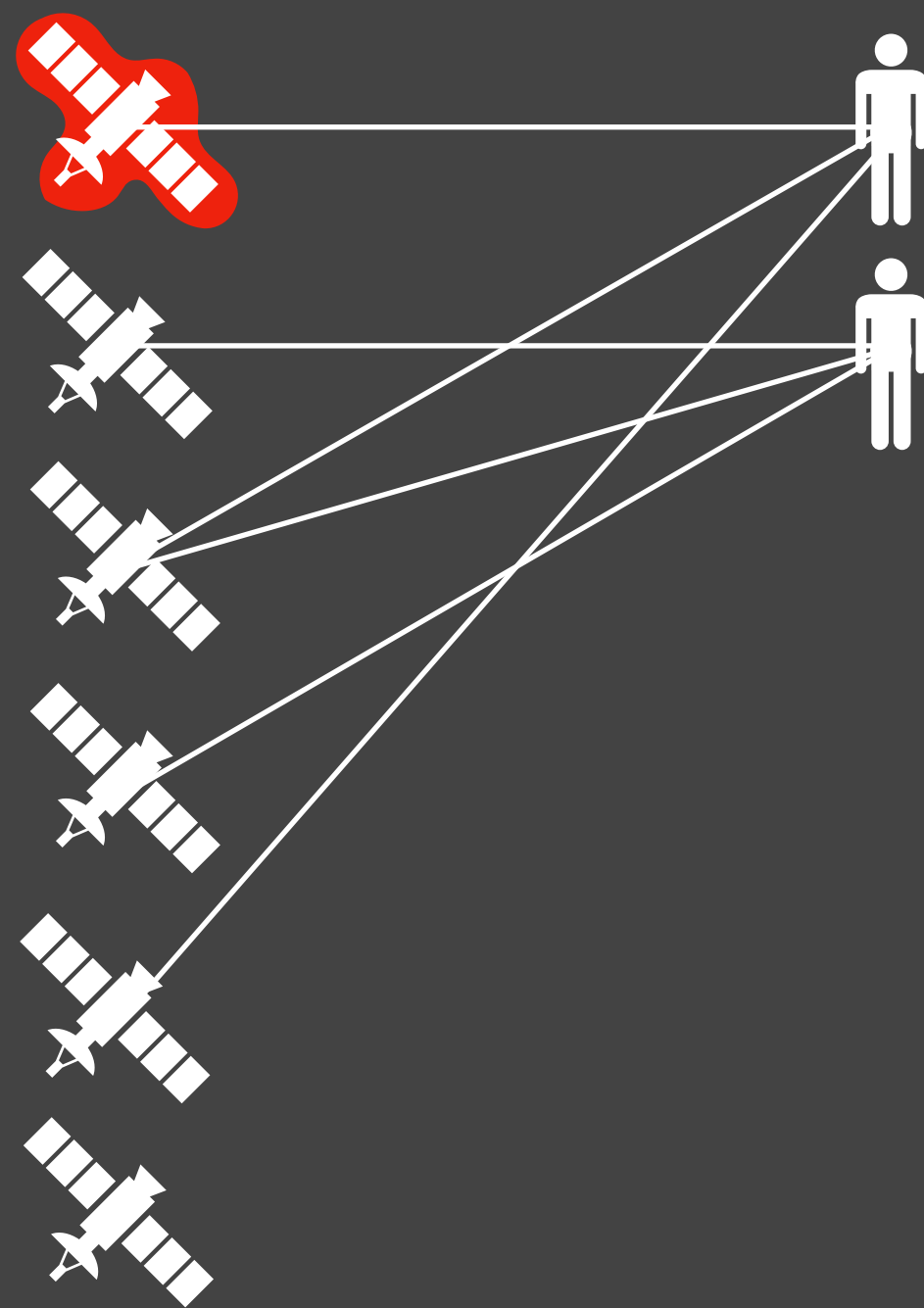
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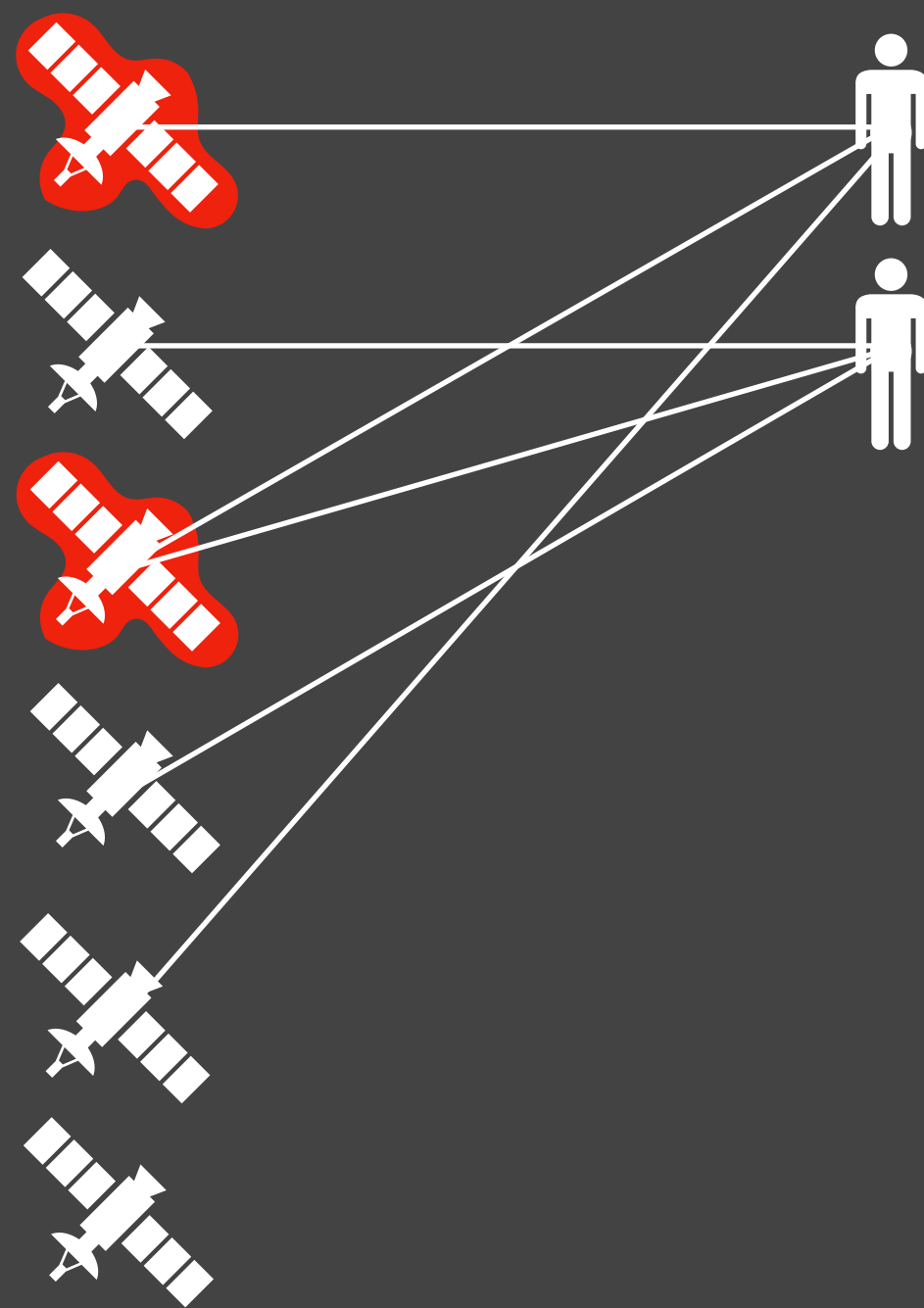
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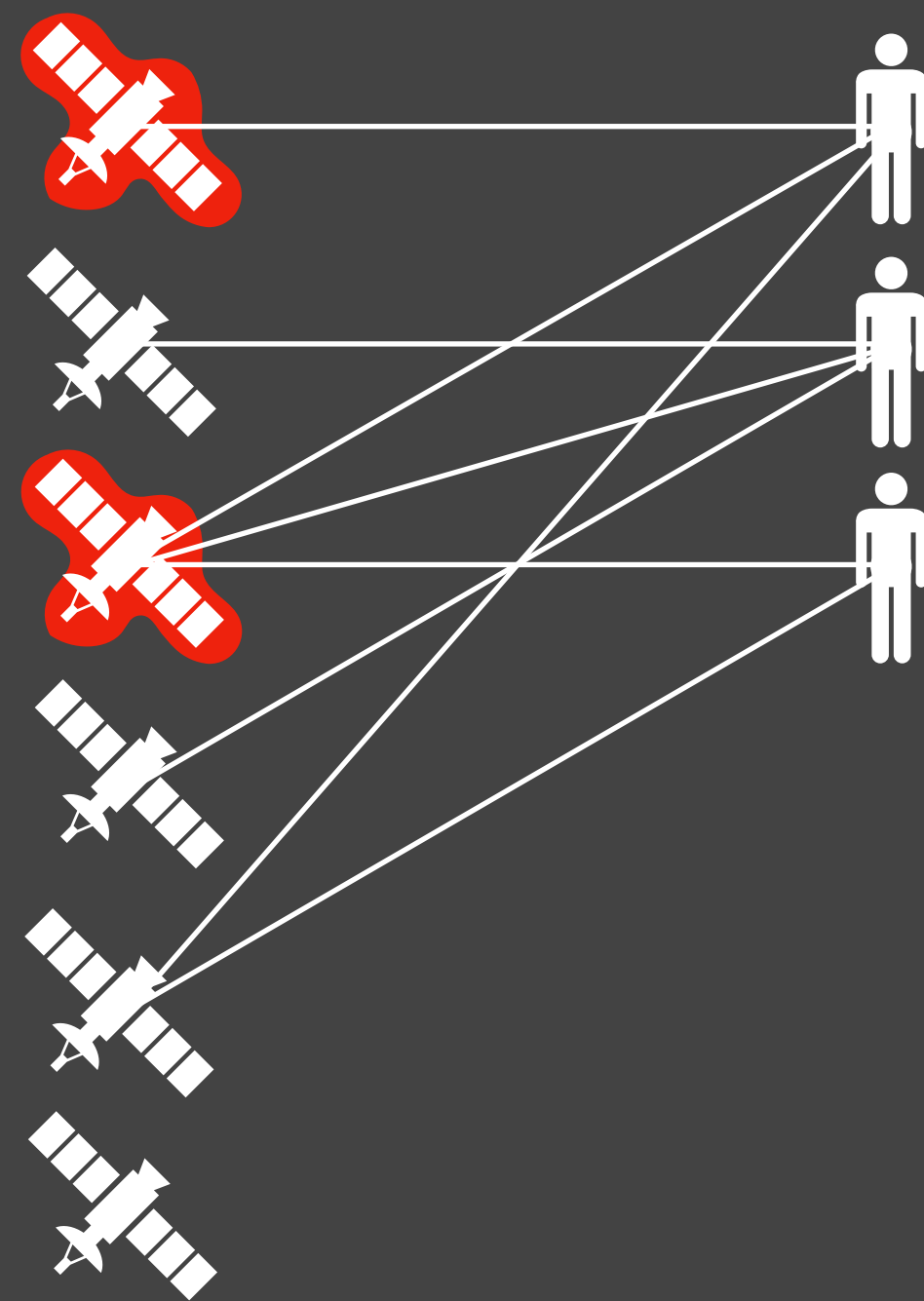
My Work

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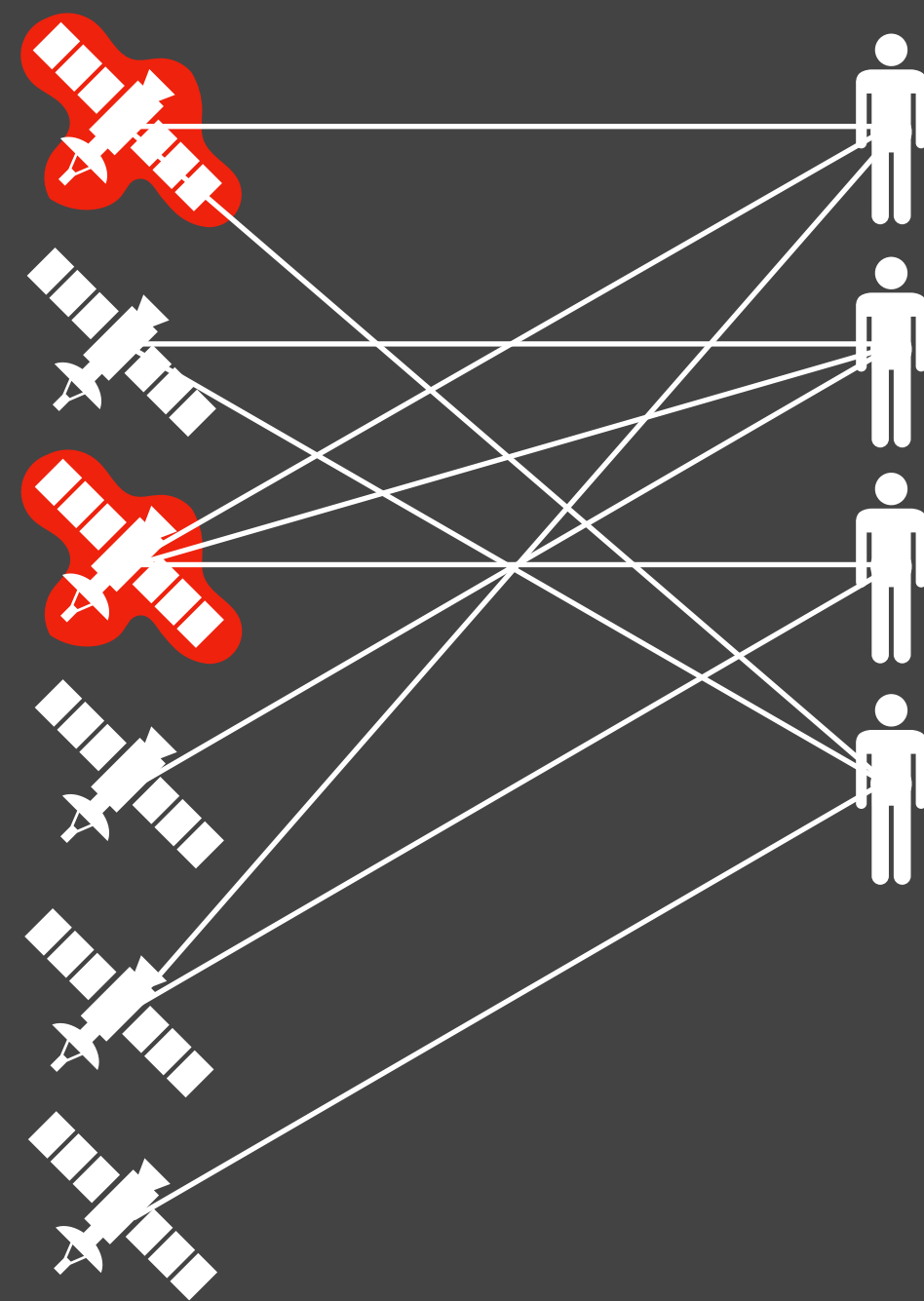
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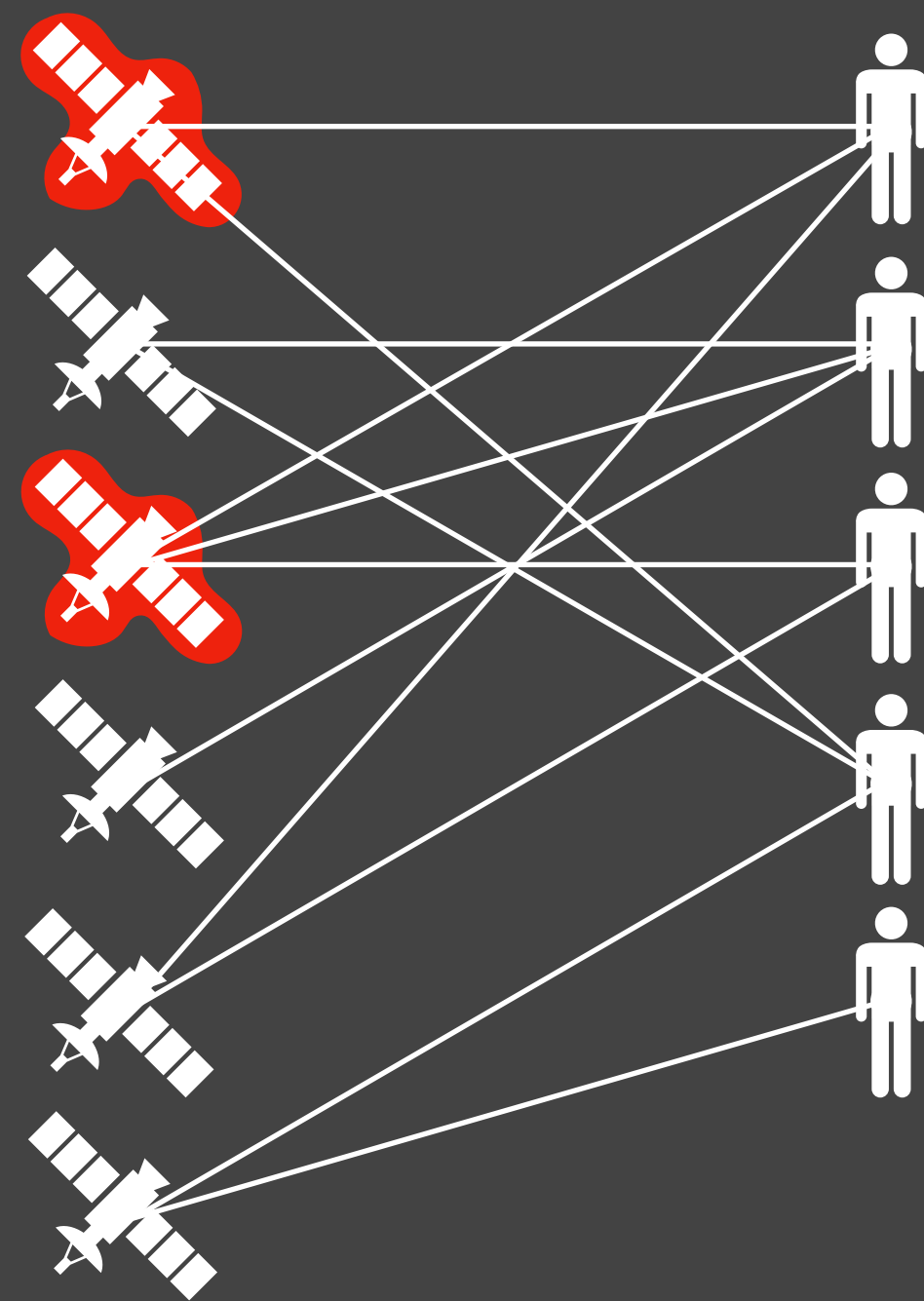
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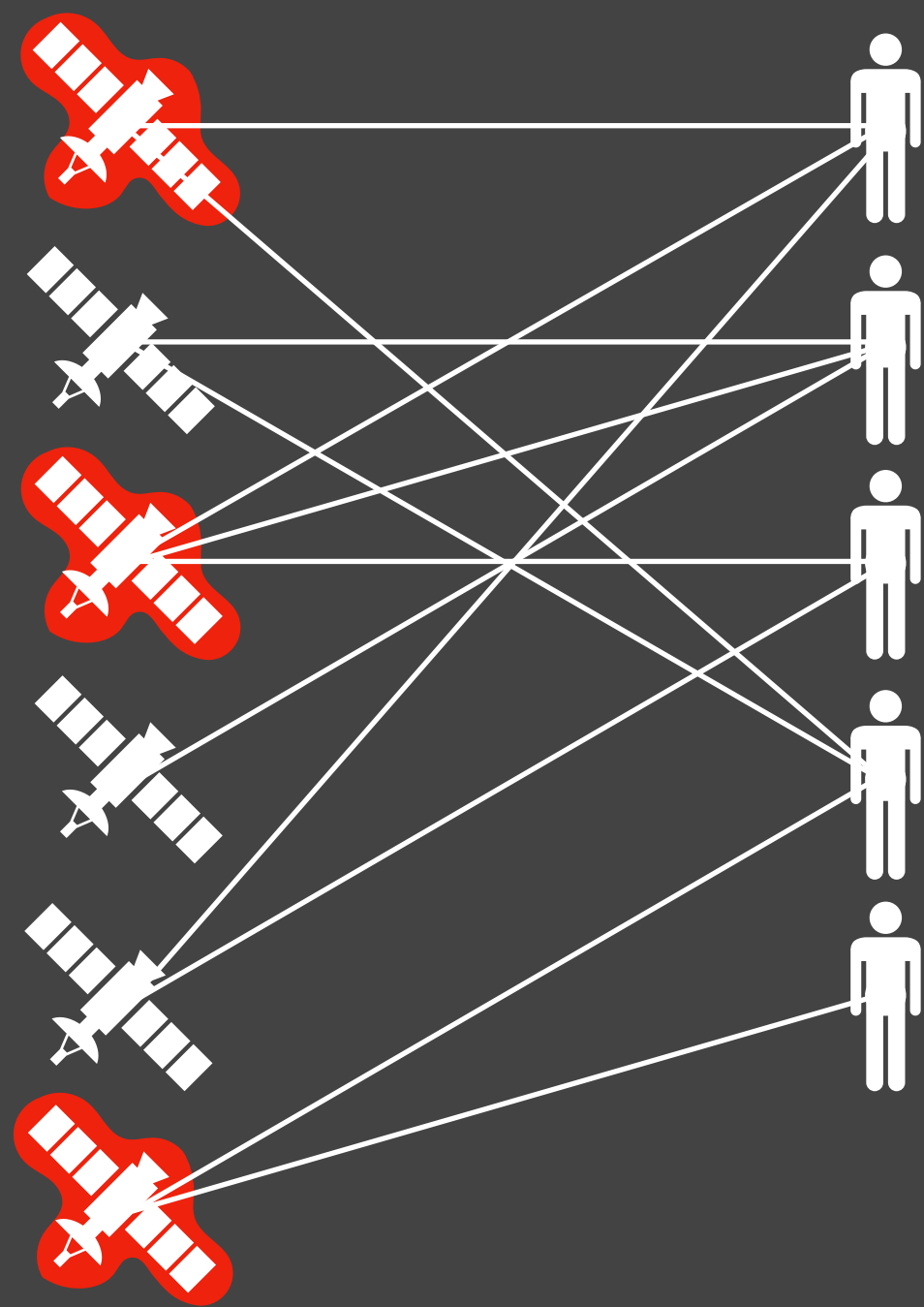
My Work

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Streaming

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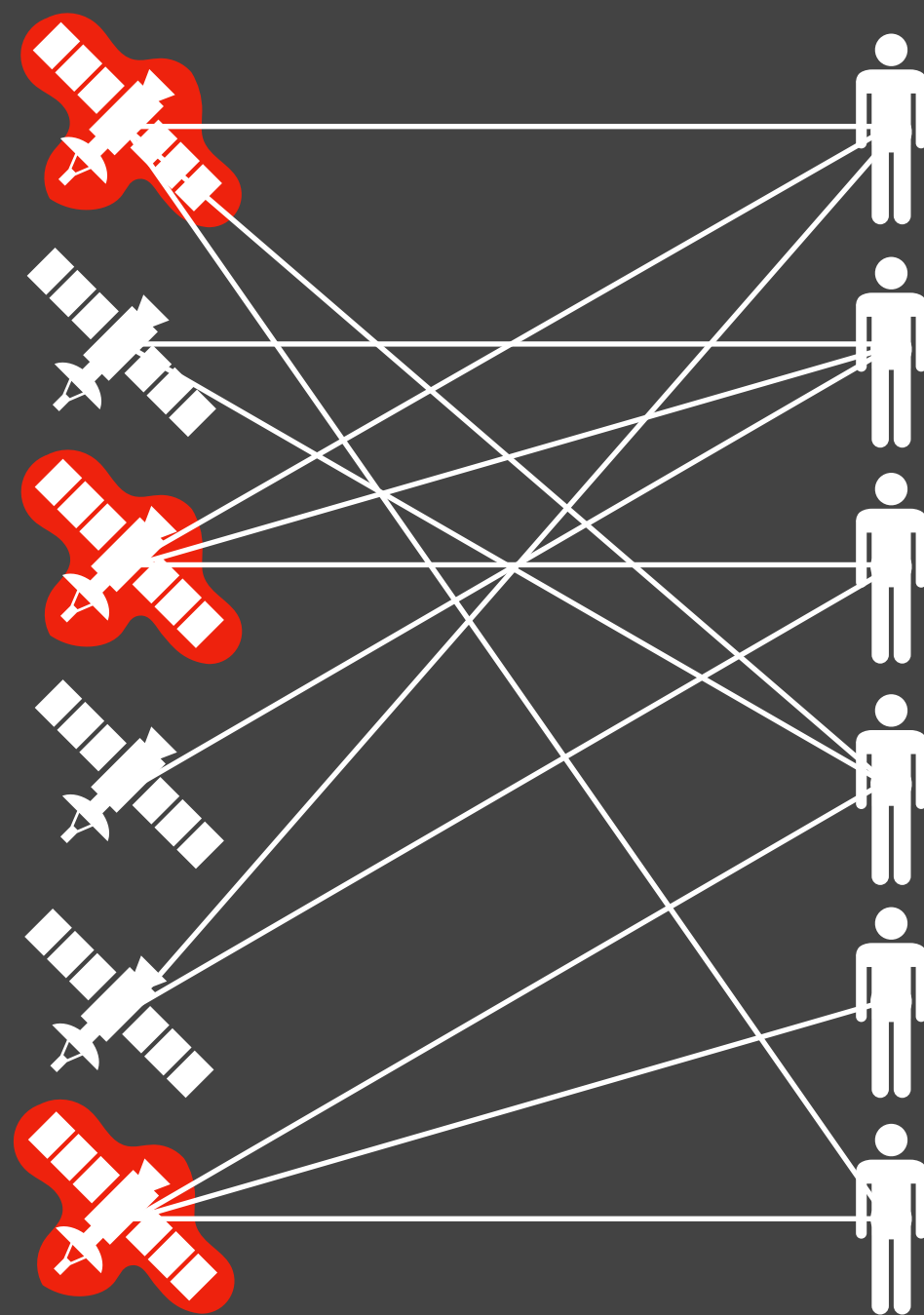
My Work

Online

Dynamic

Streaming

No take-backs



My Work

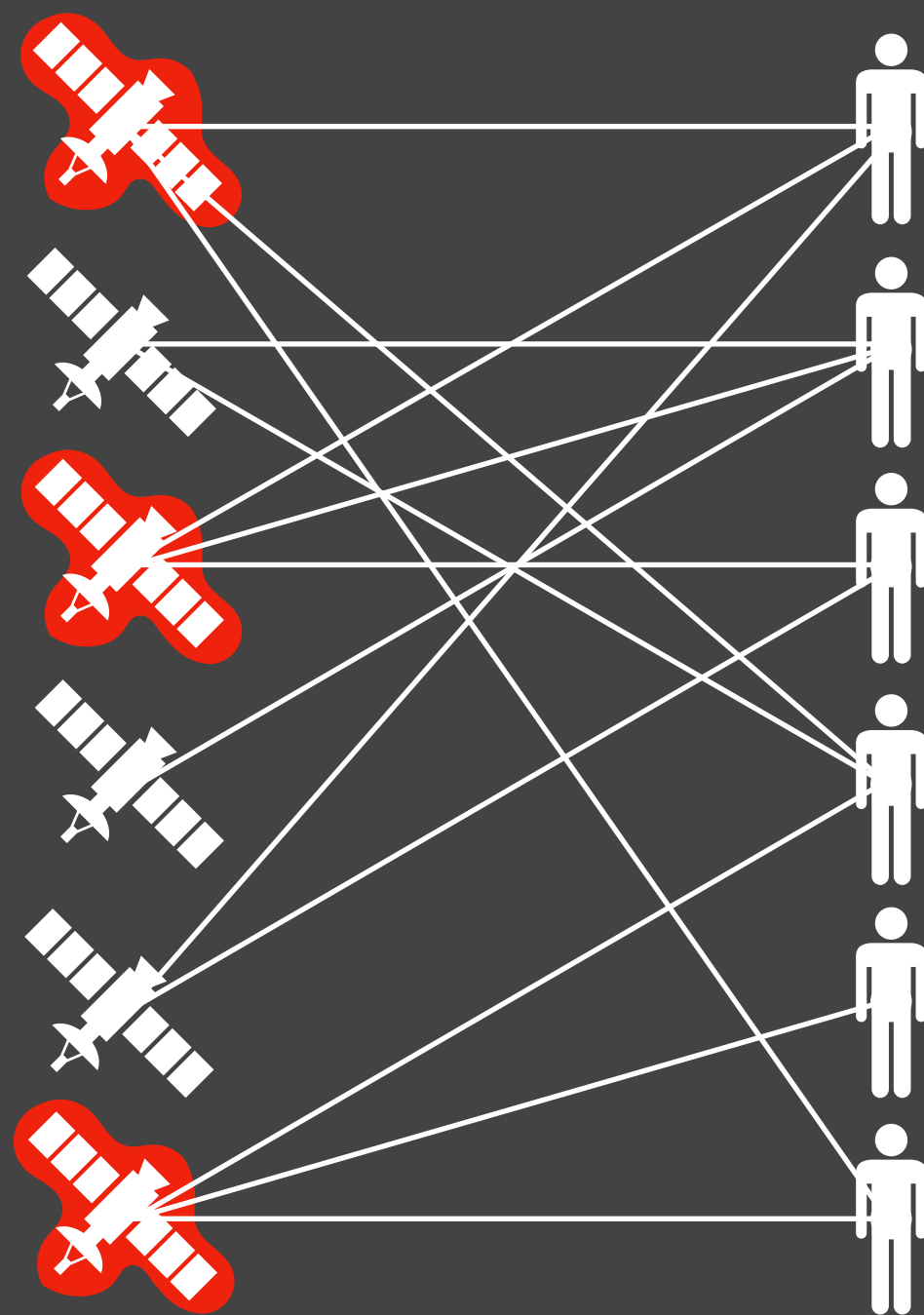
Online

Dynamic

Streaming

No take-backs

Low movement



My Work

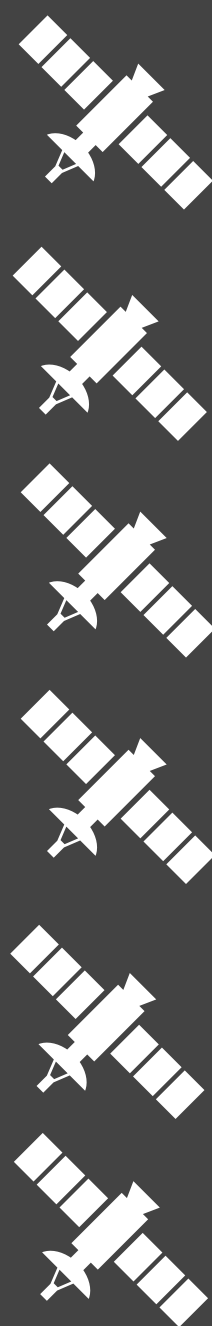
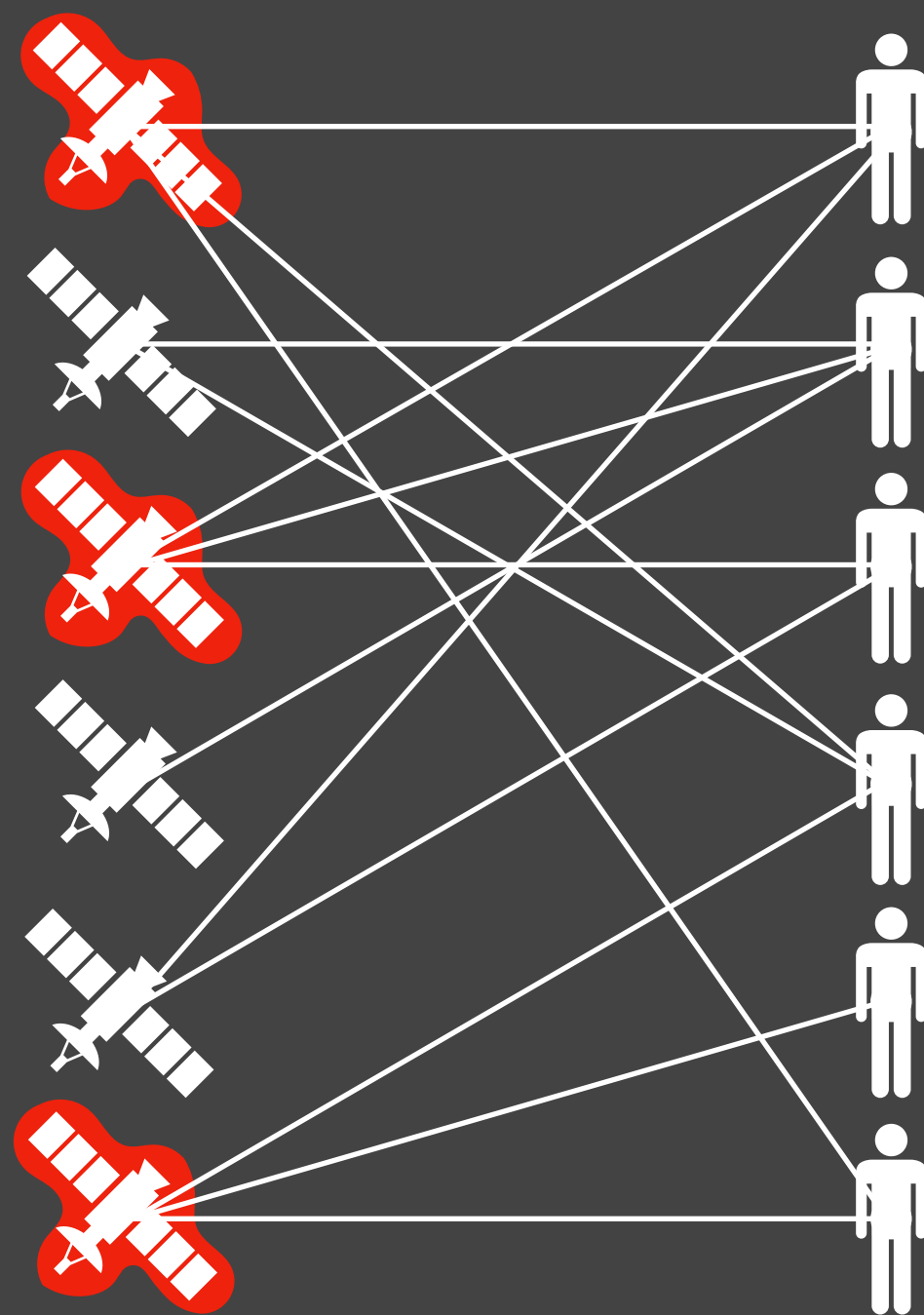
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My Work

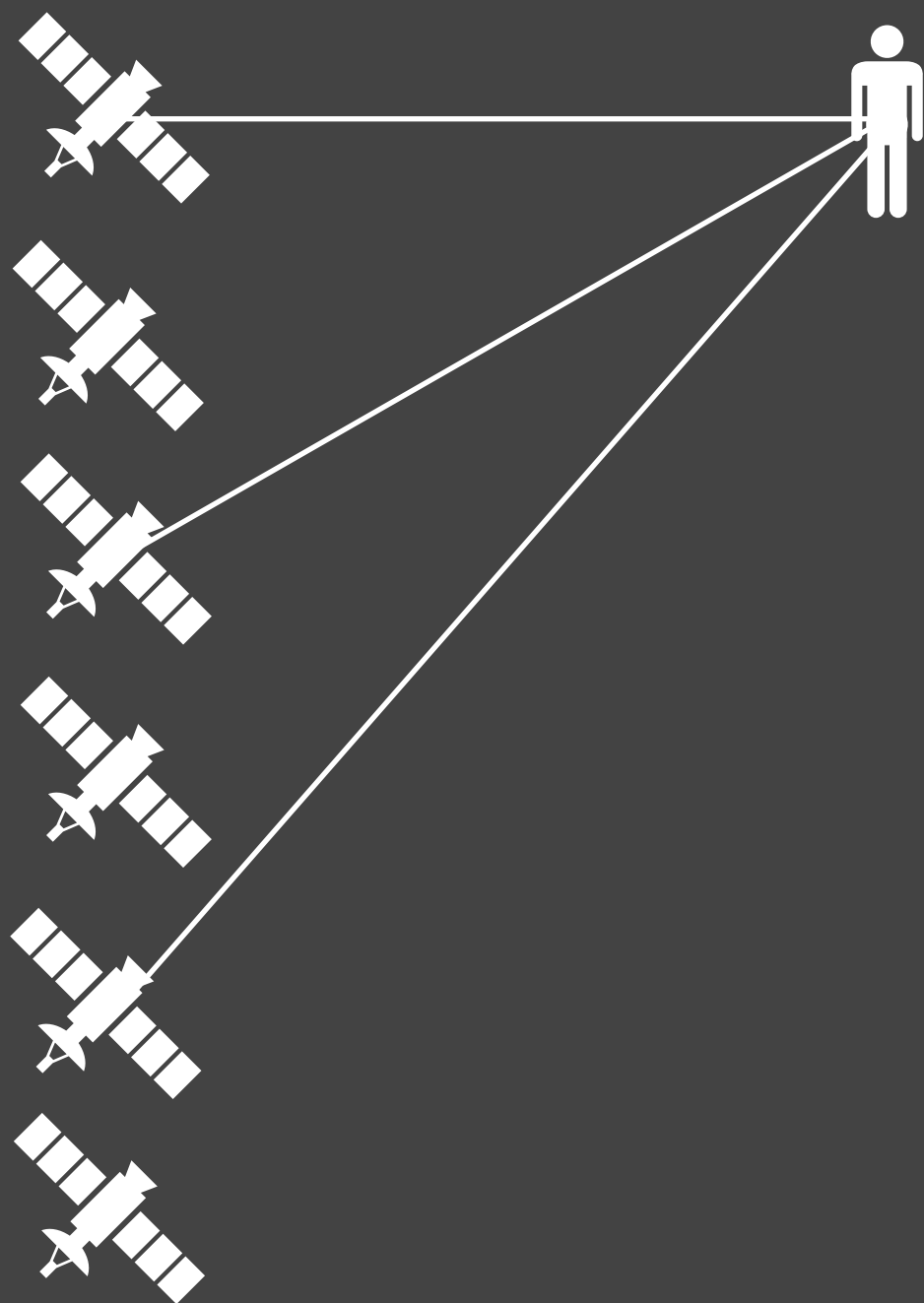
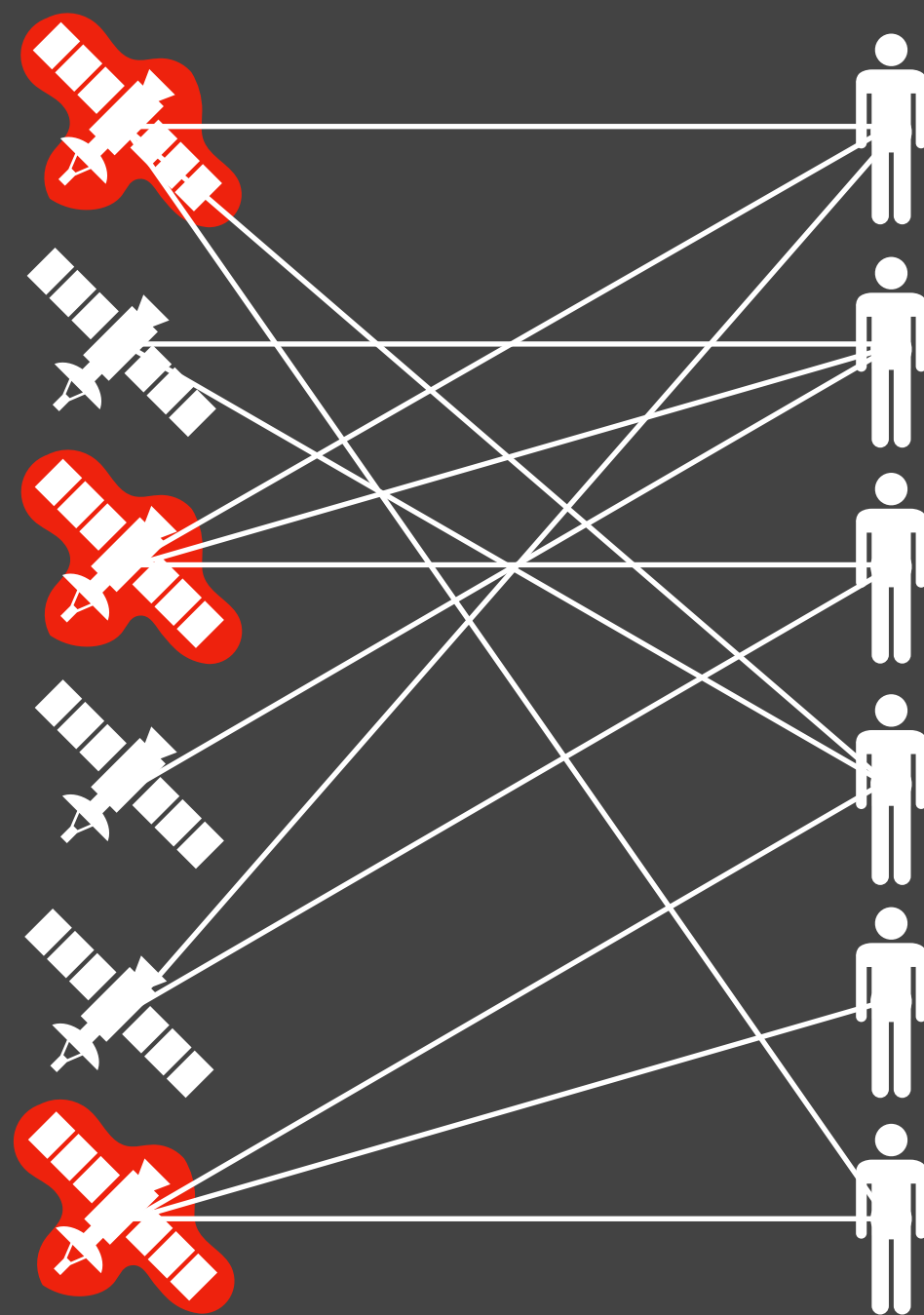
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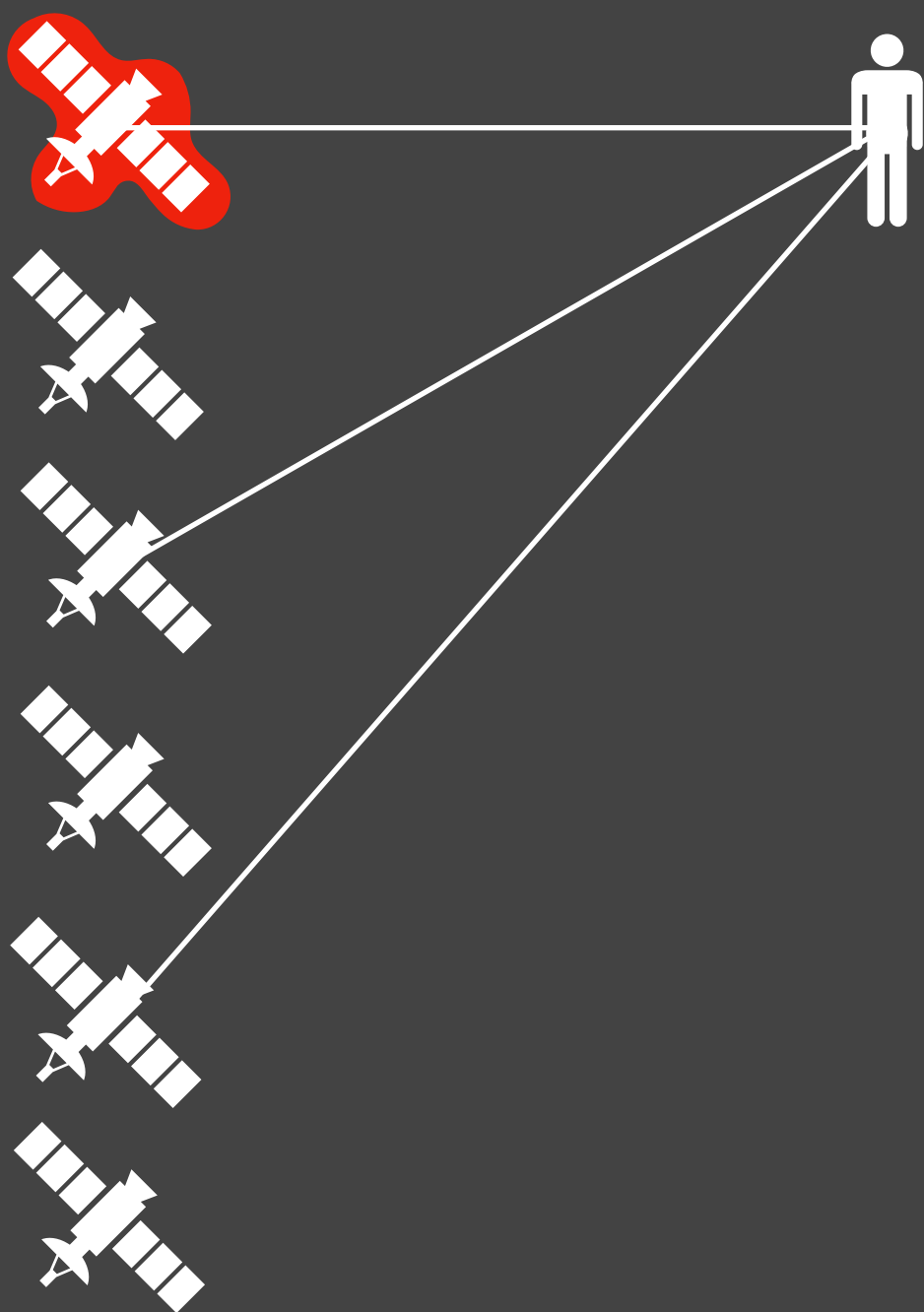
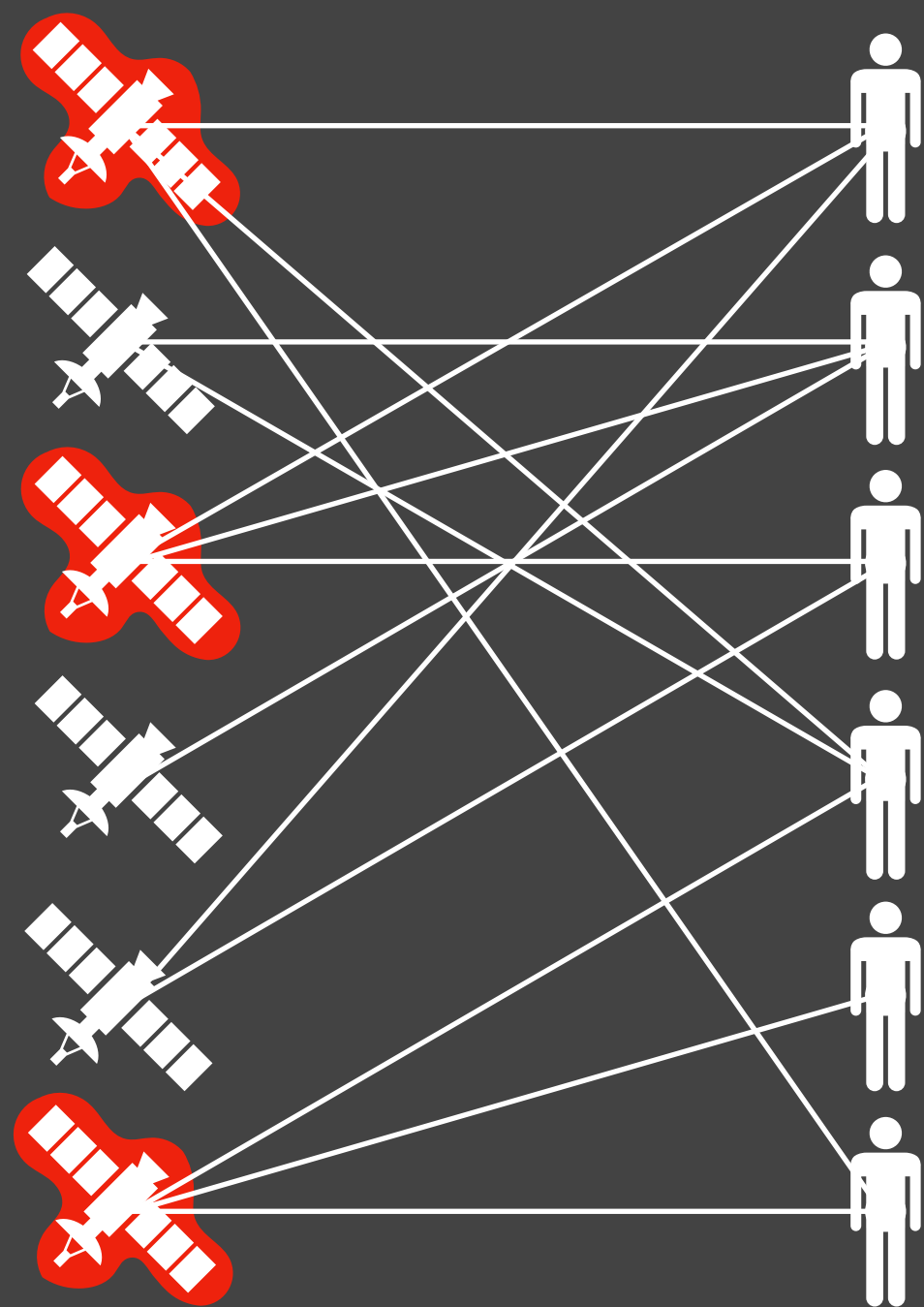
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My Work

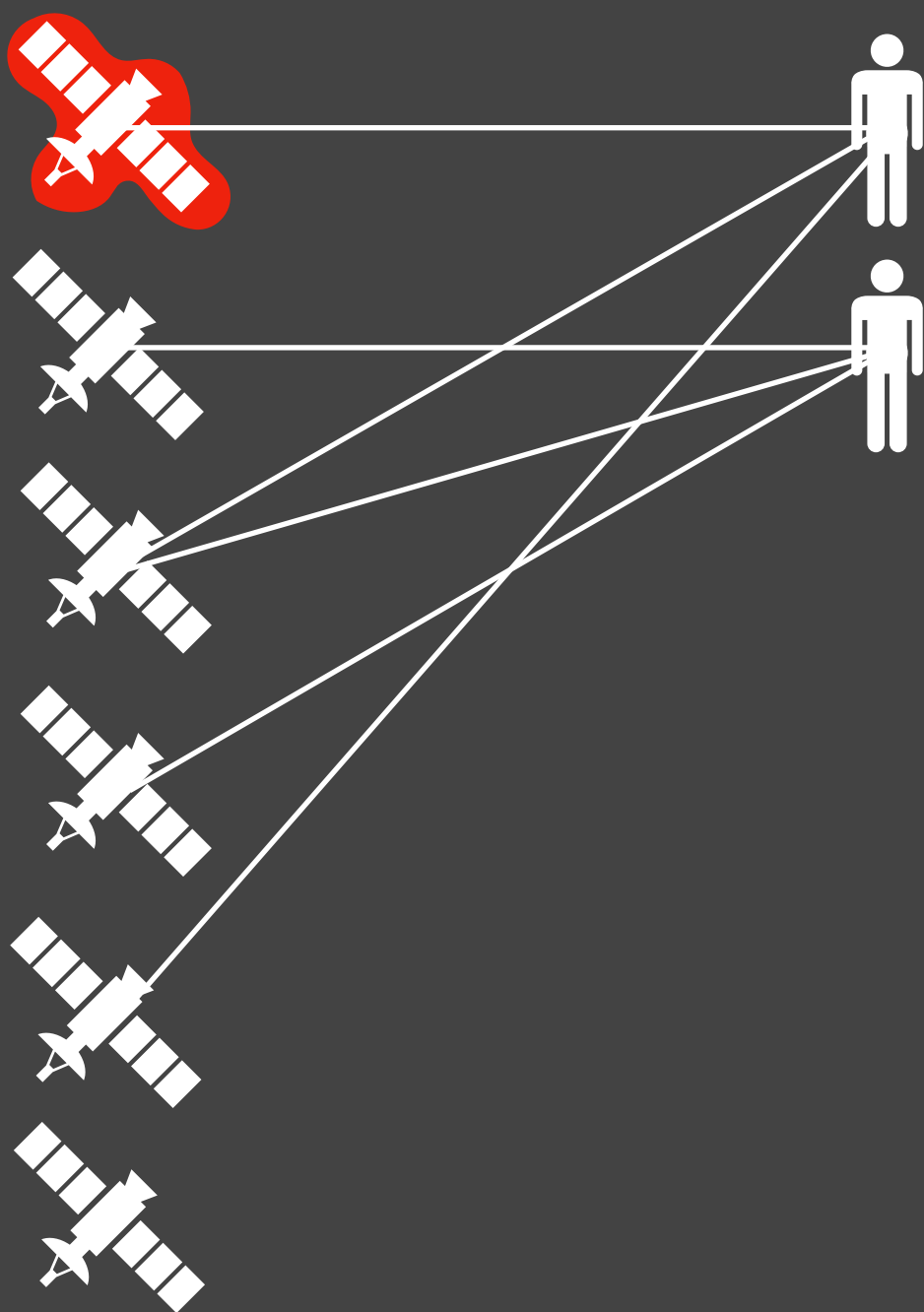
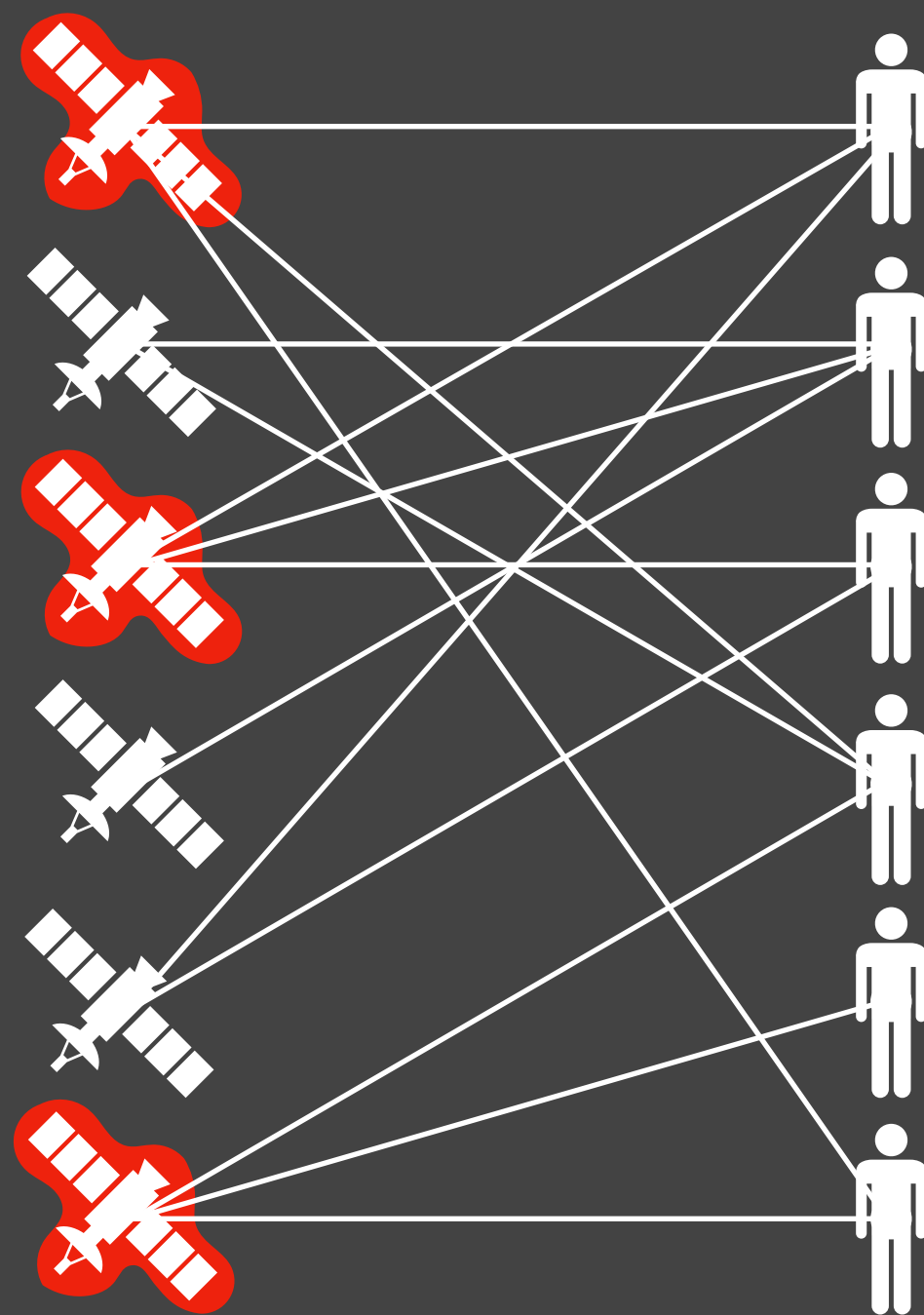
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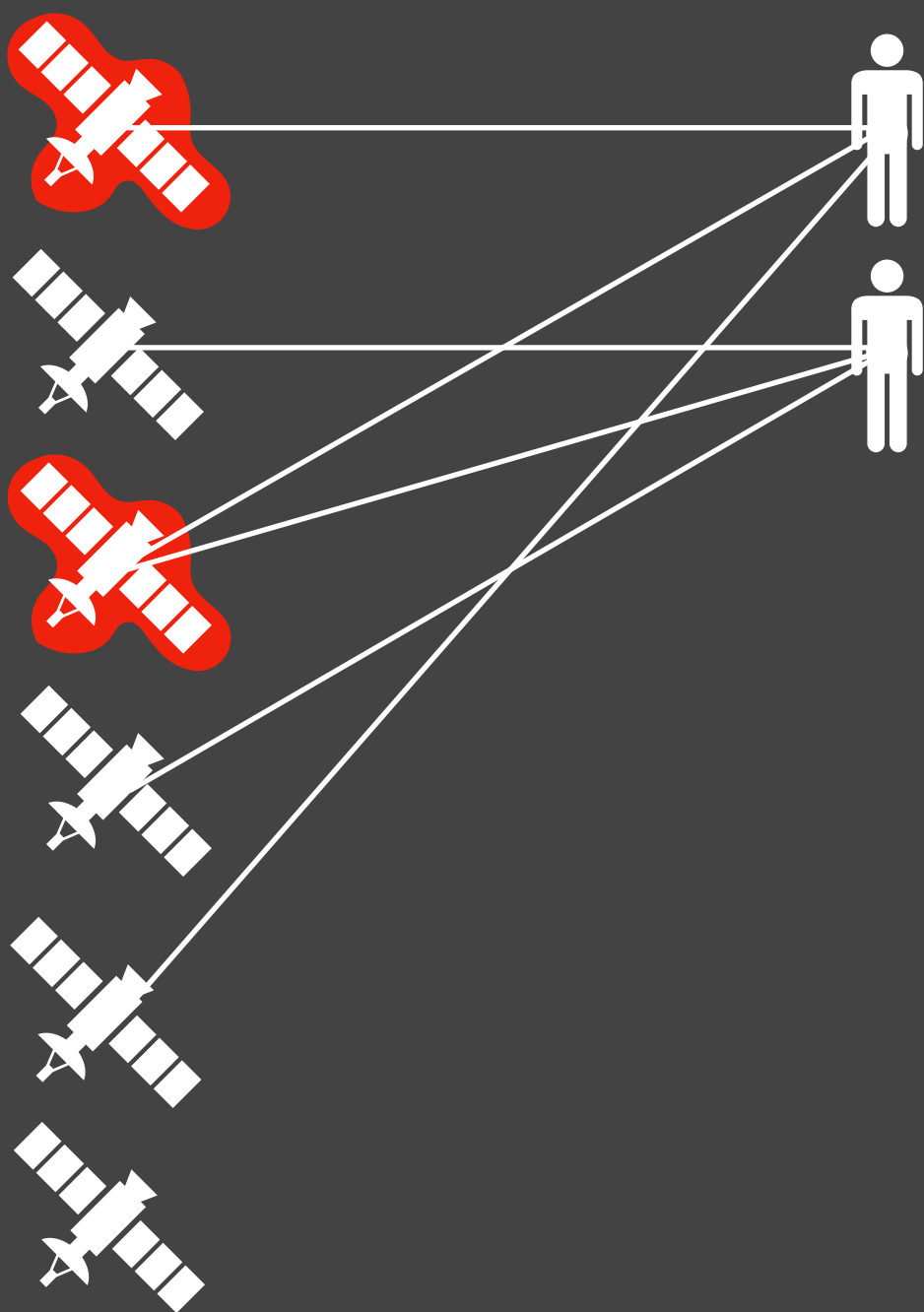
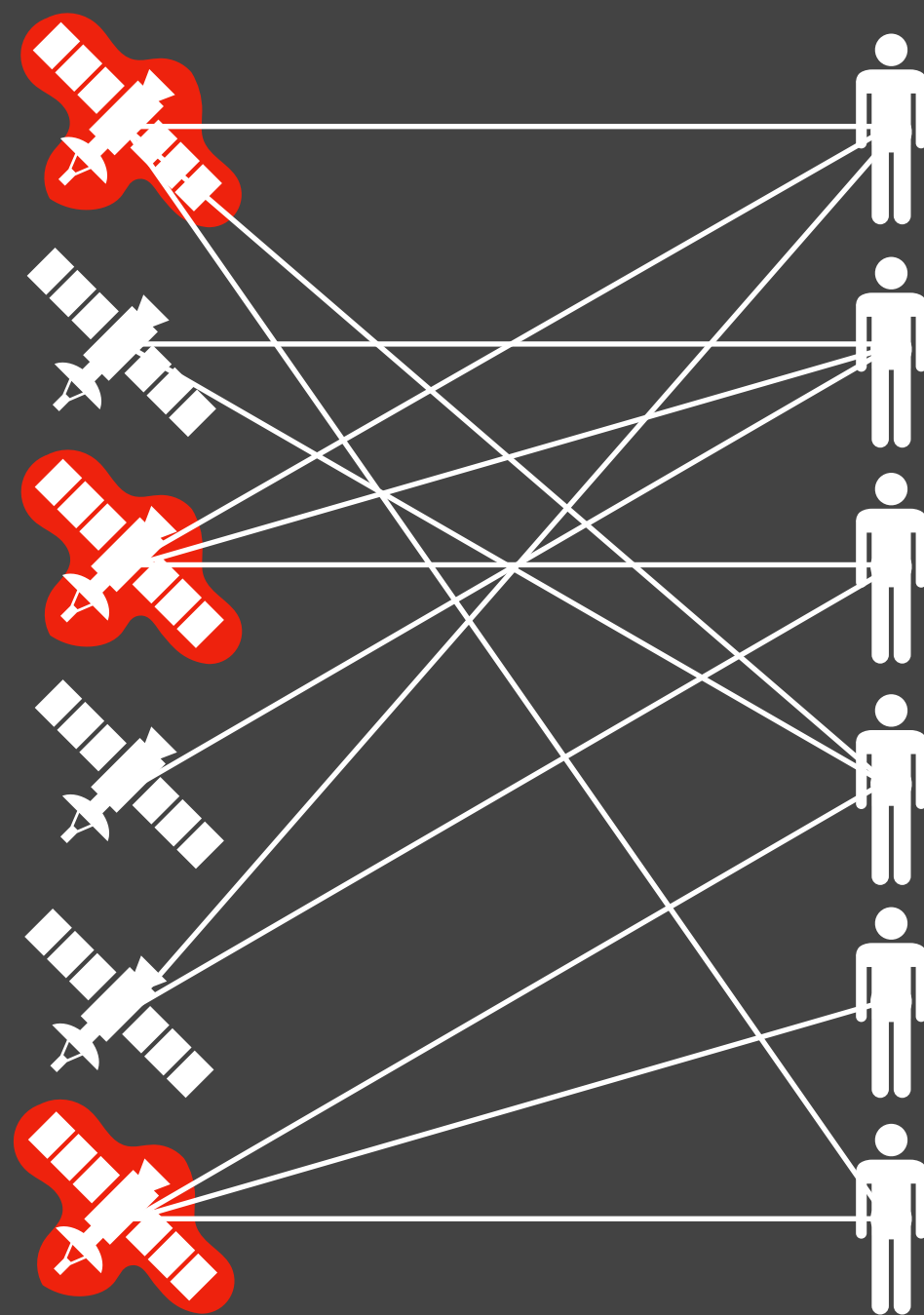
Online

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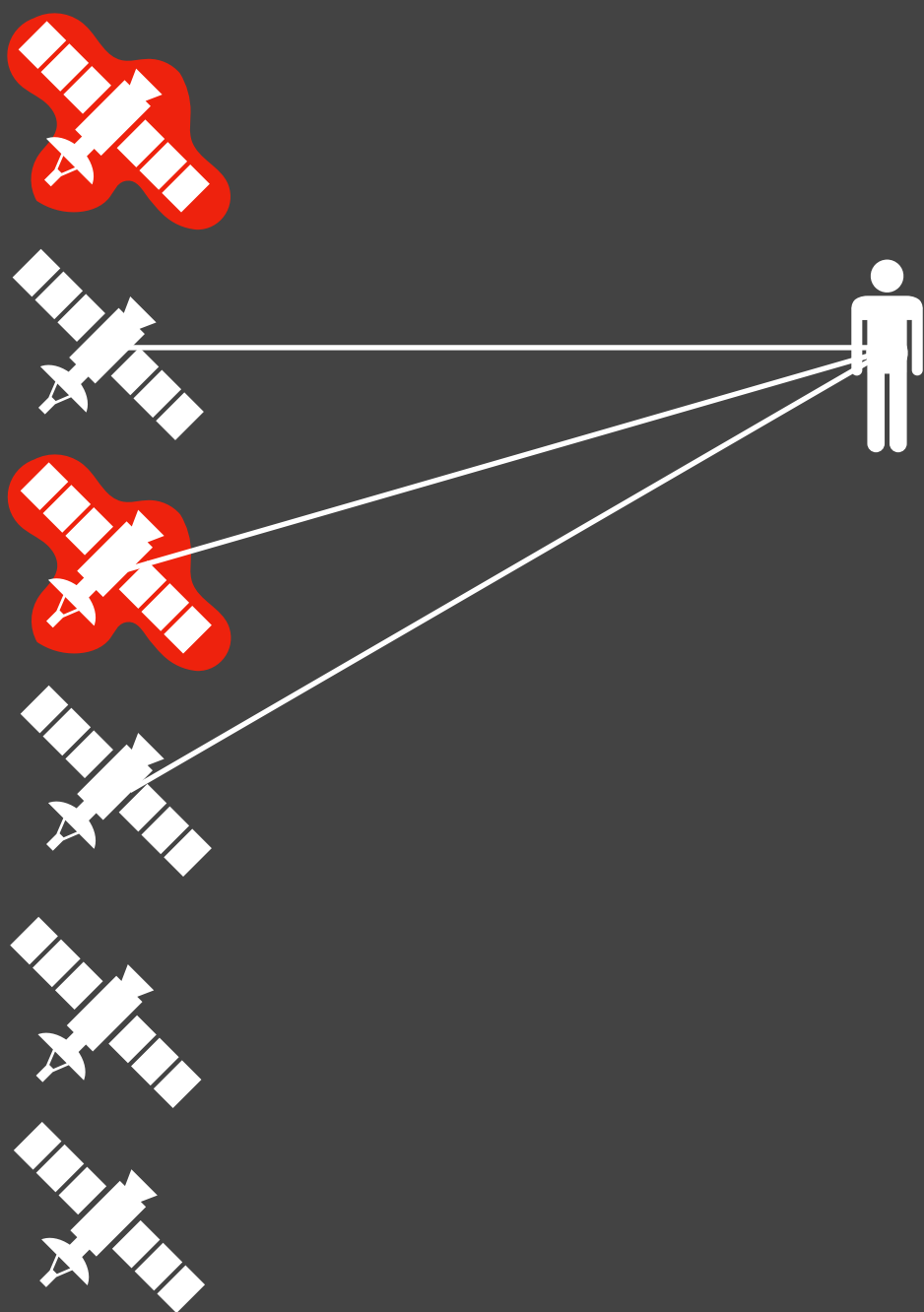
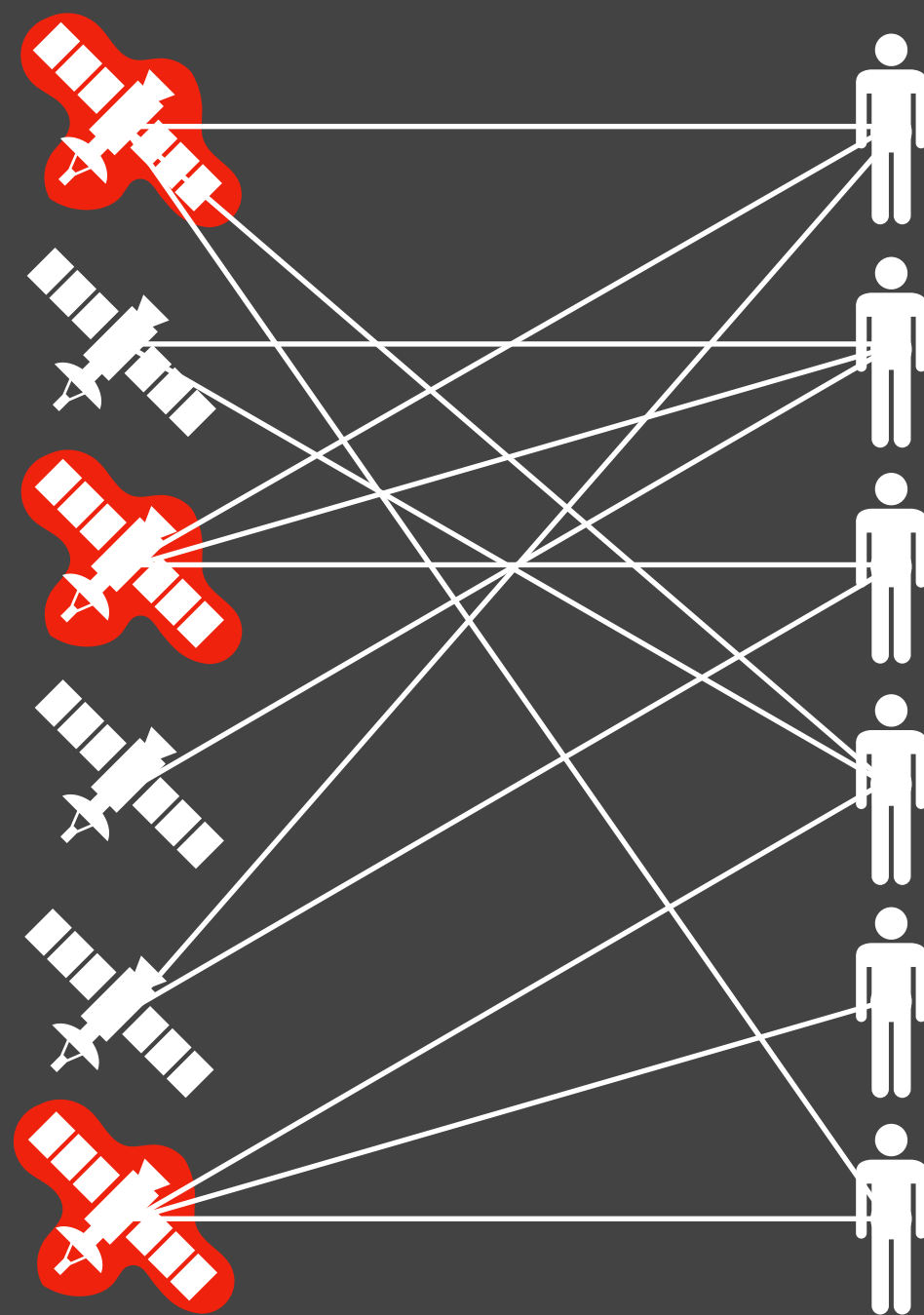
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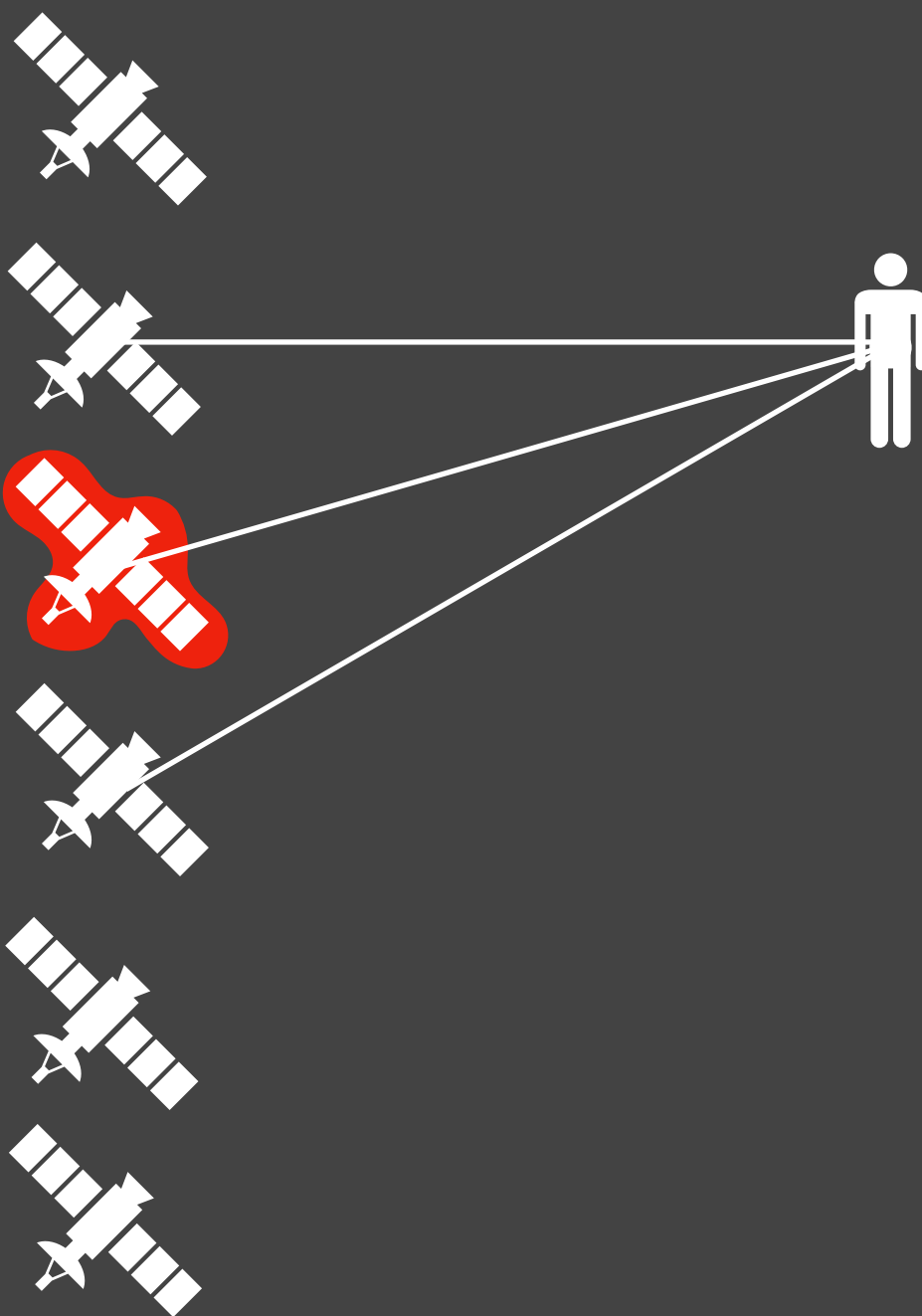
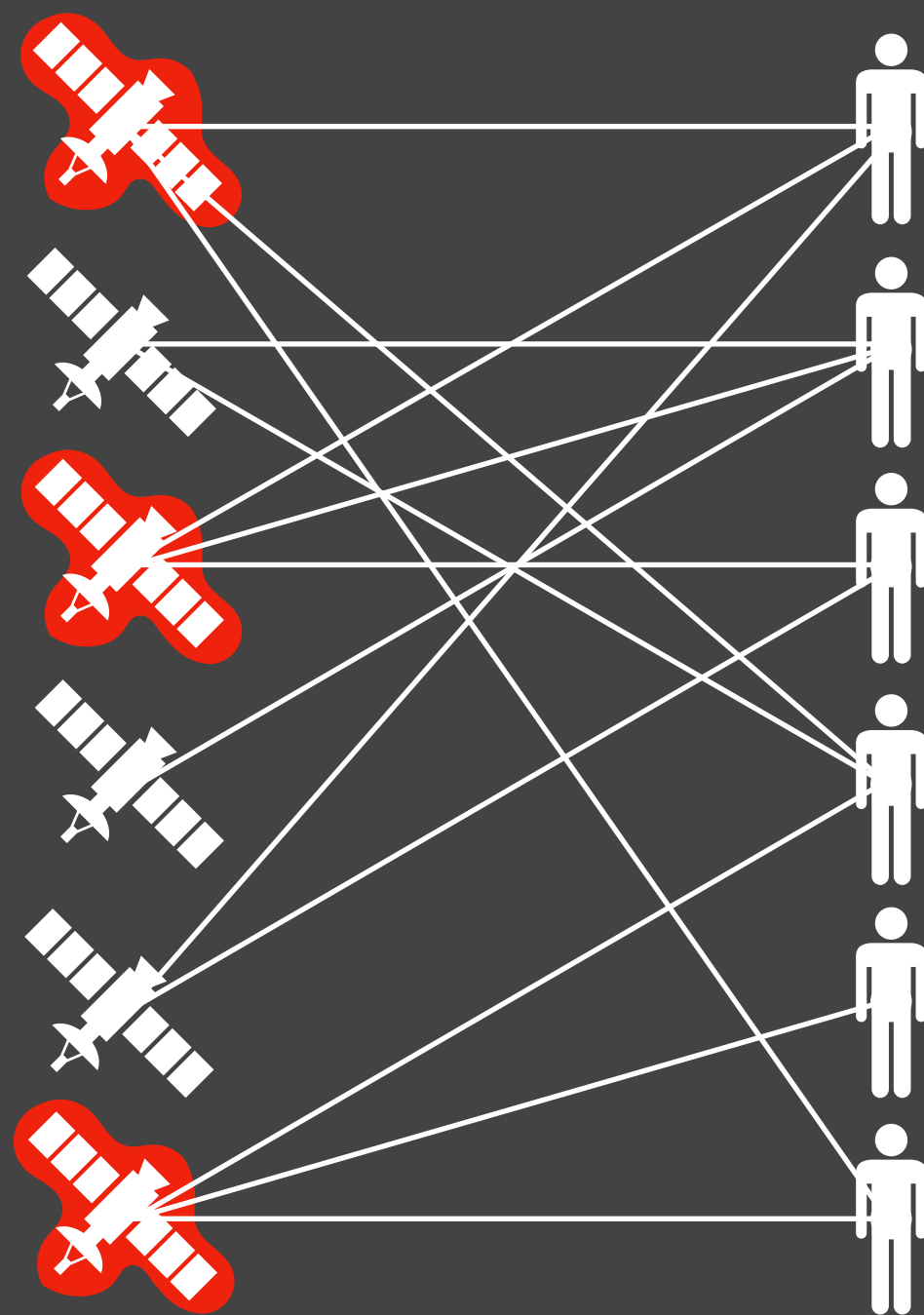
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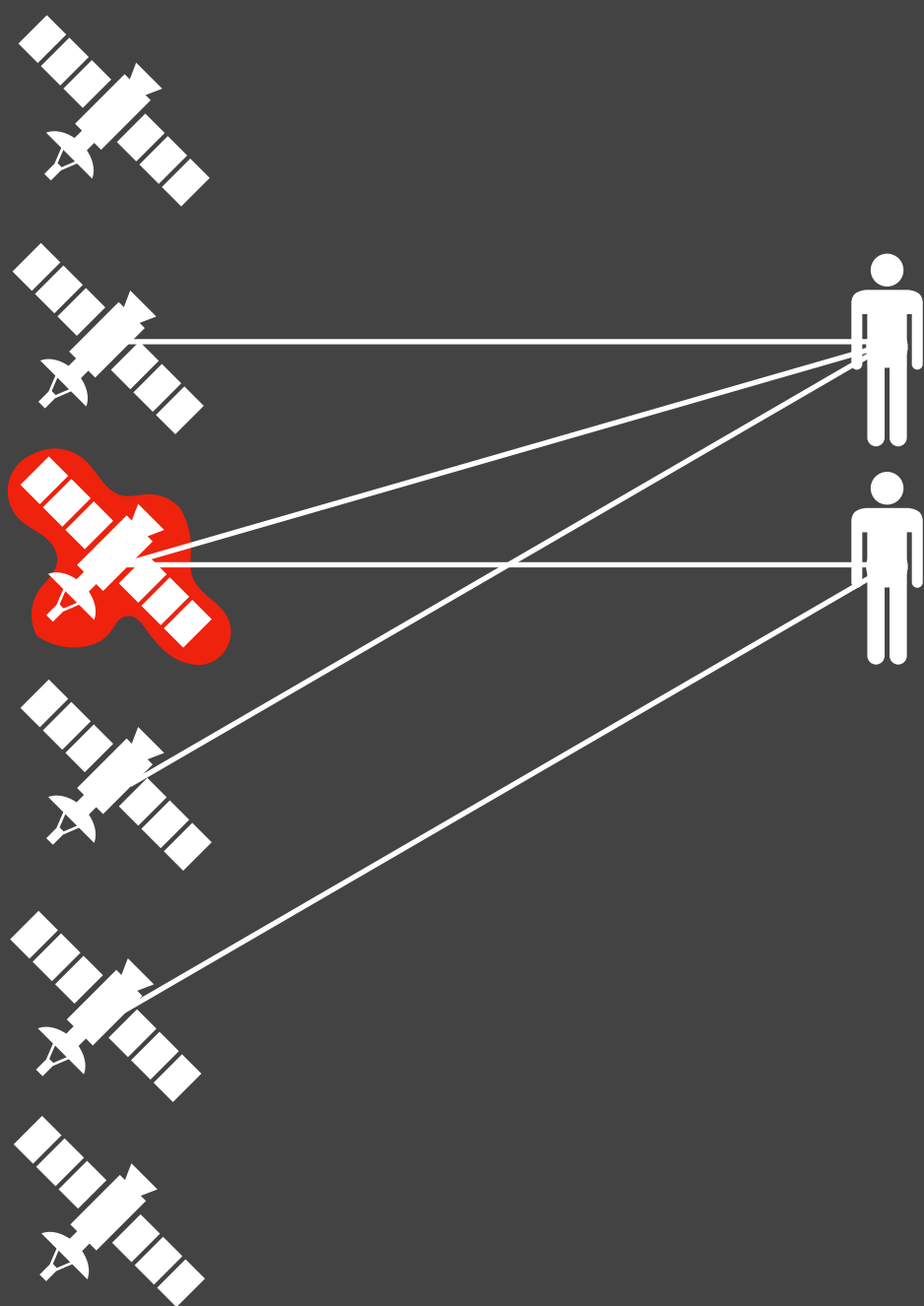
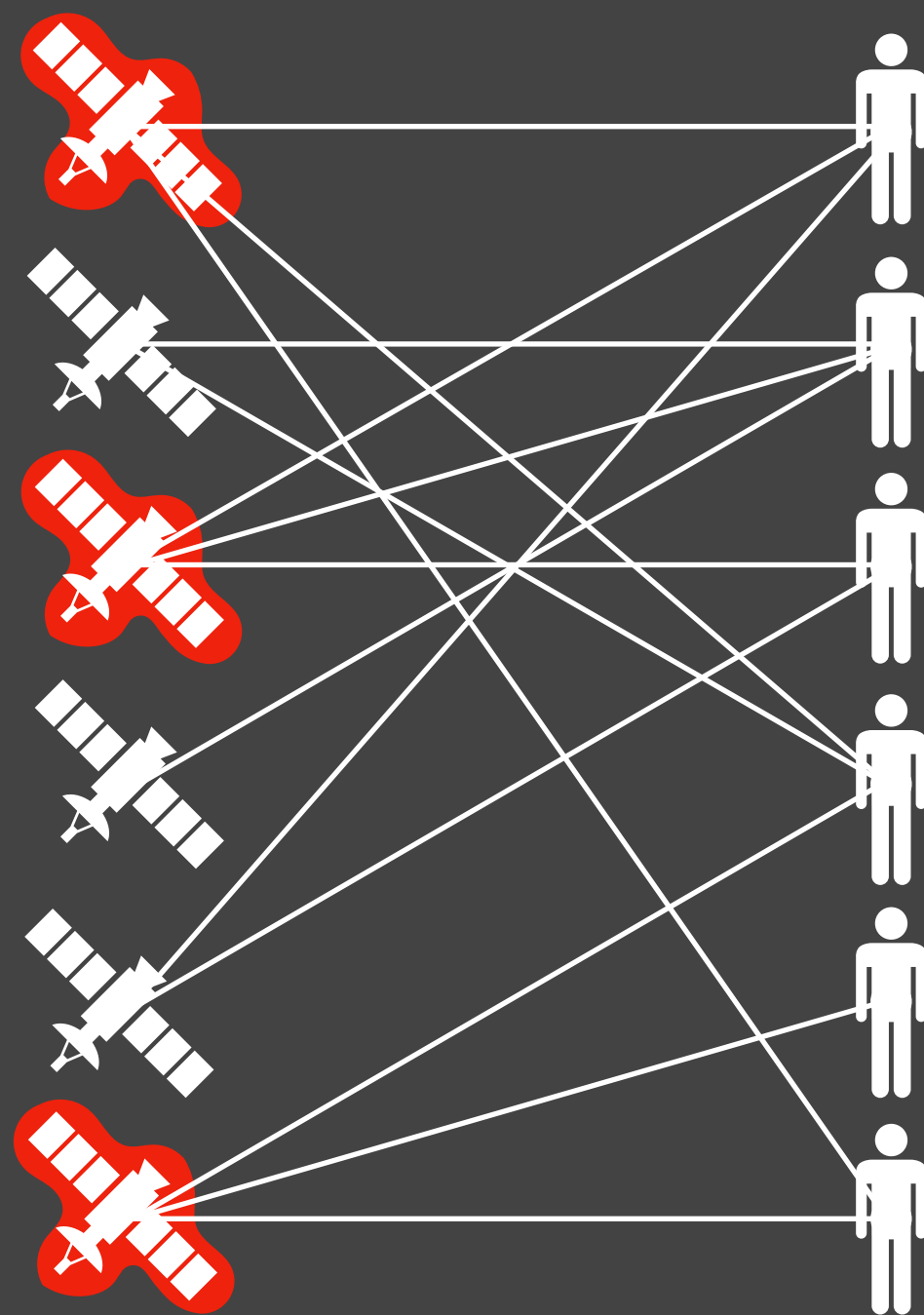
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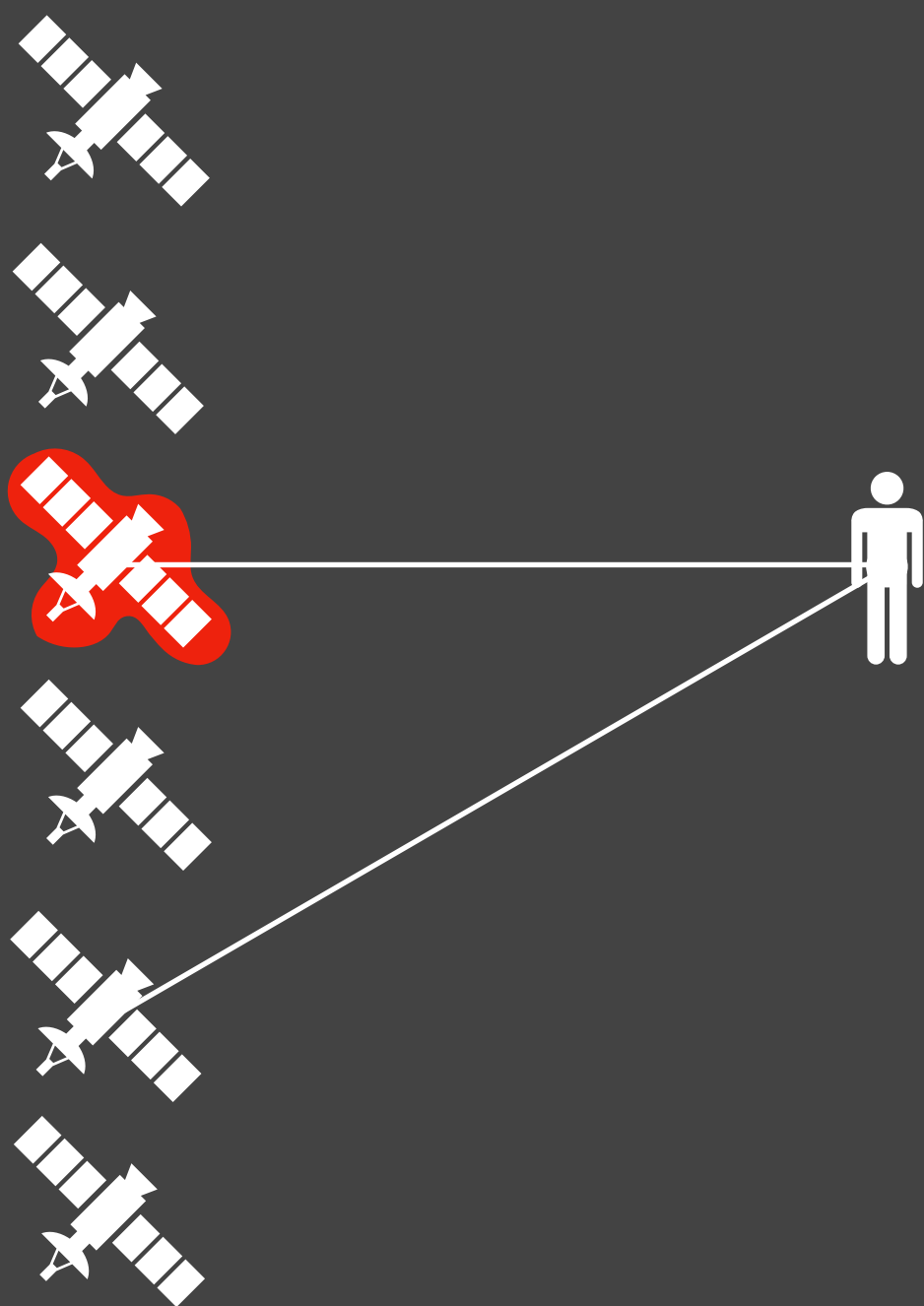
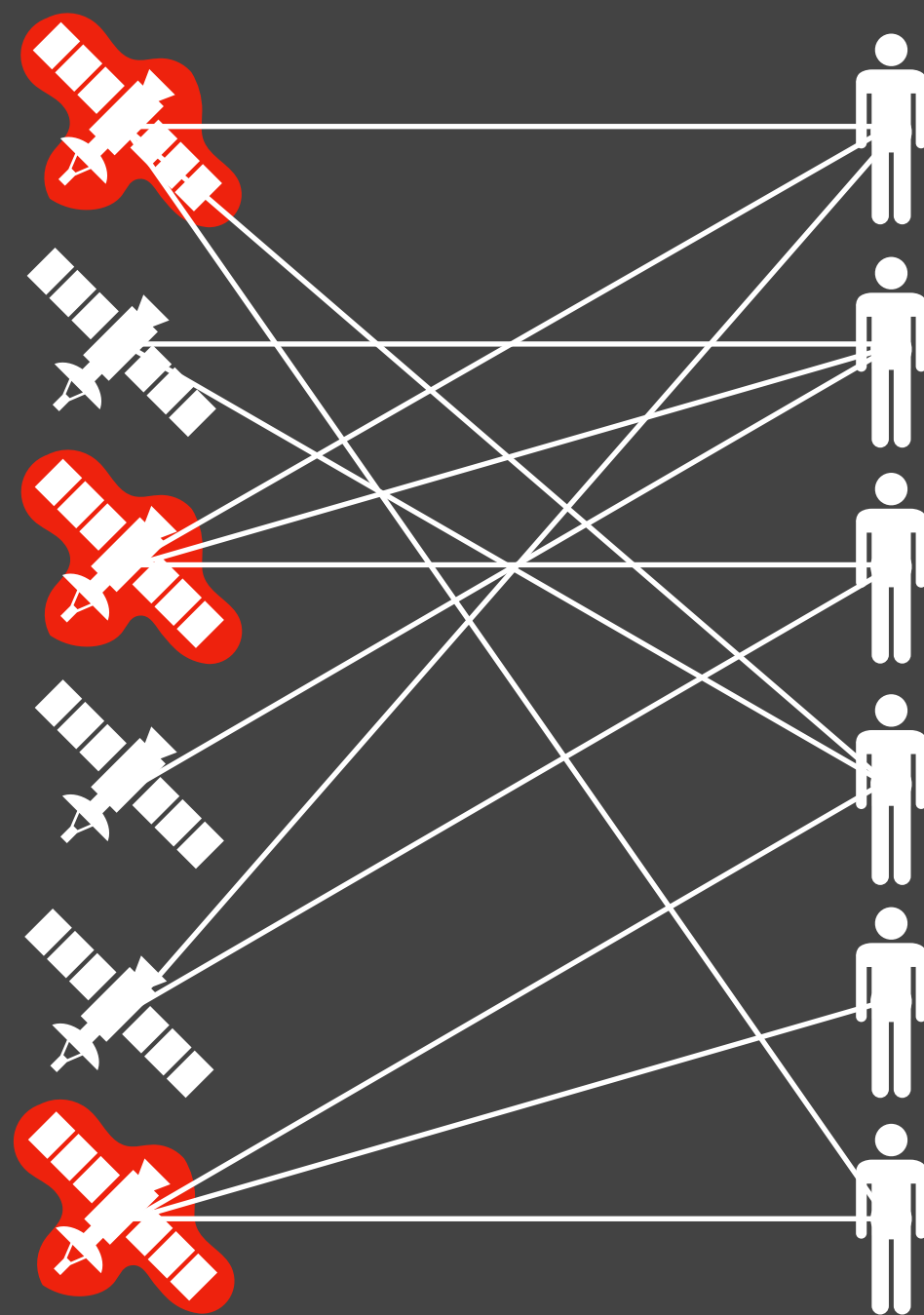
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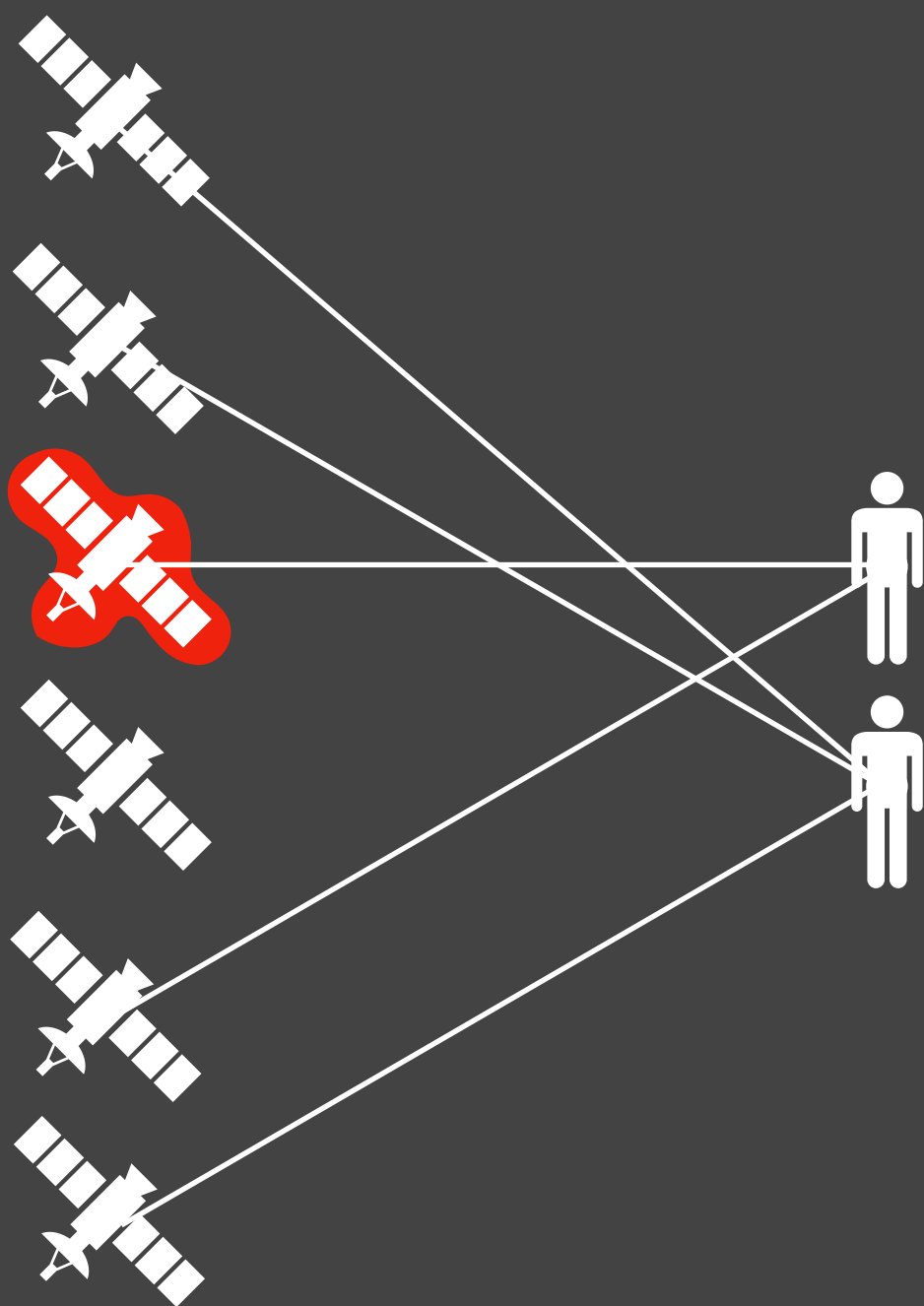
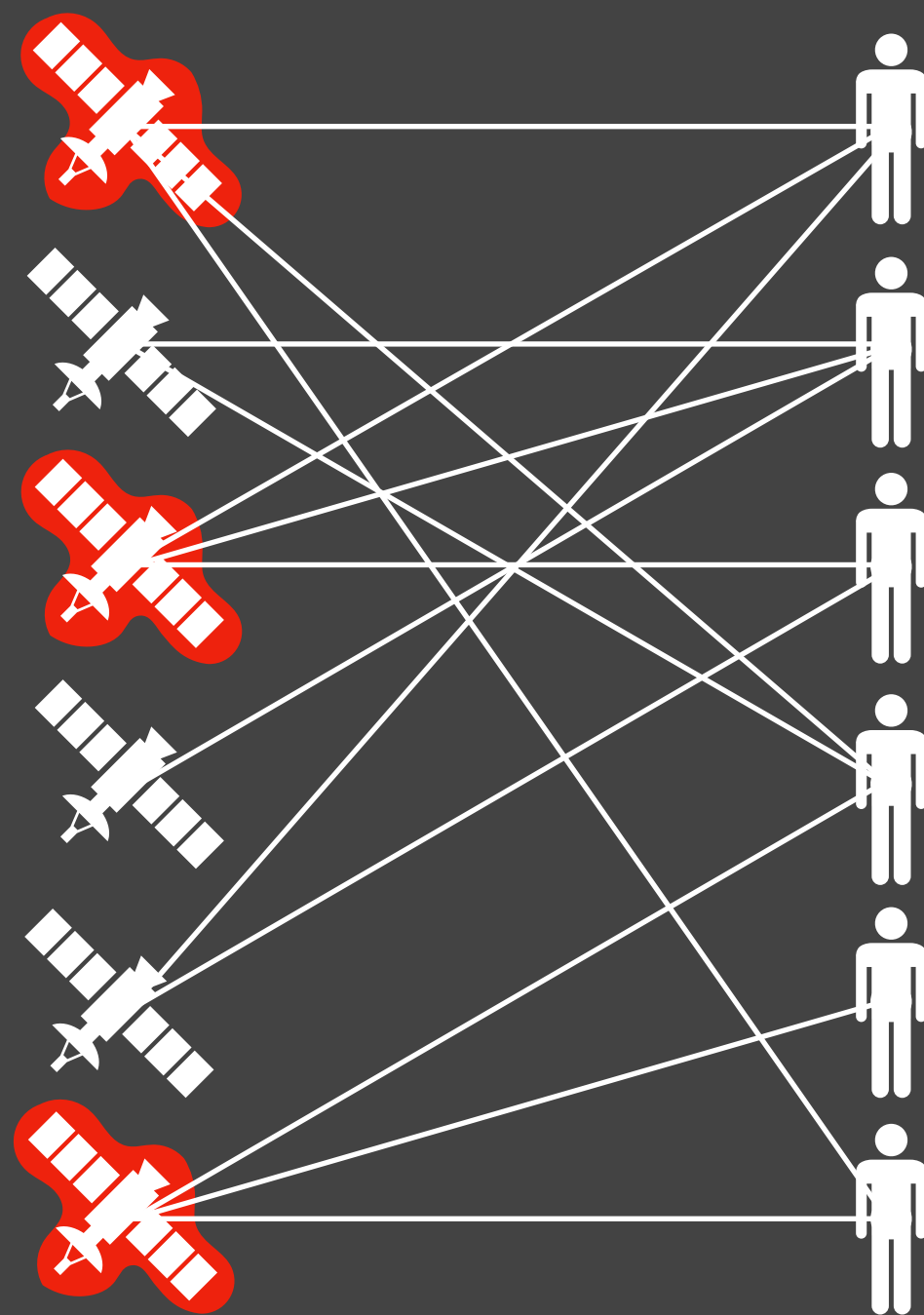
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My Work

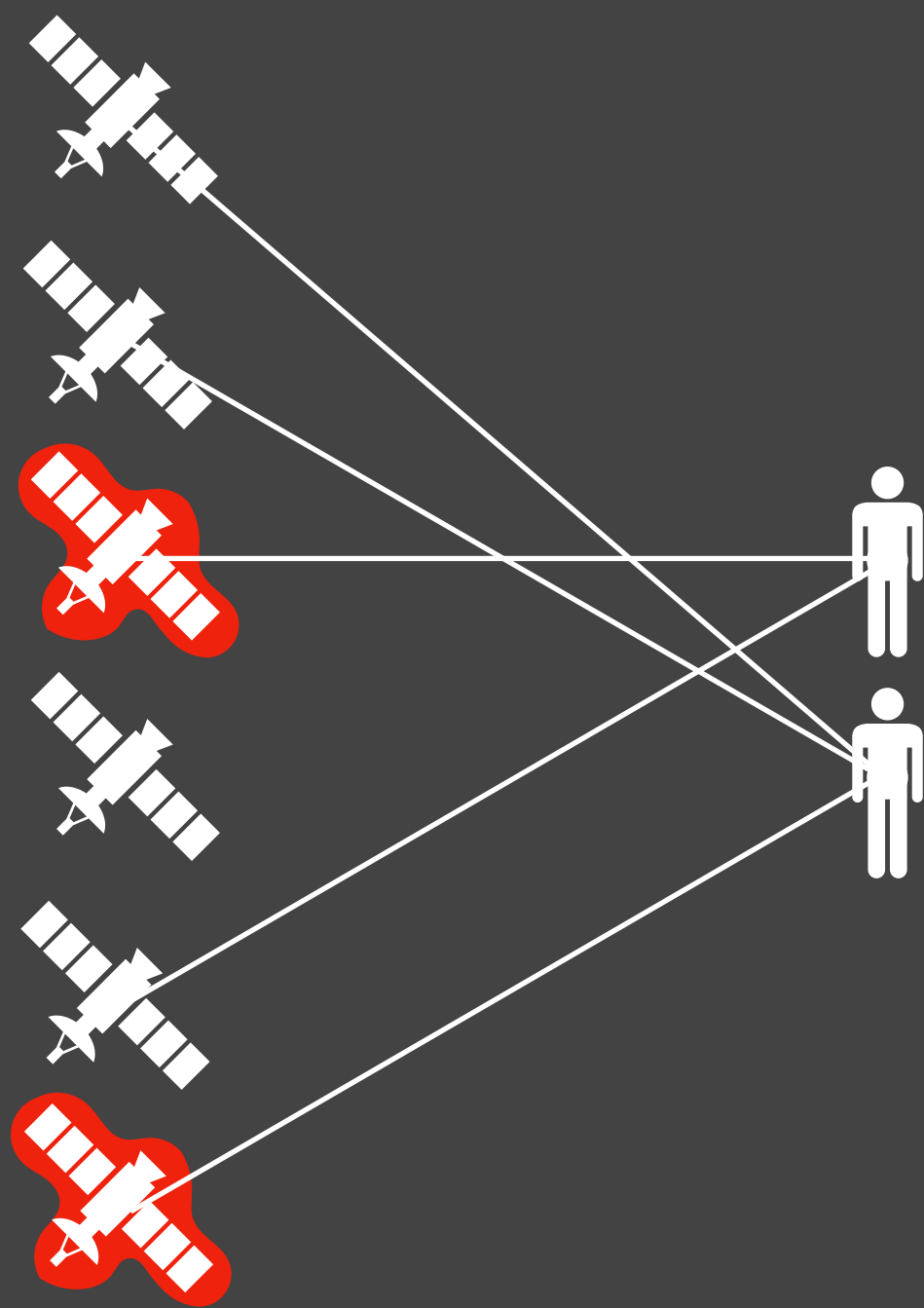
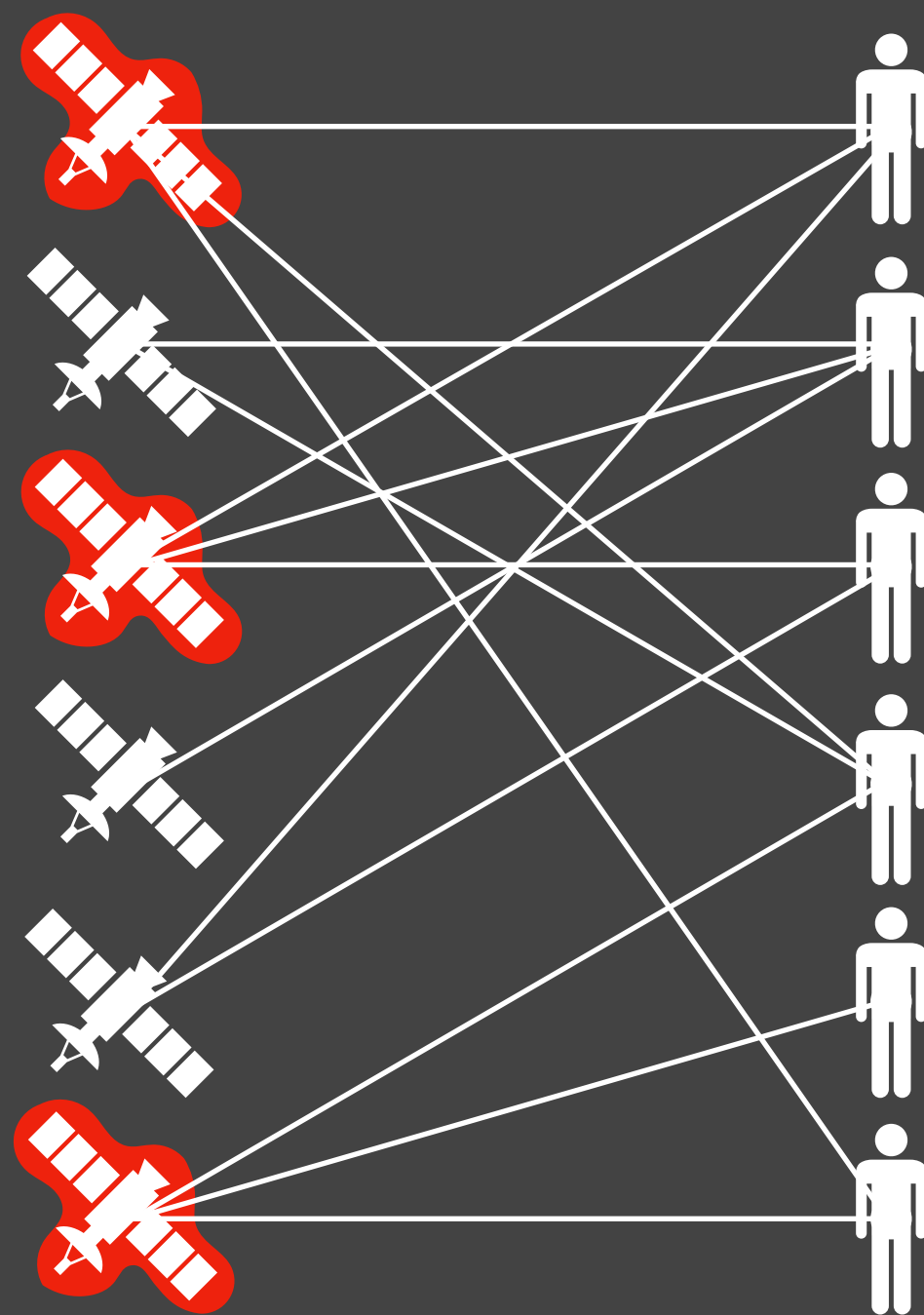
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My Work

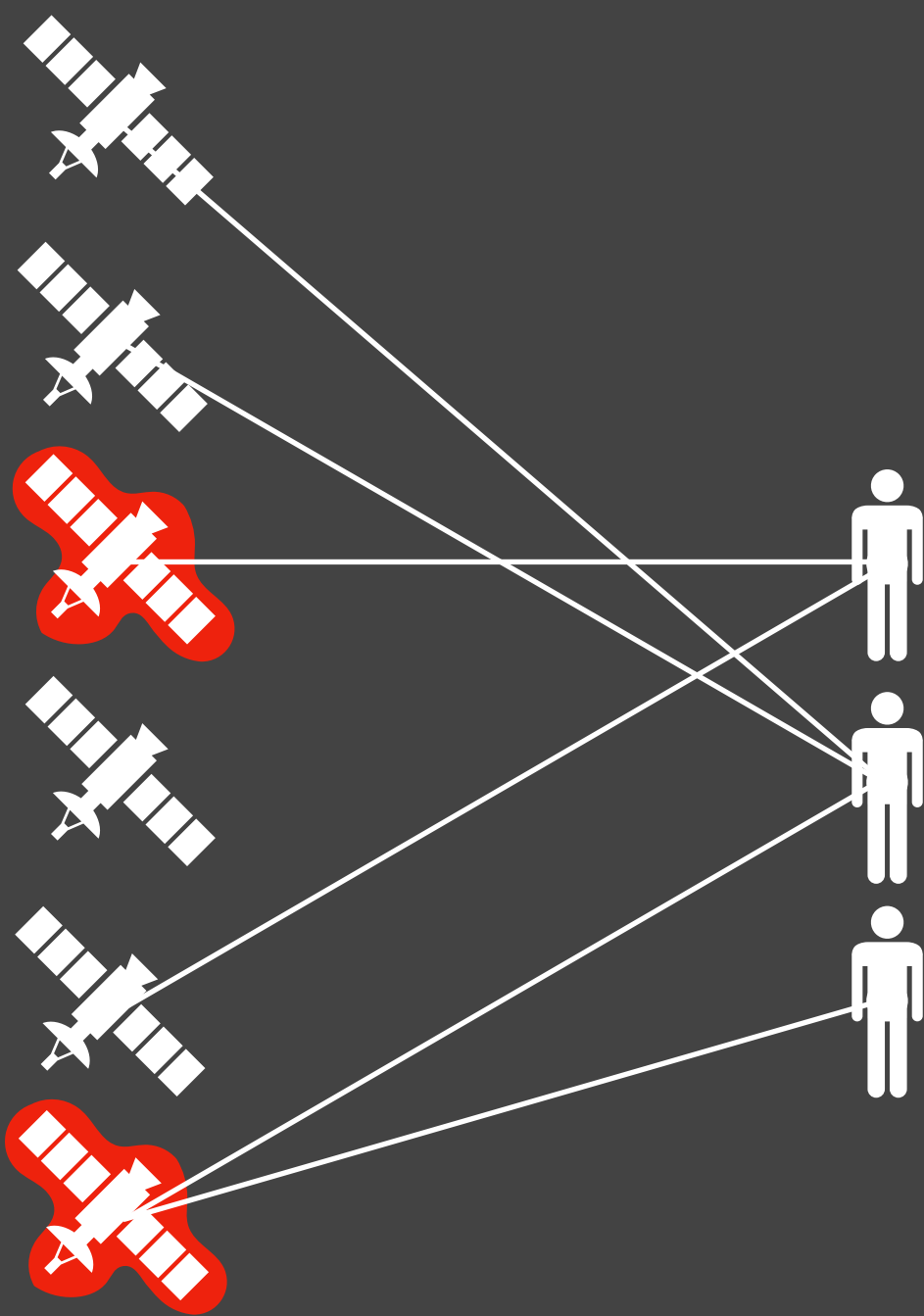
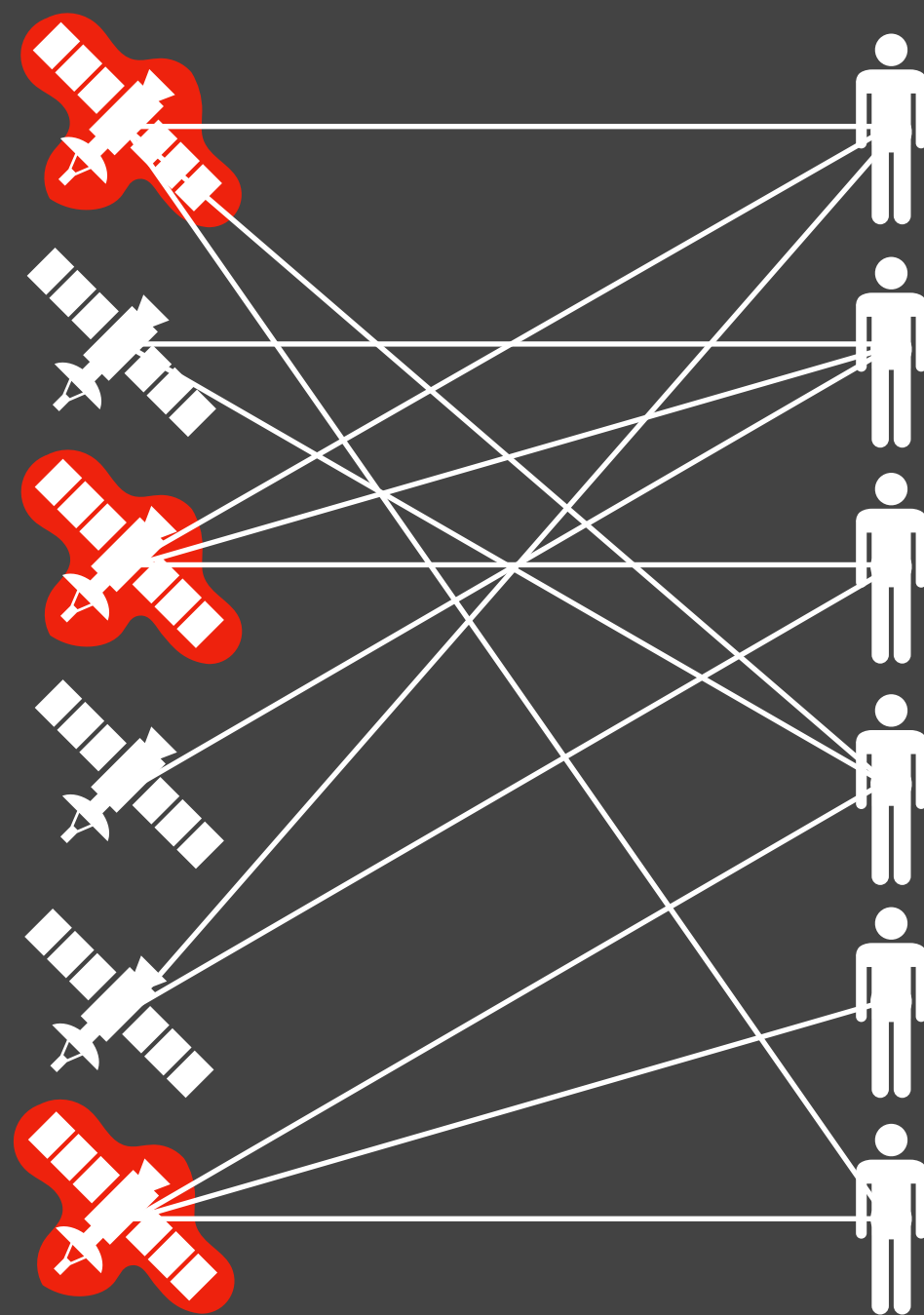
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My Work

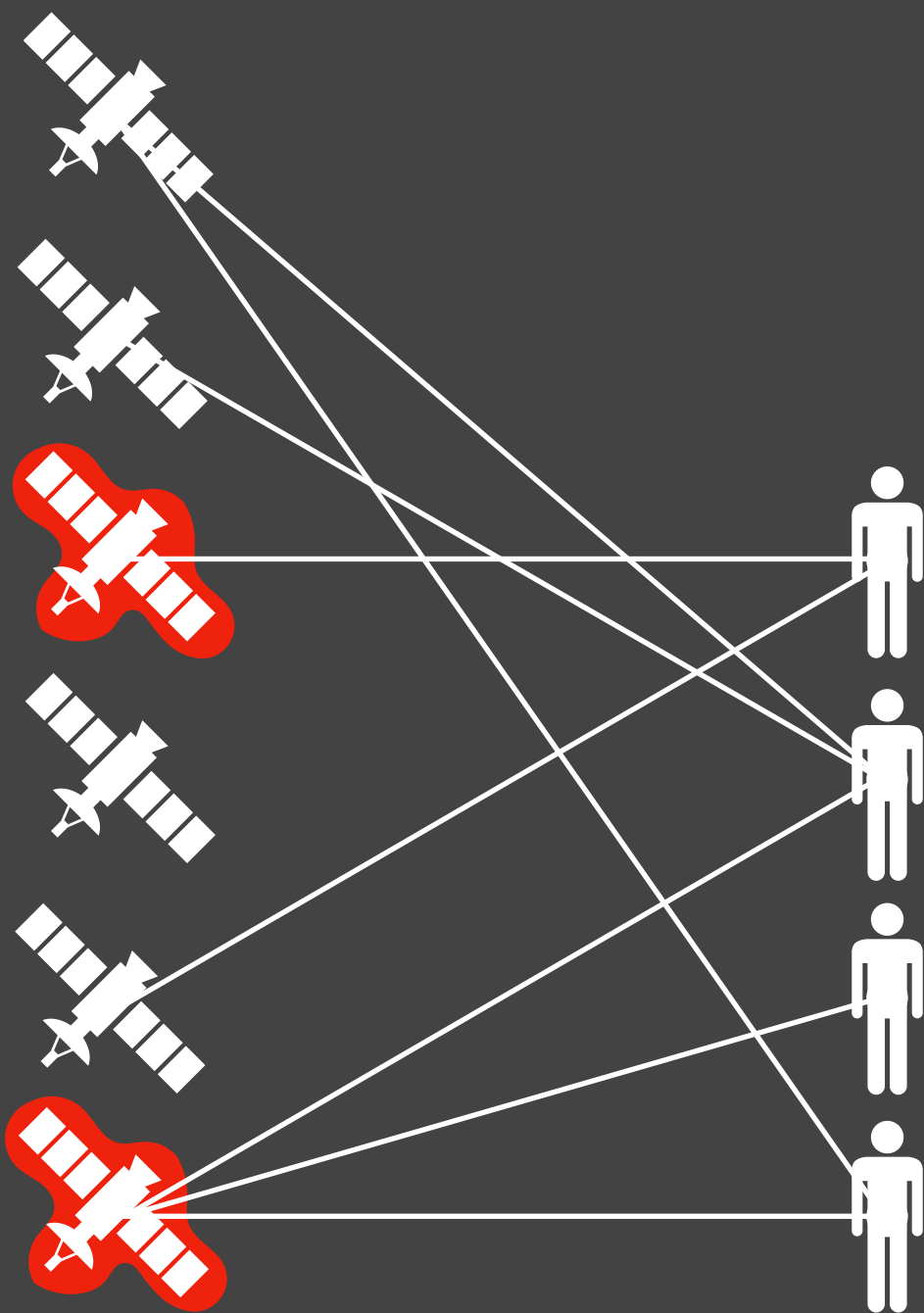
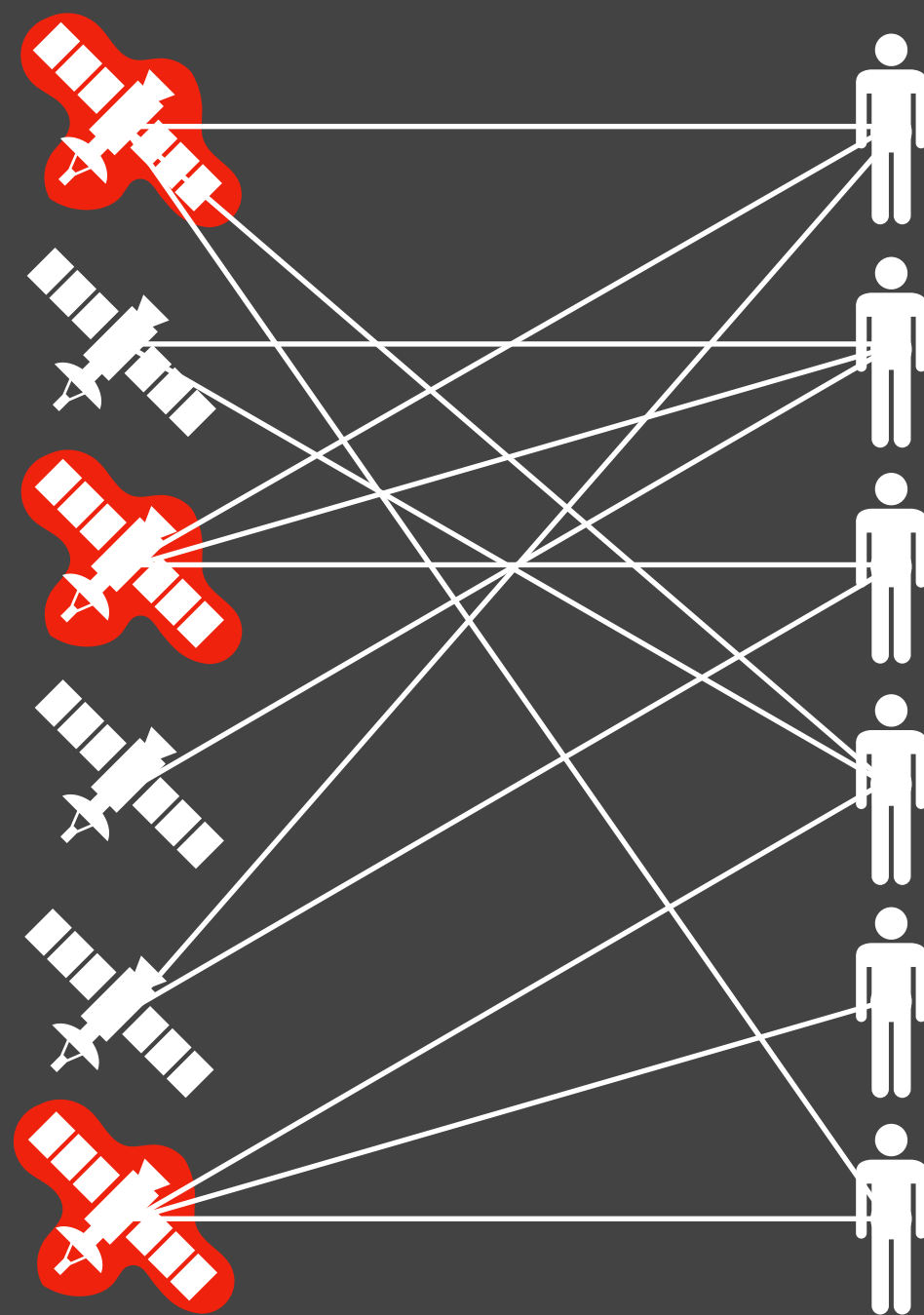
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My Work

Online

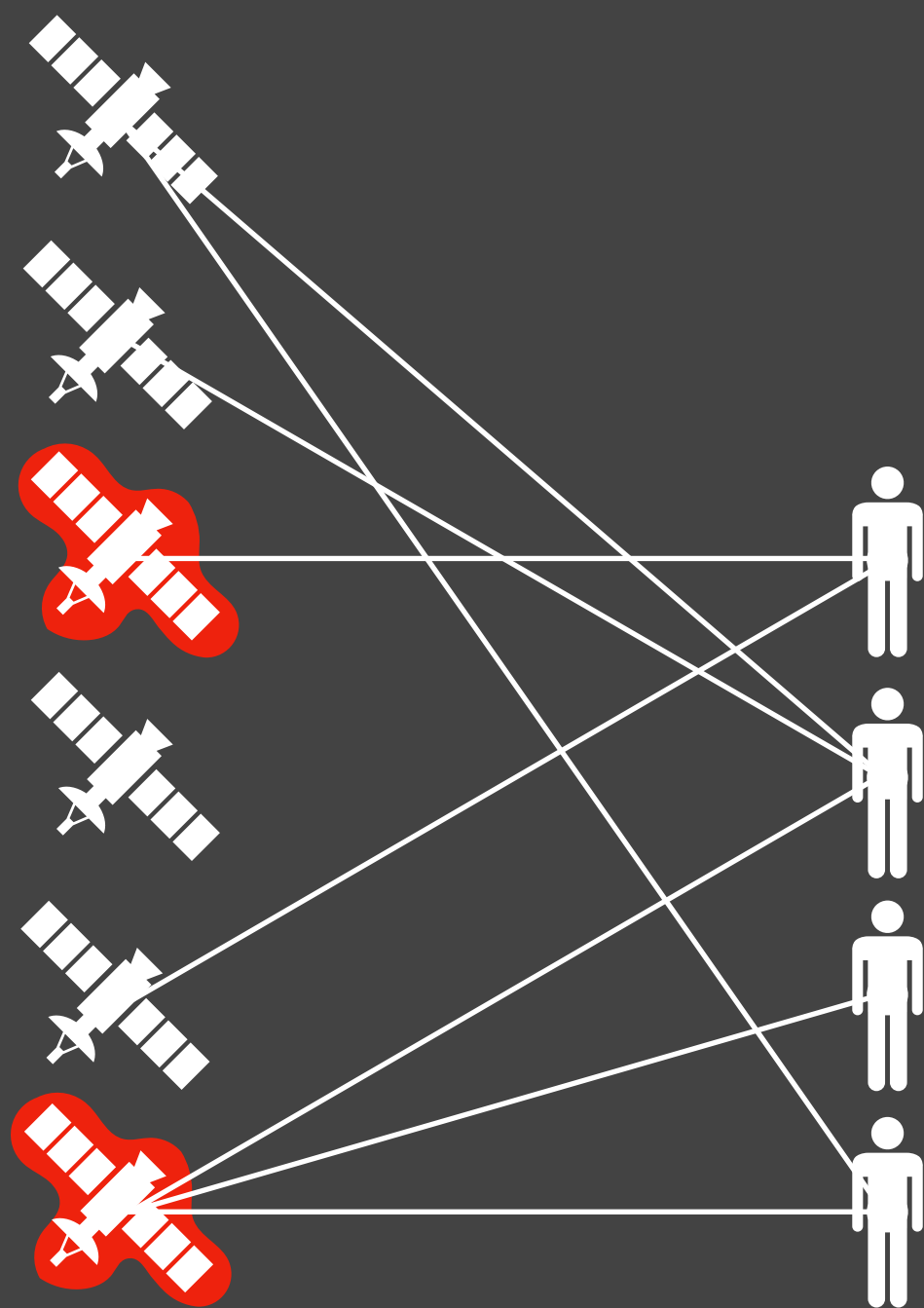
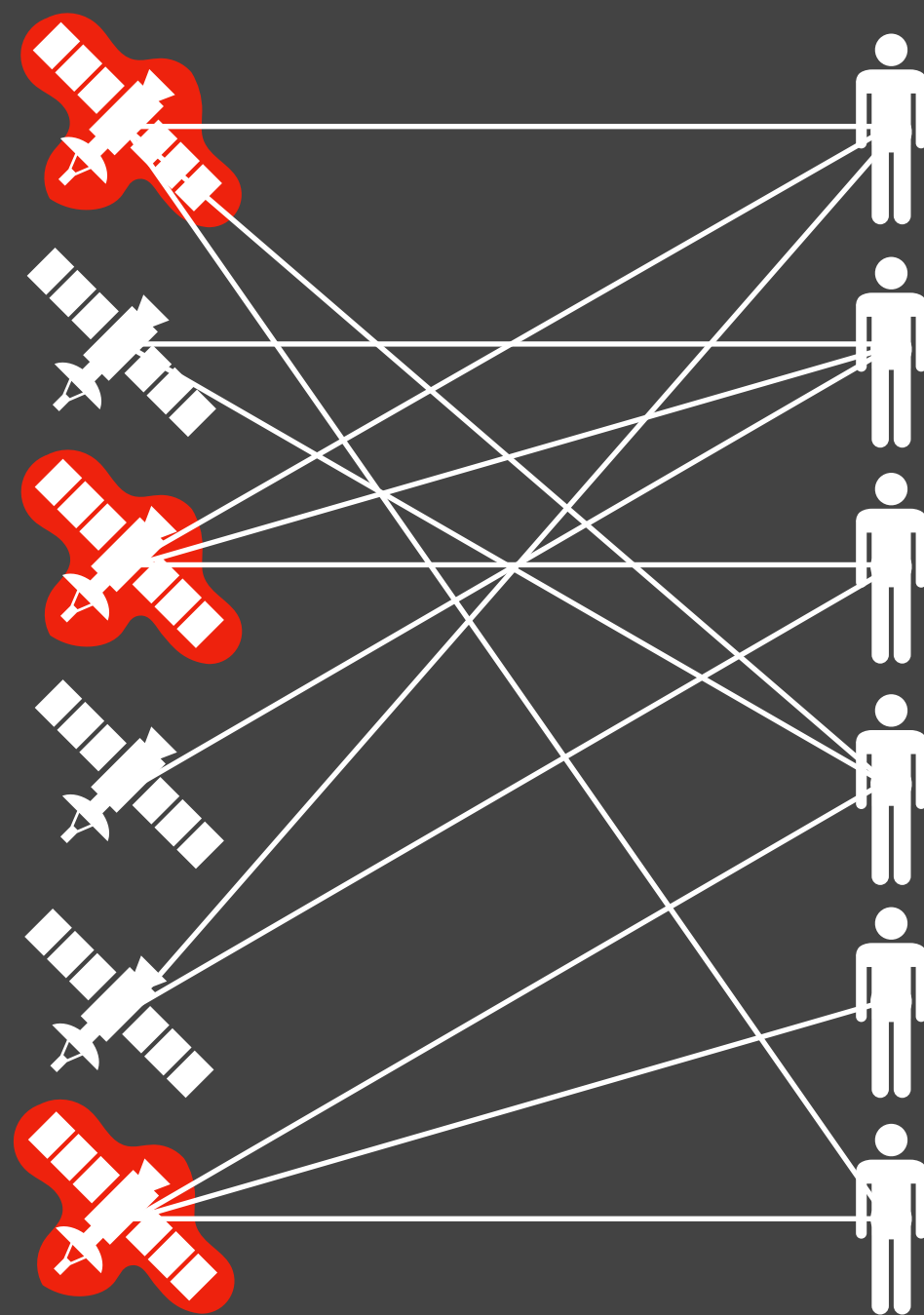
Dynamic

Streaming

No take-backs

Low movement

Low memory



My Work

Online

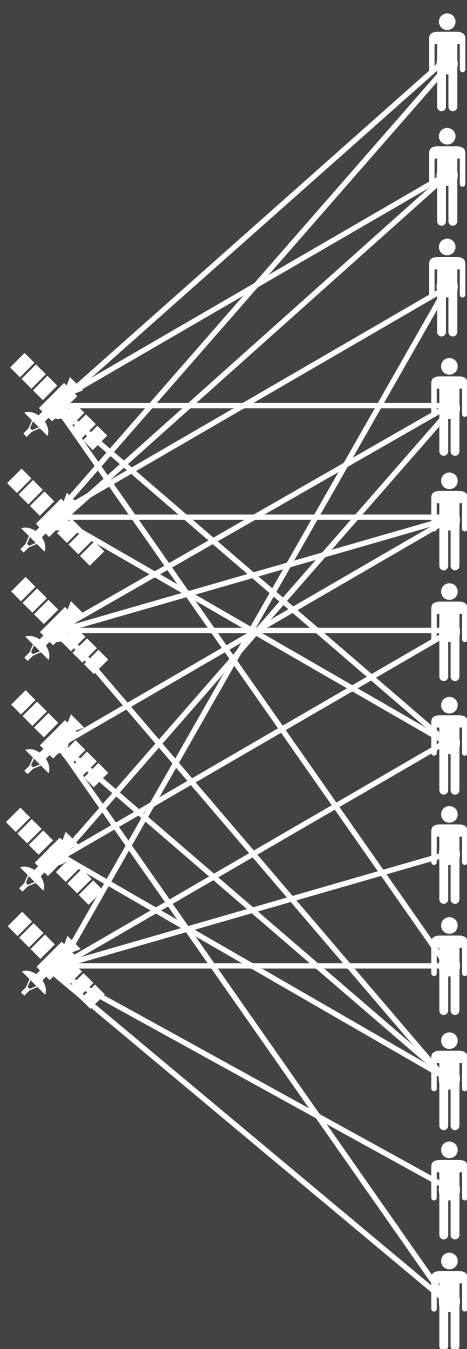
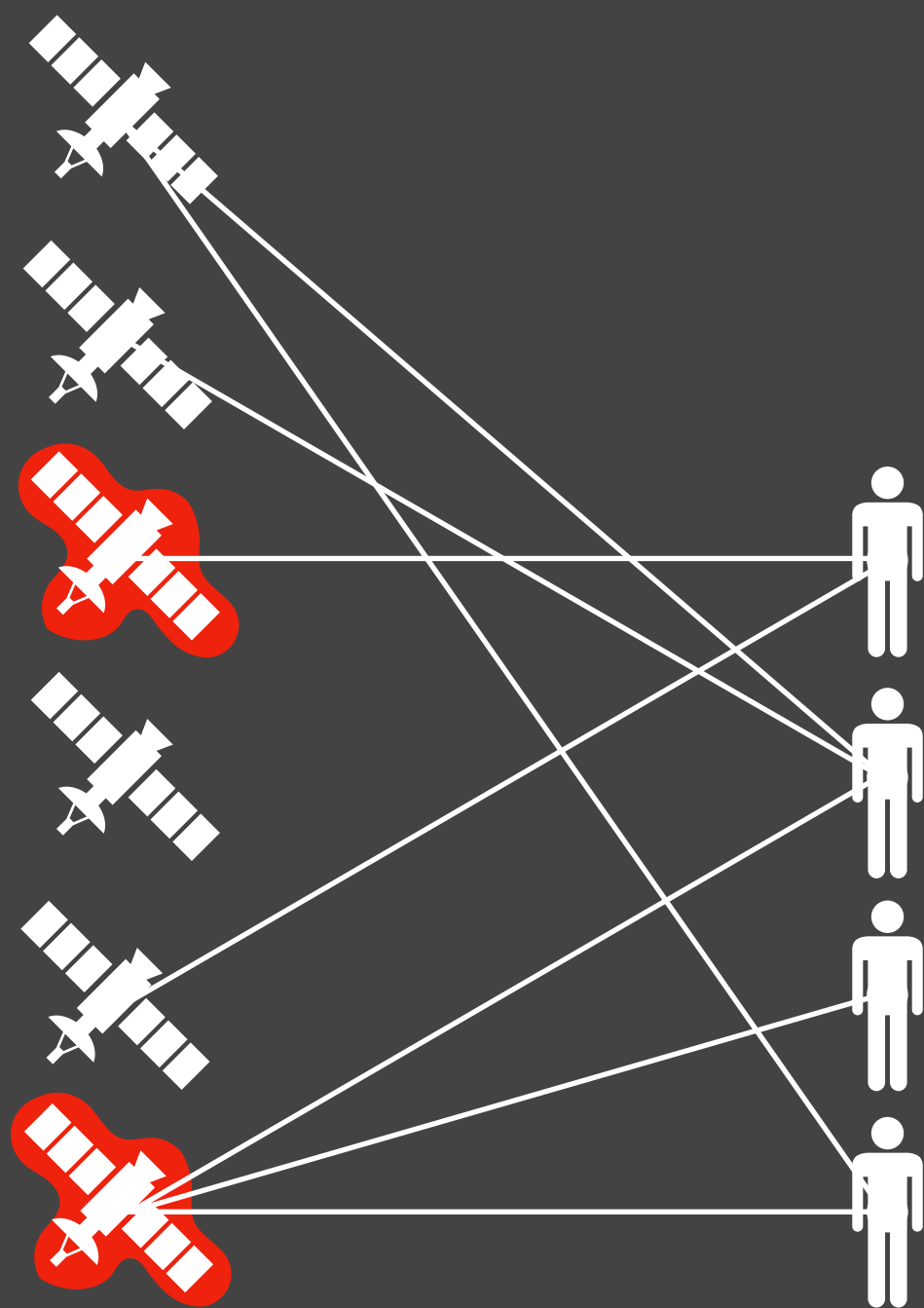
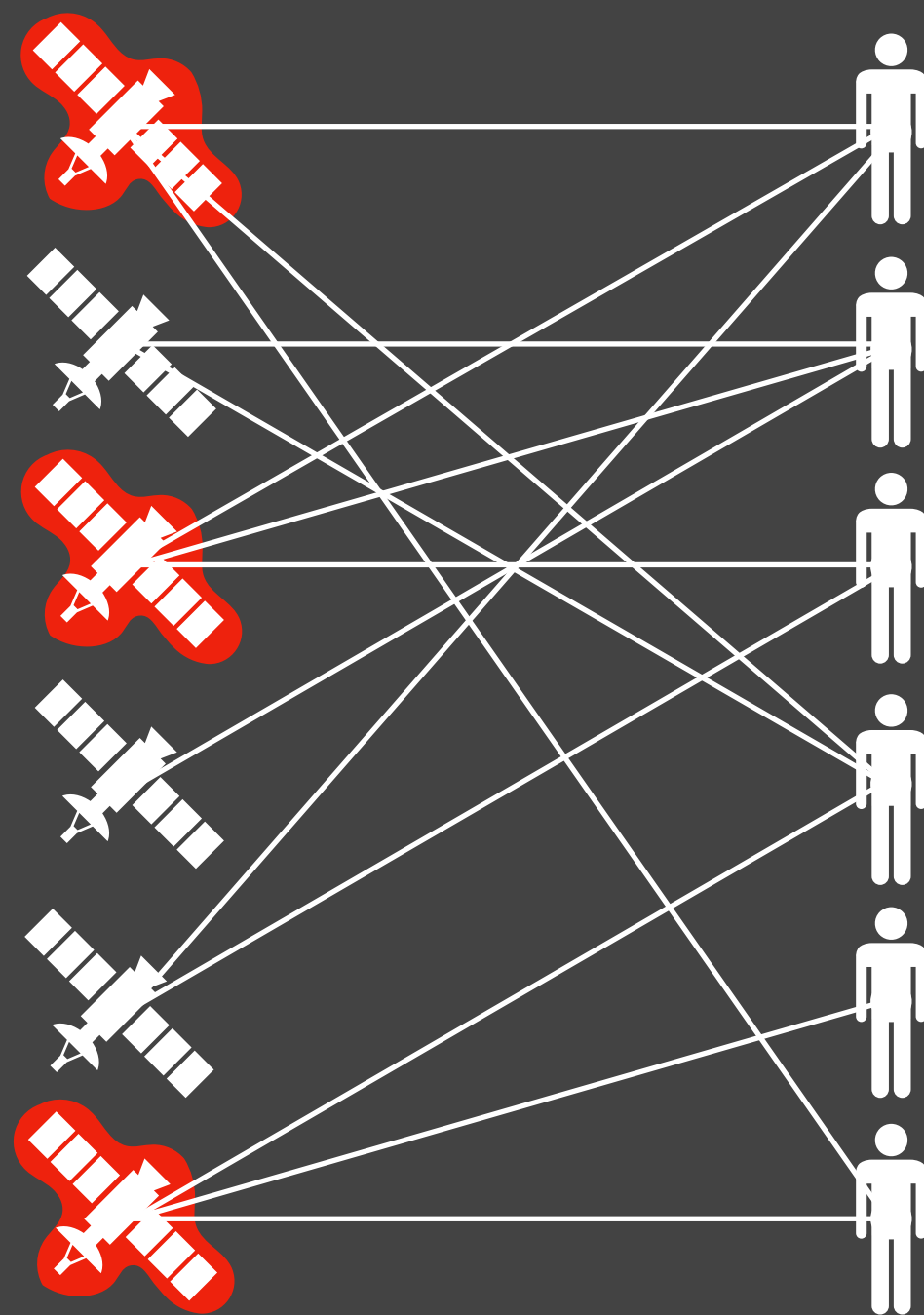
Dynamic

Streaming

No take-backs

Low movement

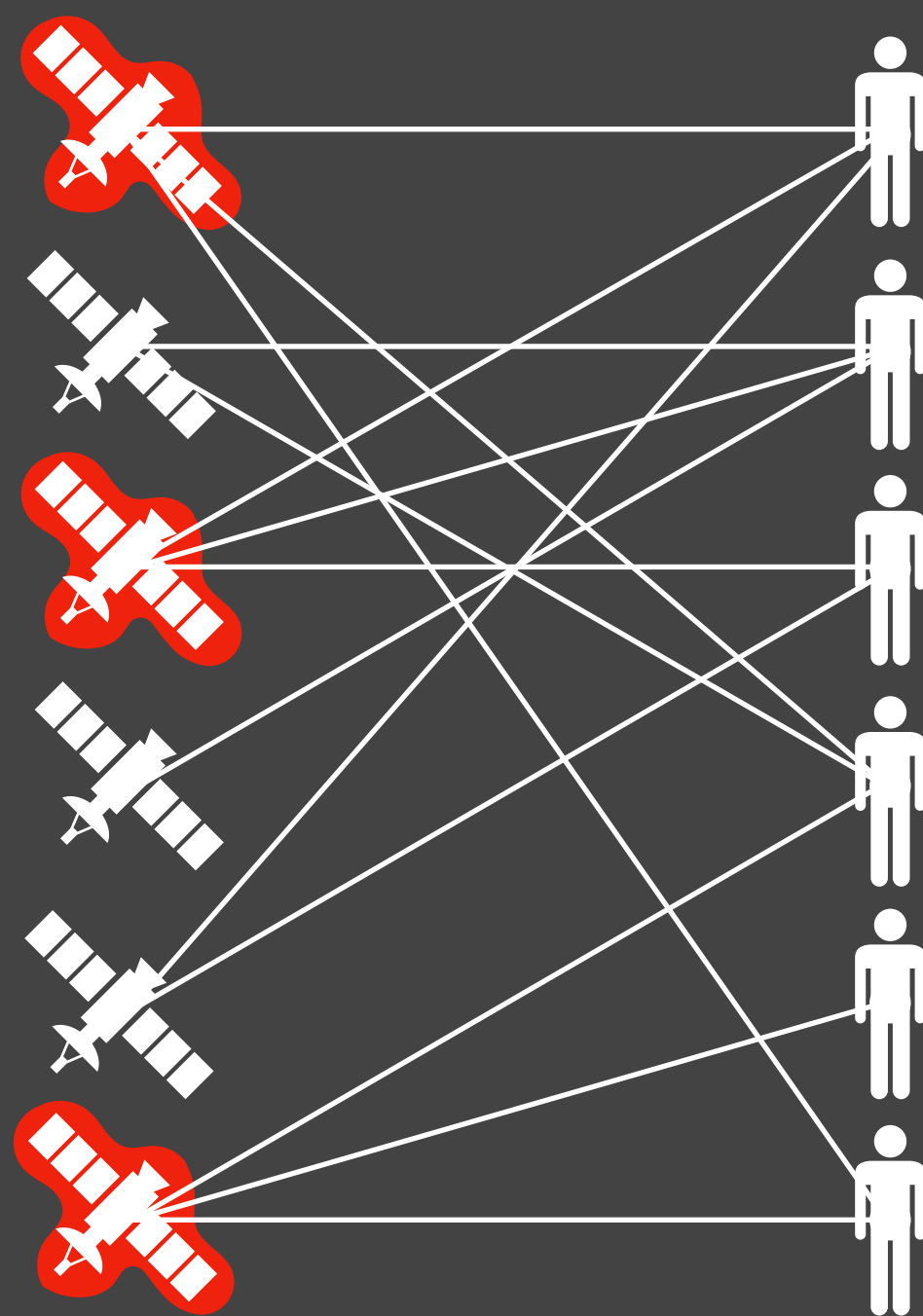
Low memory



My Work

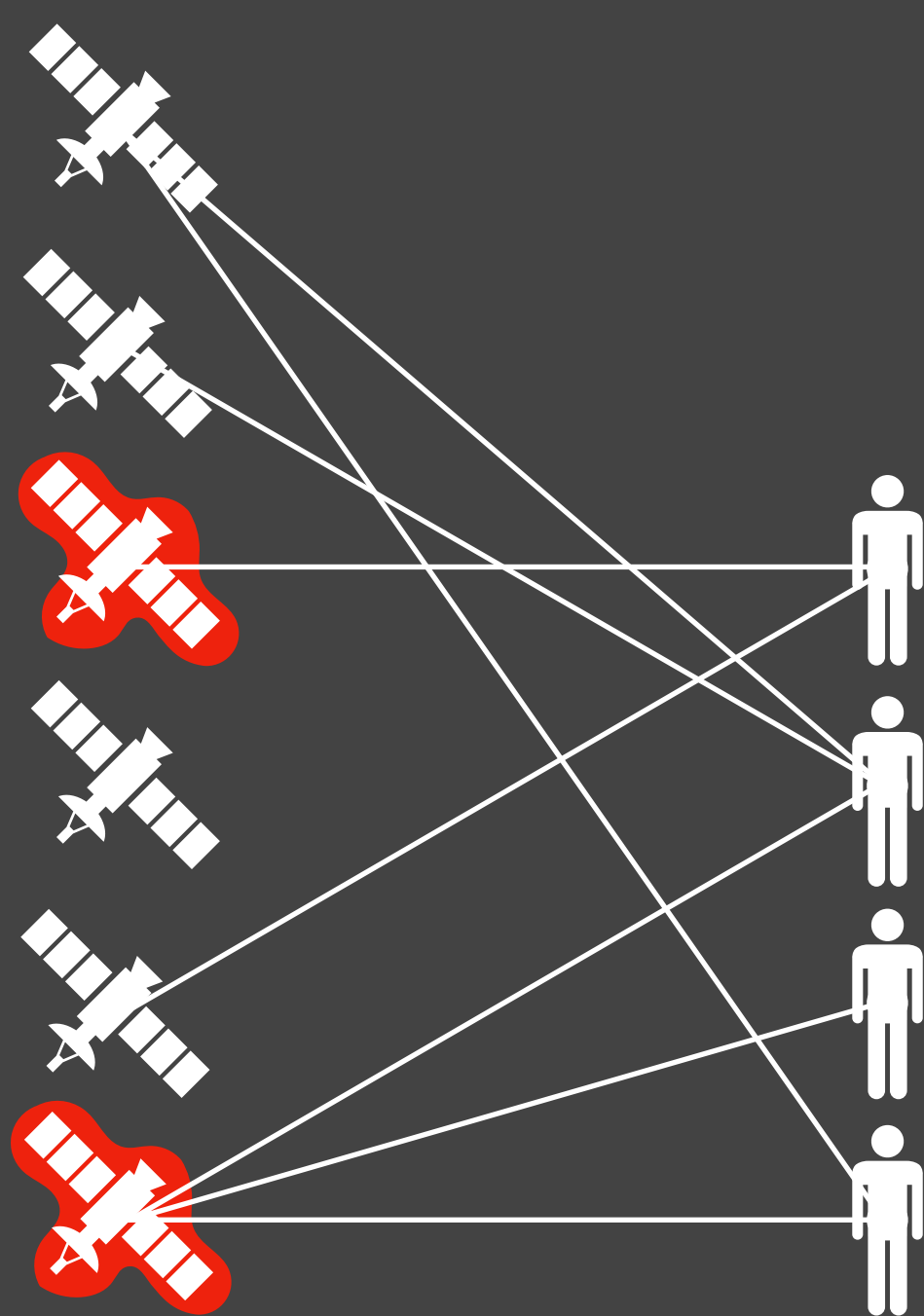
Online

No take-backs



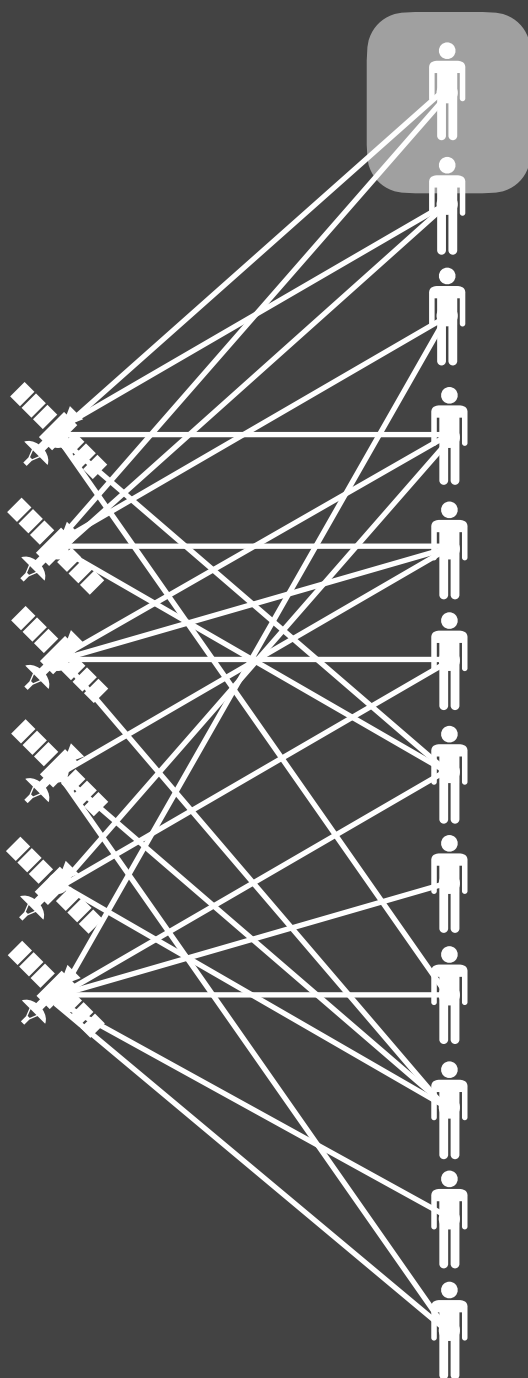
Dynamic

Low movement



Streaming

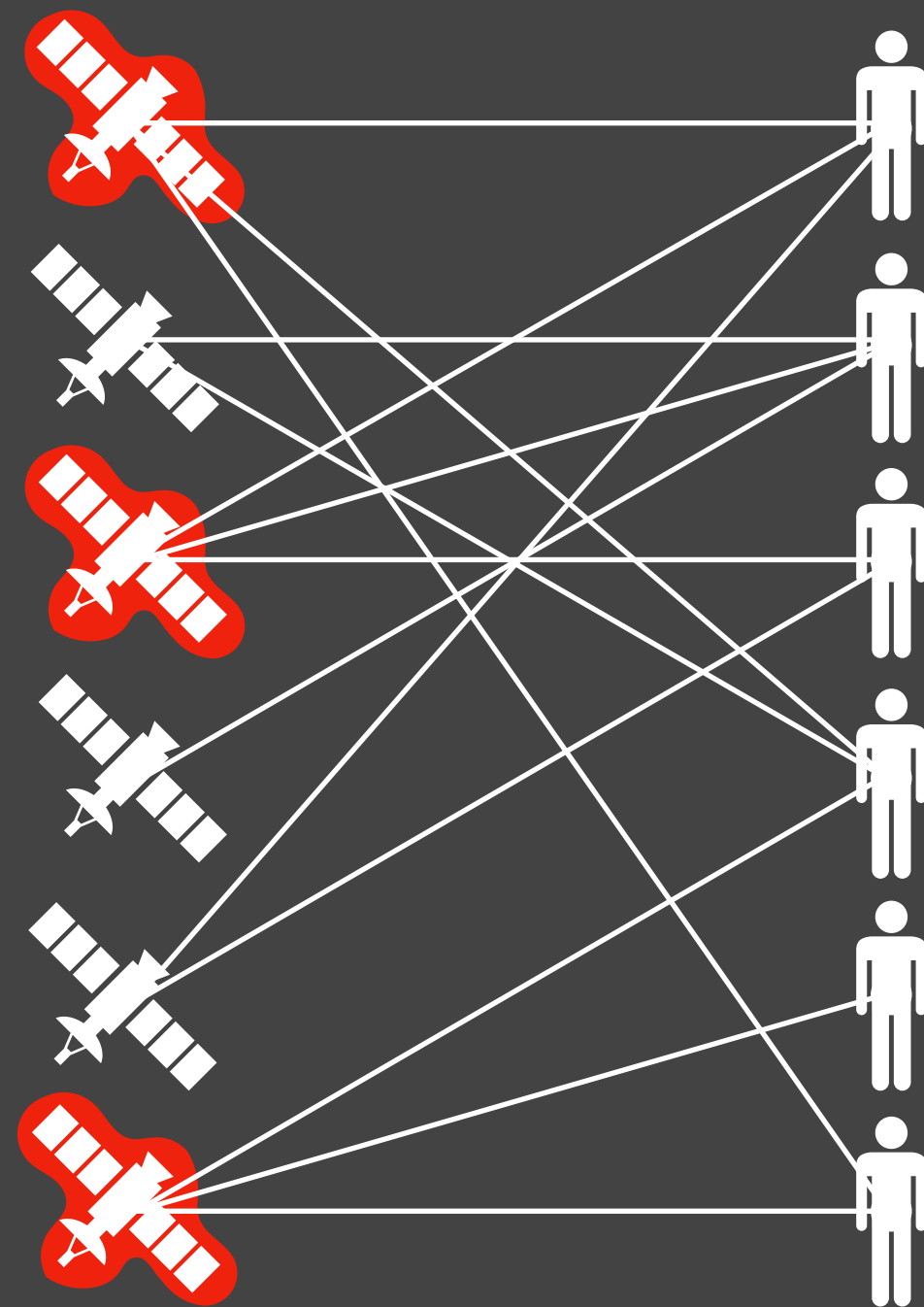
Low memory



My Work

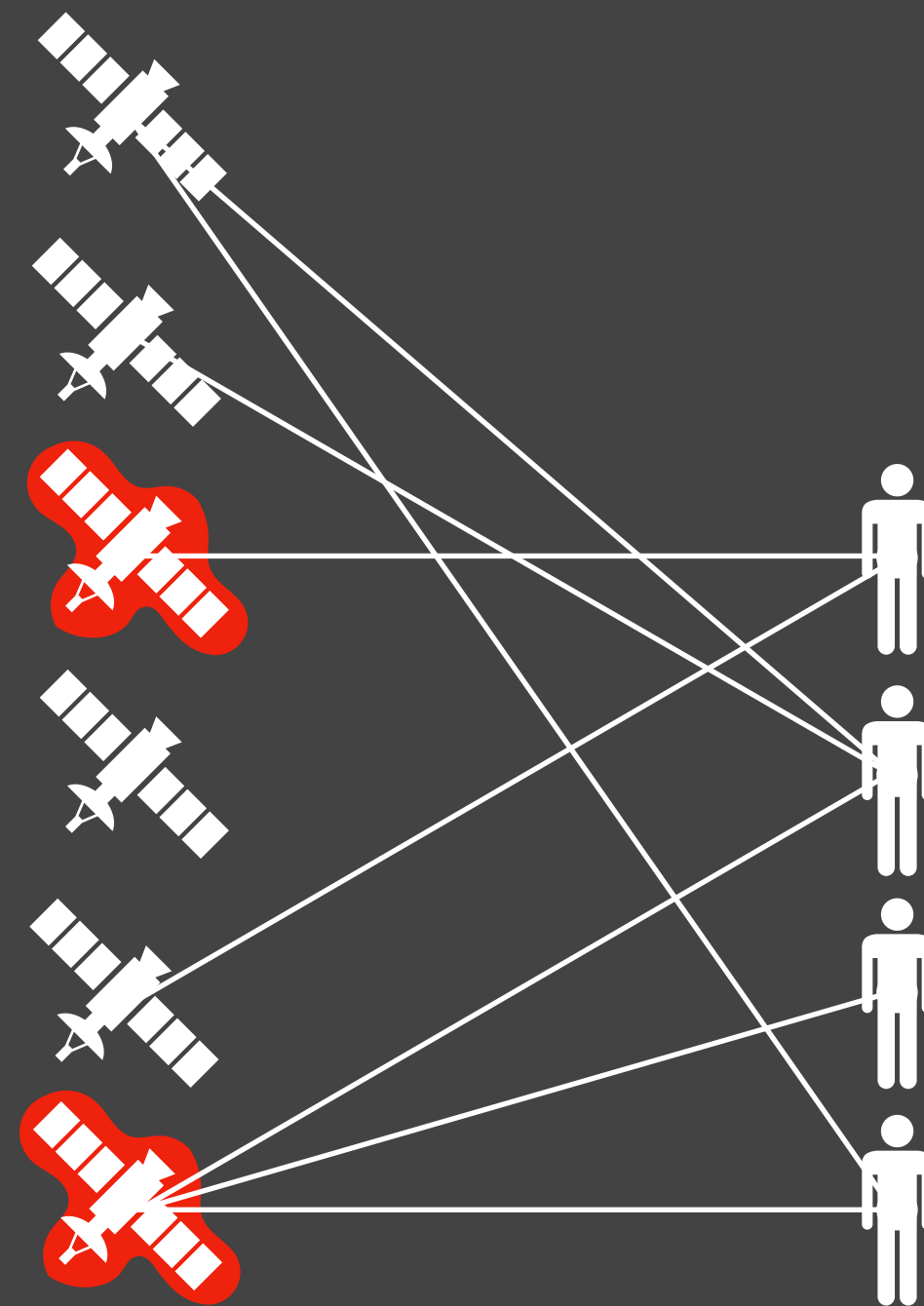
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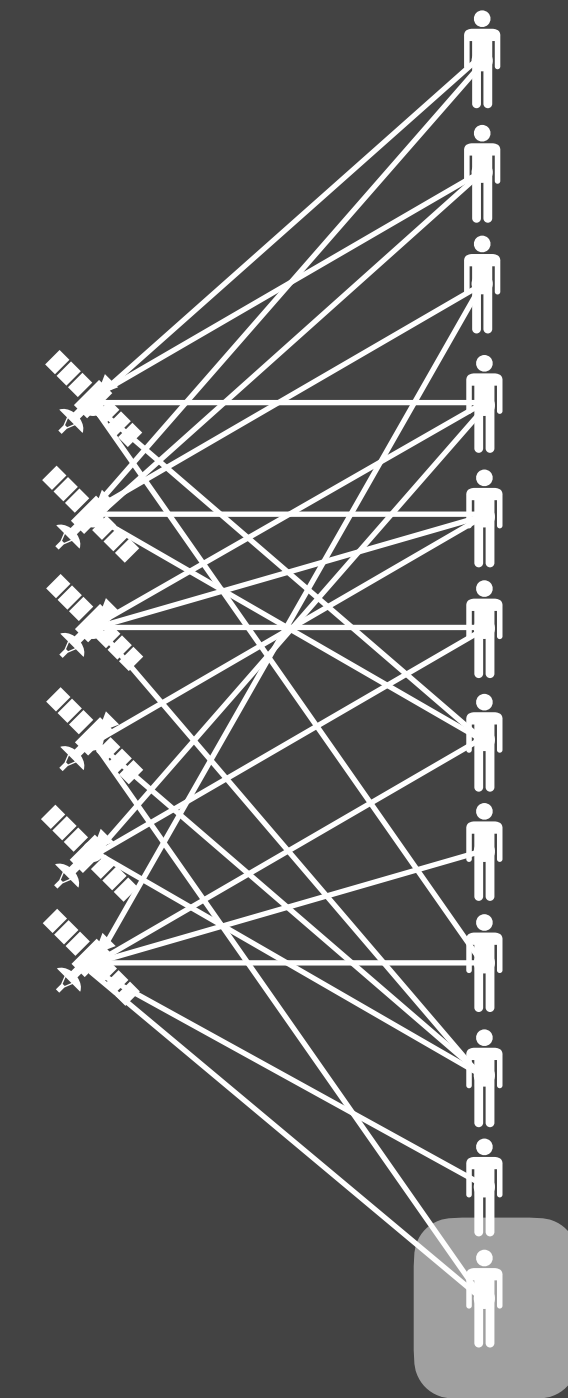
Dynamic

Low movement



Streaming

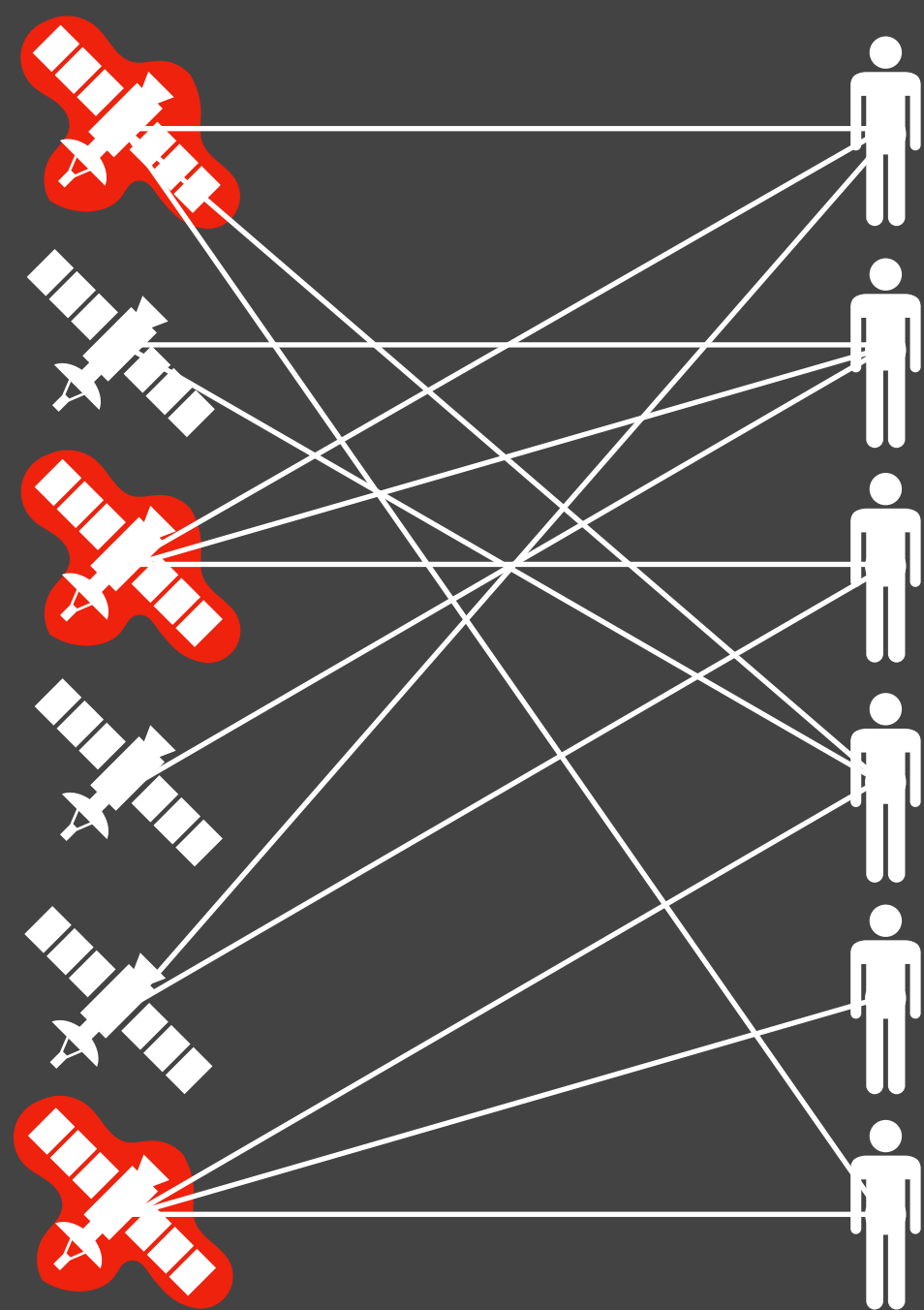
Low memory



My Work

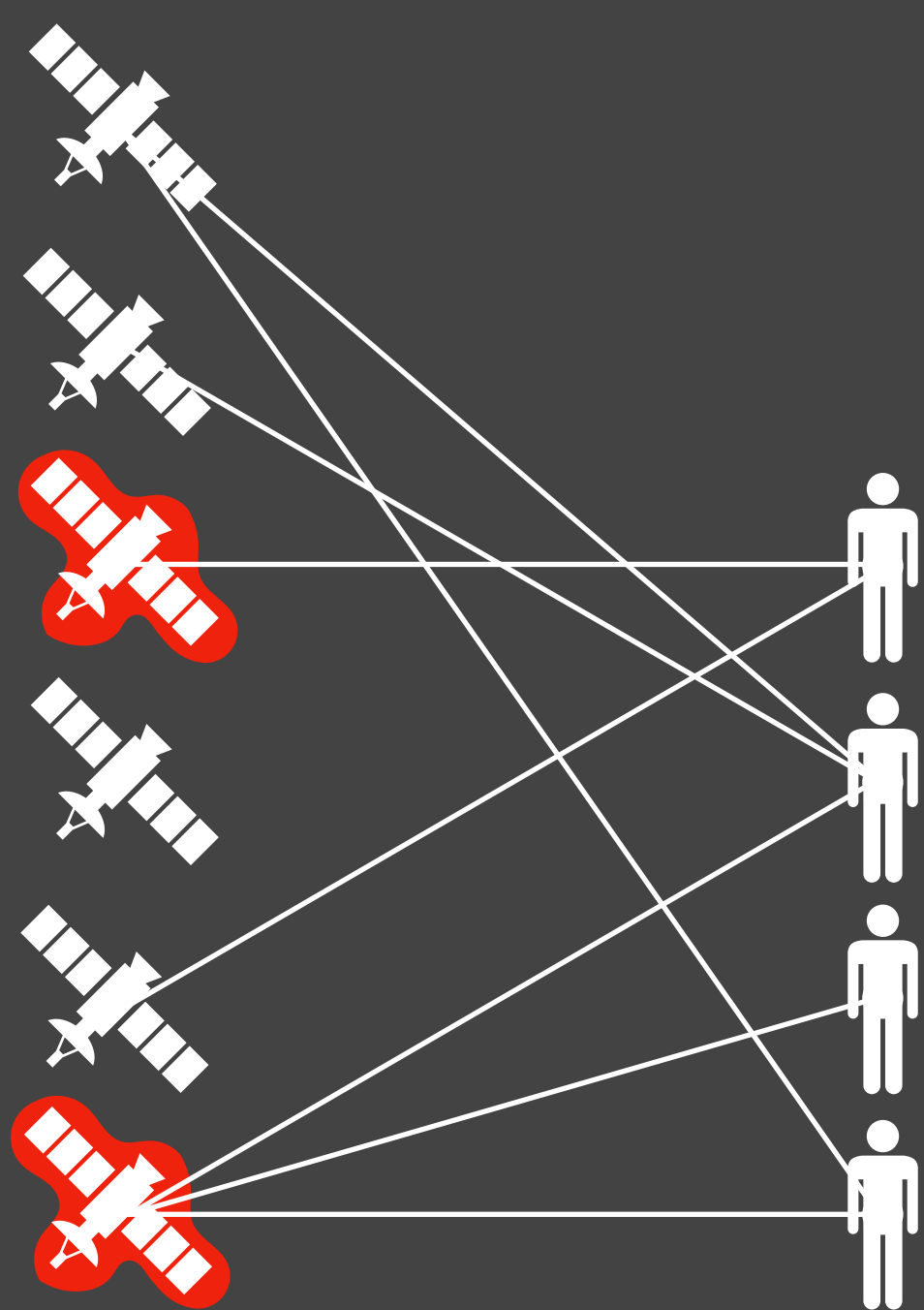
Online

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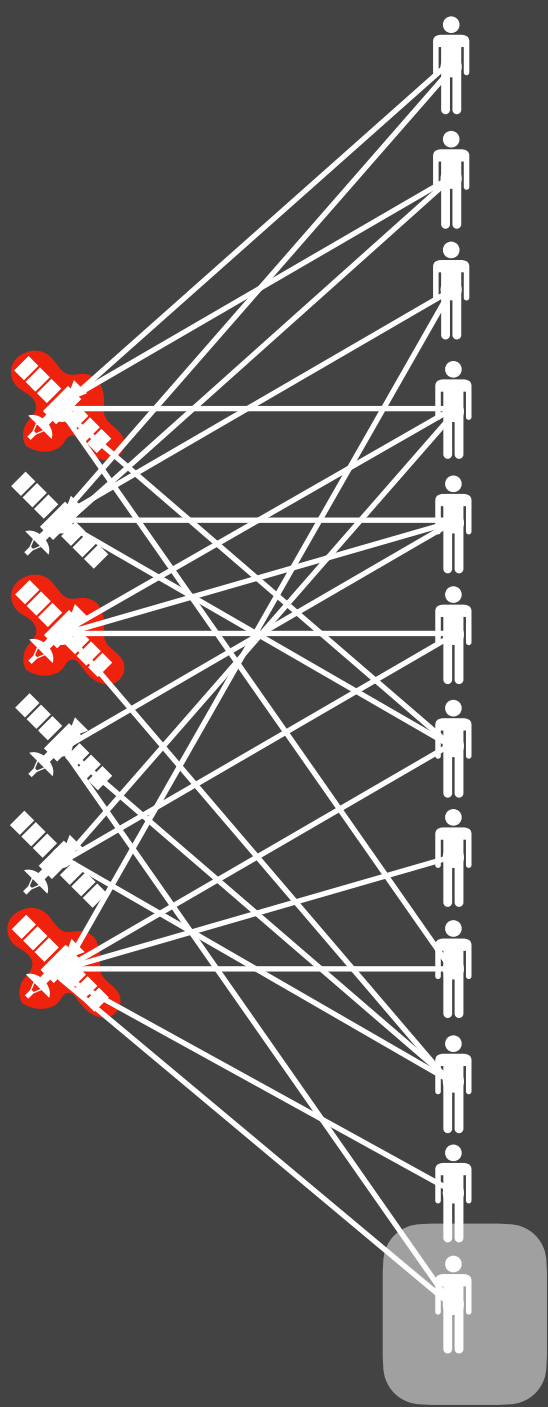
Dynamic

Low movement

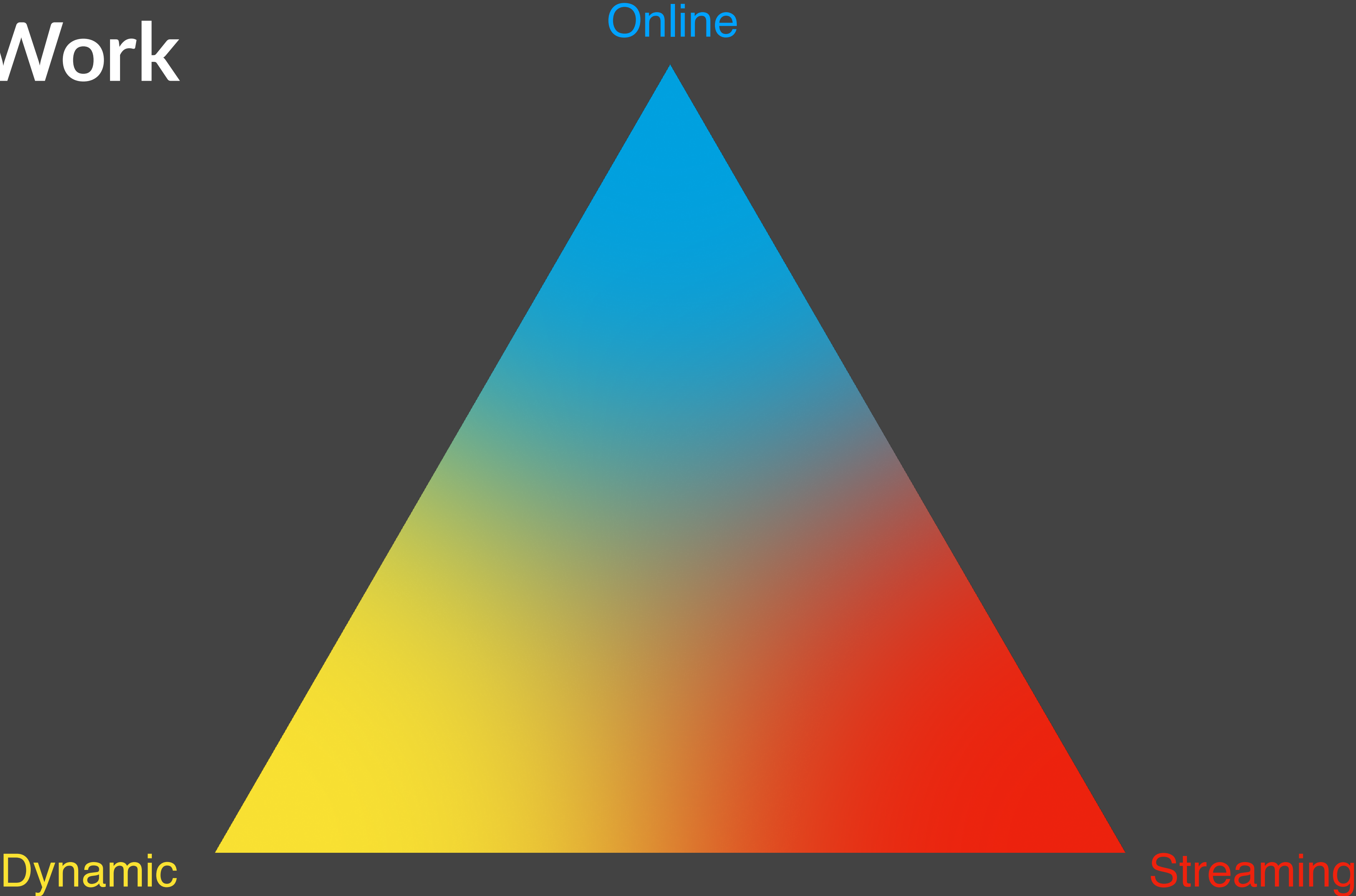


Streaming

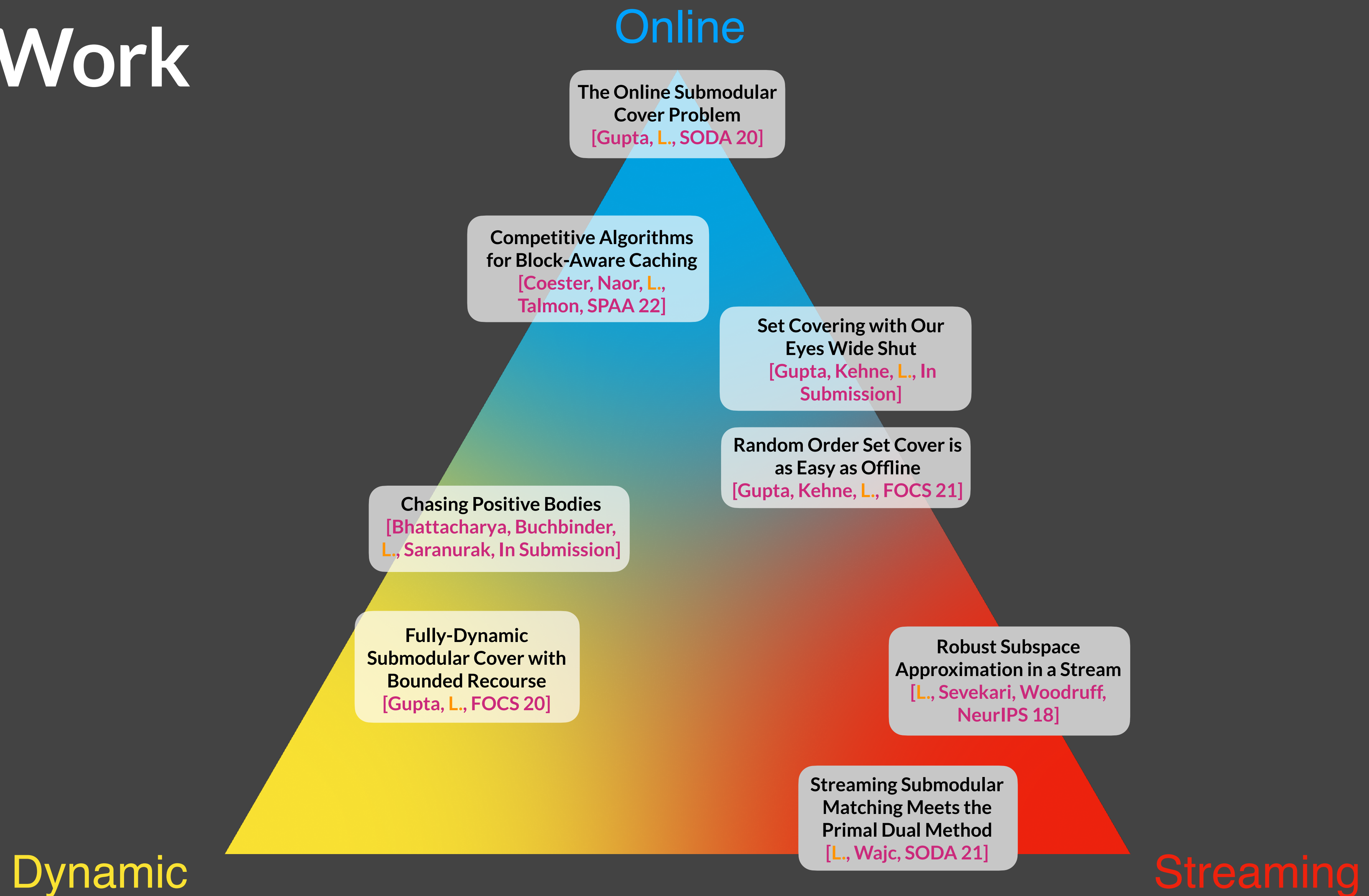
Low memory



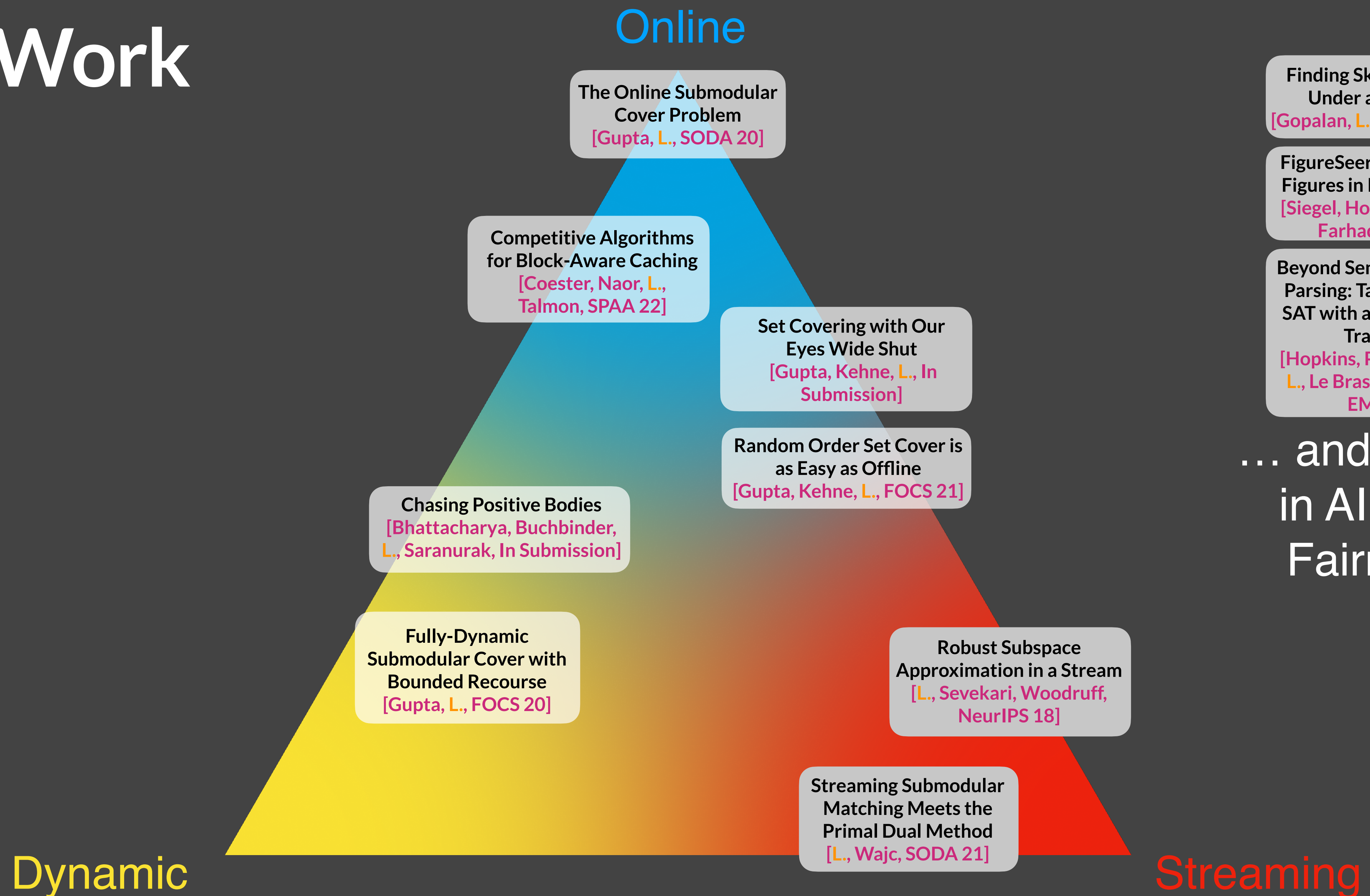
My Work



My Work



My Work



Finding Skewed Subcubes Under a Distribution
[Gopalan, L., Wieder, ITCS 20]

FigureSeer: Parsing Result-Figures in Research Papers
[Siegel, Horvitz, L., Divvala, Farhadi, ECCV 16]

Beyond Sentential Semantic Parsing: Tackling the Math SAT with a Cascade of Tree Transducers
[Hopkins, Petrscu-Prahova, L., Le Bras, Herrasti, Joshi, EMNLP 17]

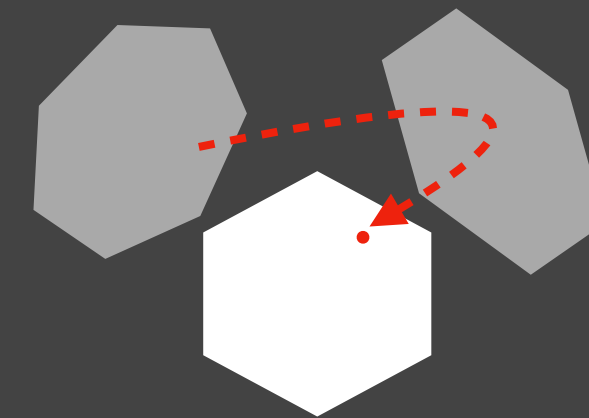
... and others
in AI, ML,
Fairness

Outline

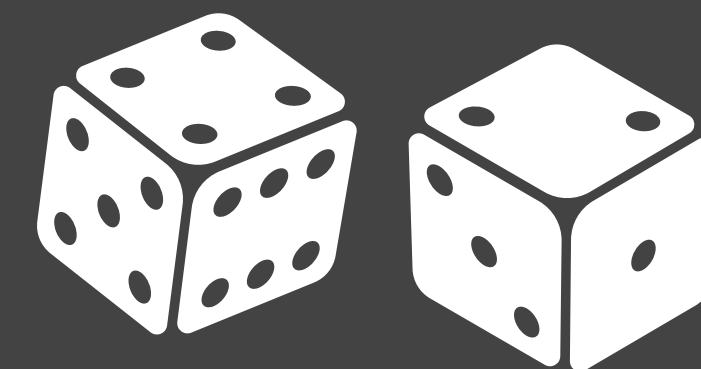
Theme I — Submodular Optimization

$$f(\text{🍕} \mid \text{🥕}) \geq f(\text{🍕} \mid \text{🥕}, \text{🍩})$$

Theme II — Stable Algorithms



Theme III — Beyond Worst-Case Analysis



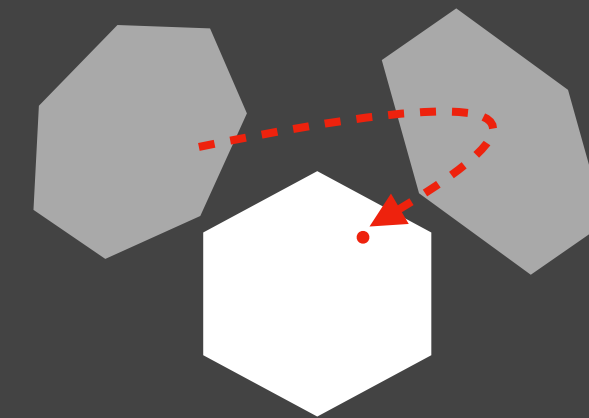
Conclusion

Outline

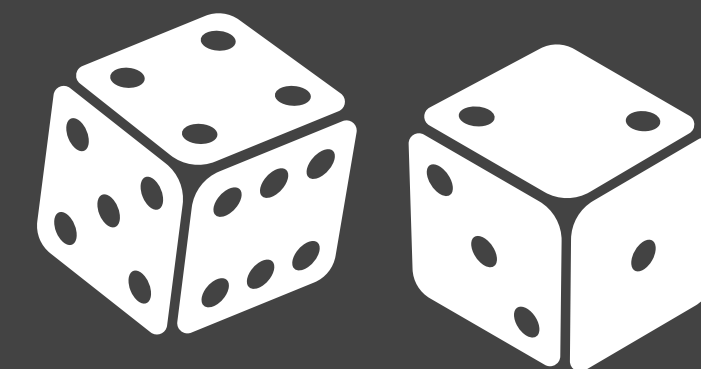
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Conclusion

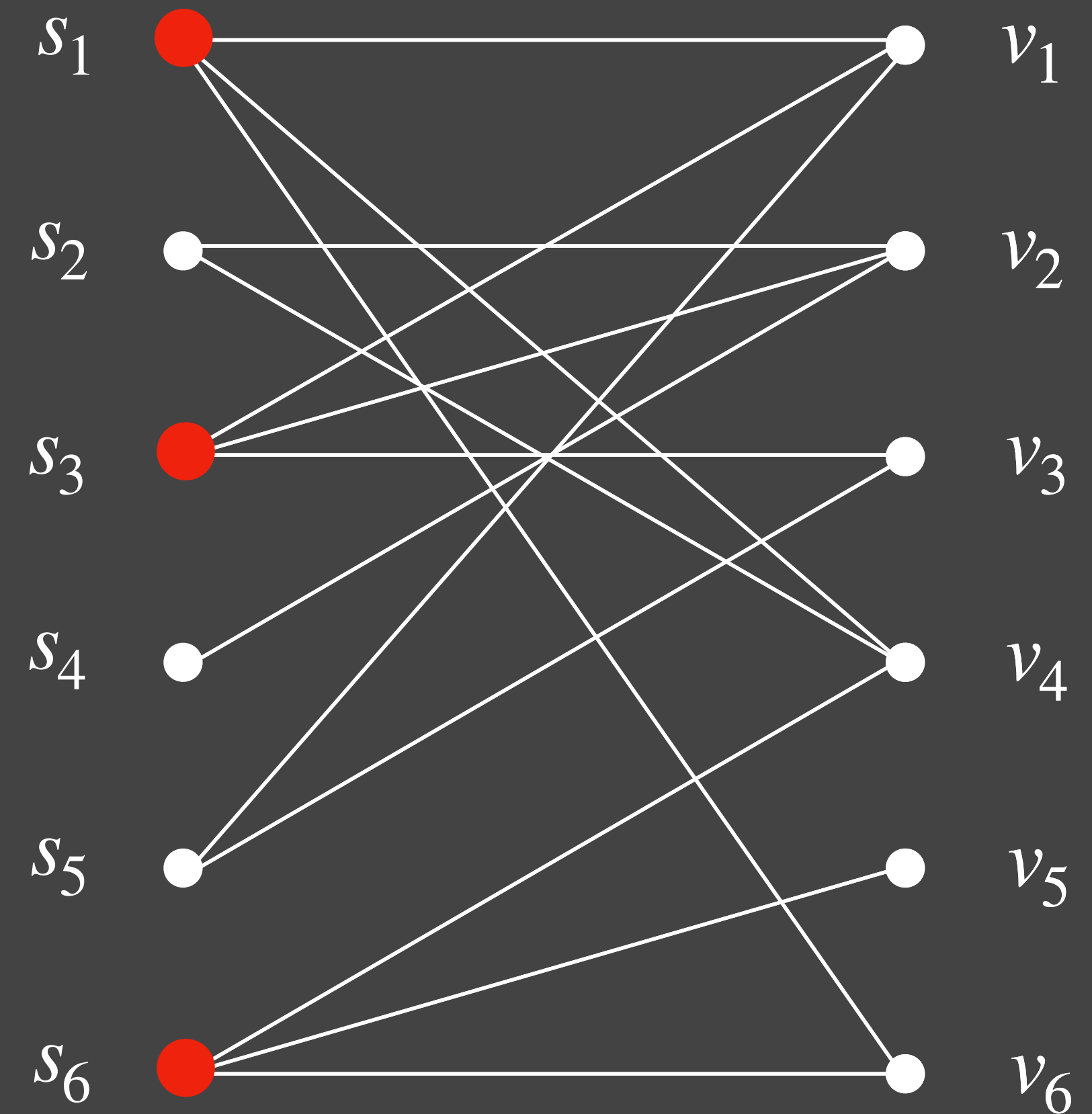
Theme I — Submodular Optimization

Beyond Set Cover

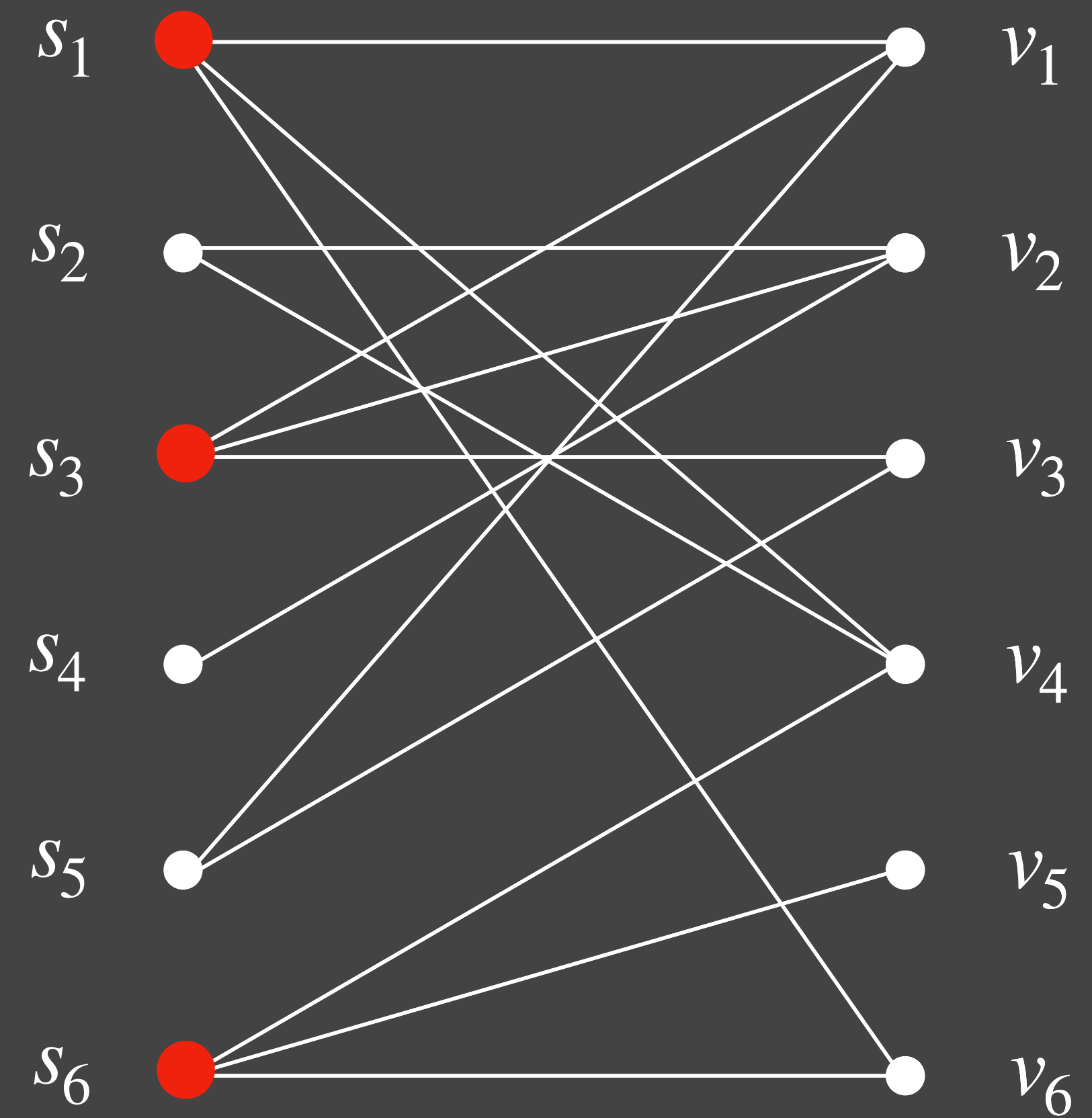
Q: What **general** classes of optimization problems can we solve **online**?

Beyond Set Cover

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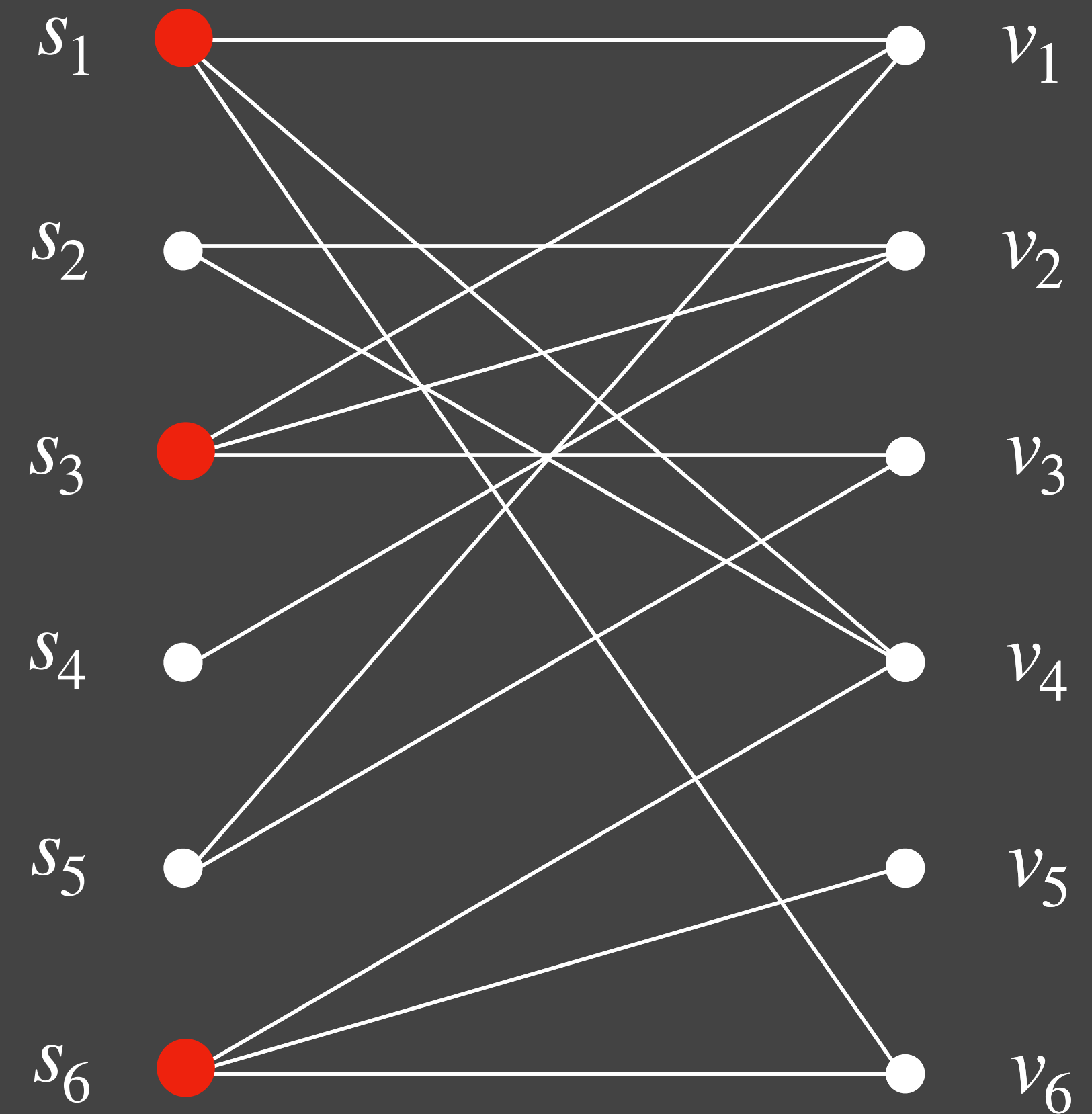


Abstracting the Problem



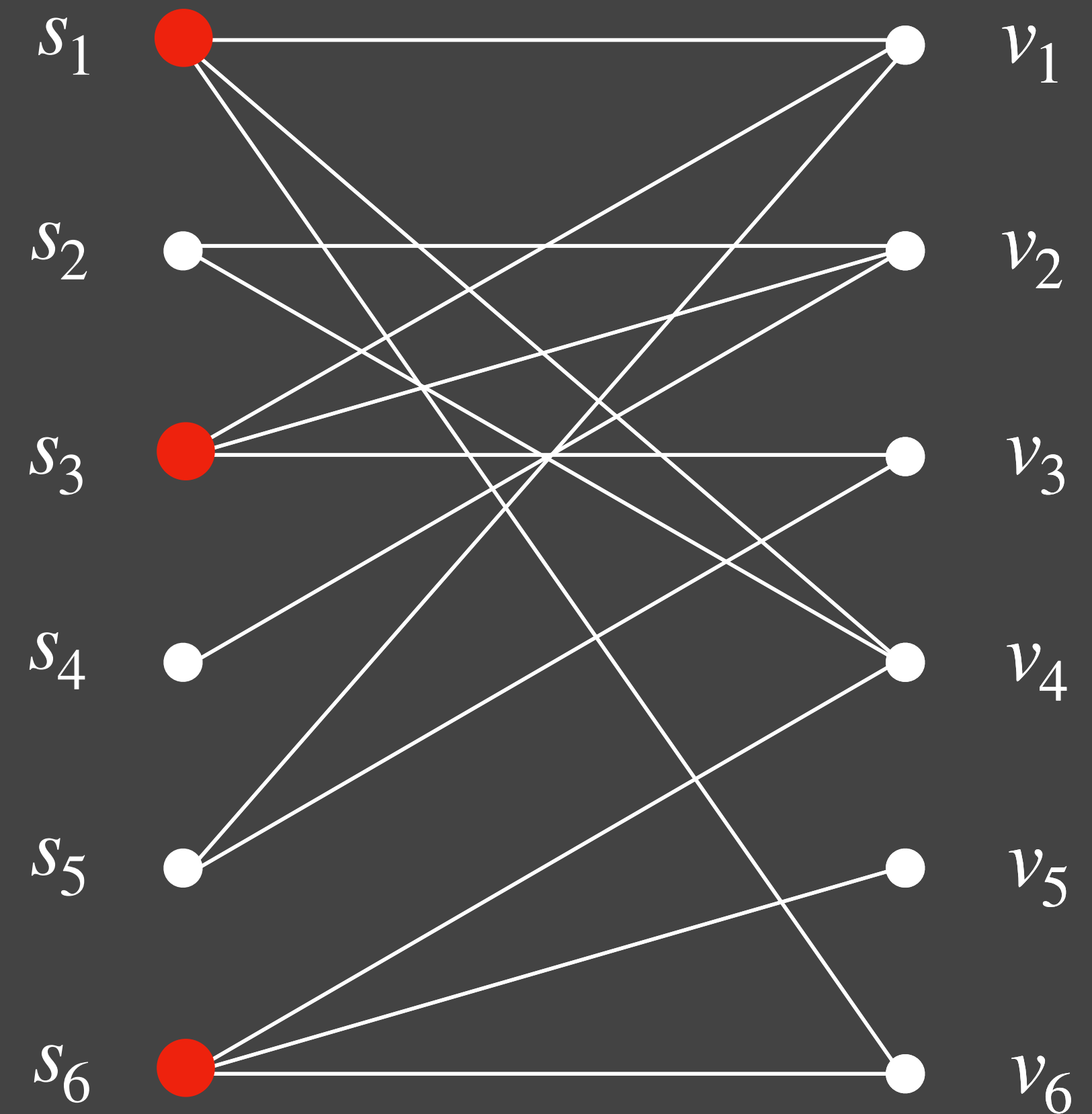
Abstracting the Problem

• Universe of choices: $\mathcal{S} = \{s_1, s_2, \dots, s_n\}$



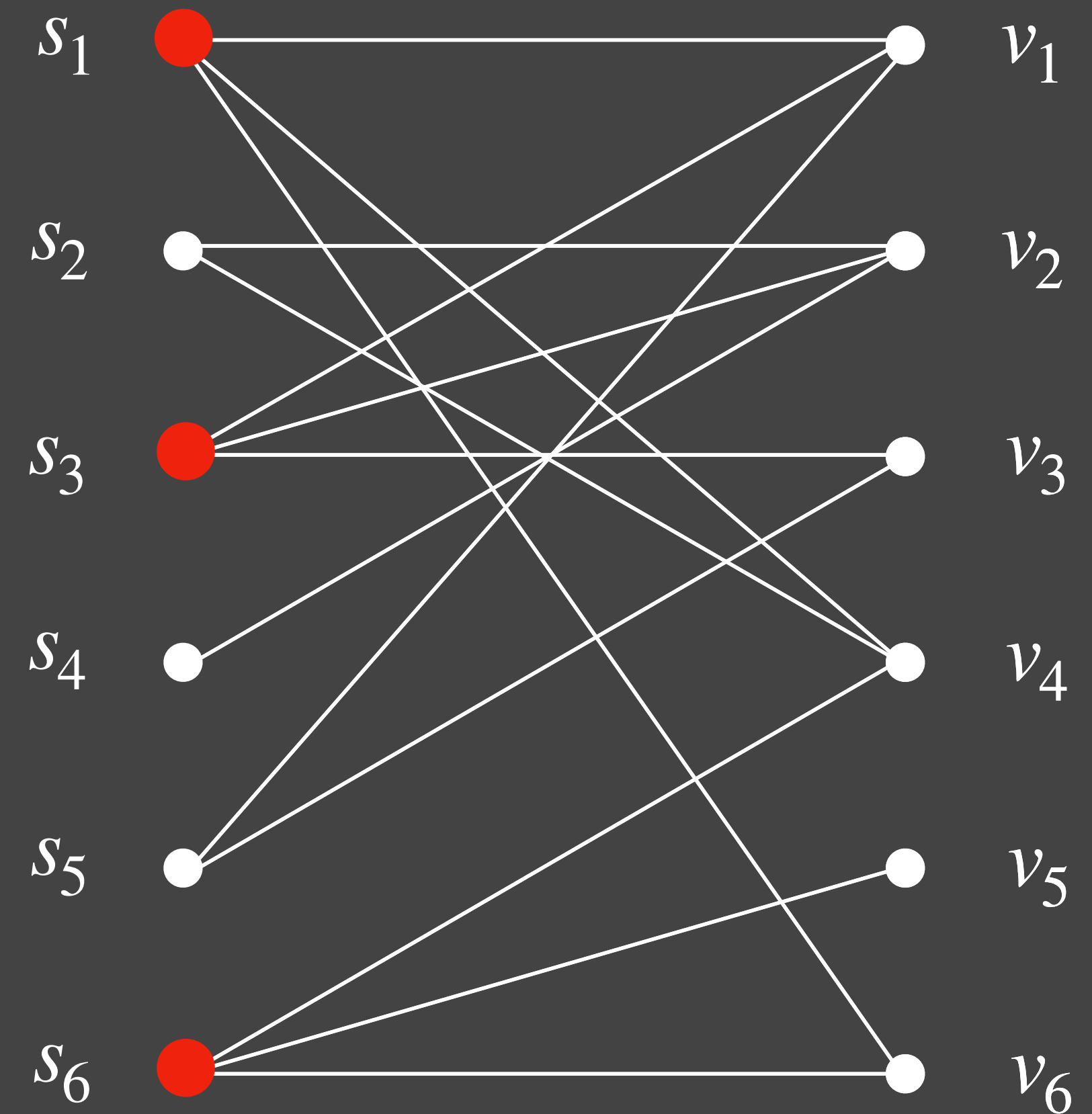
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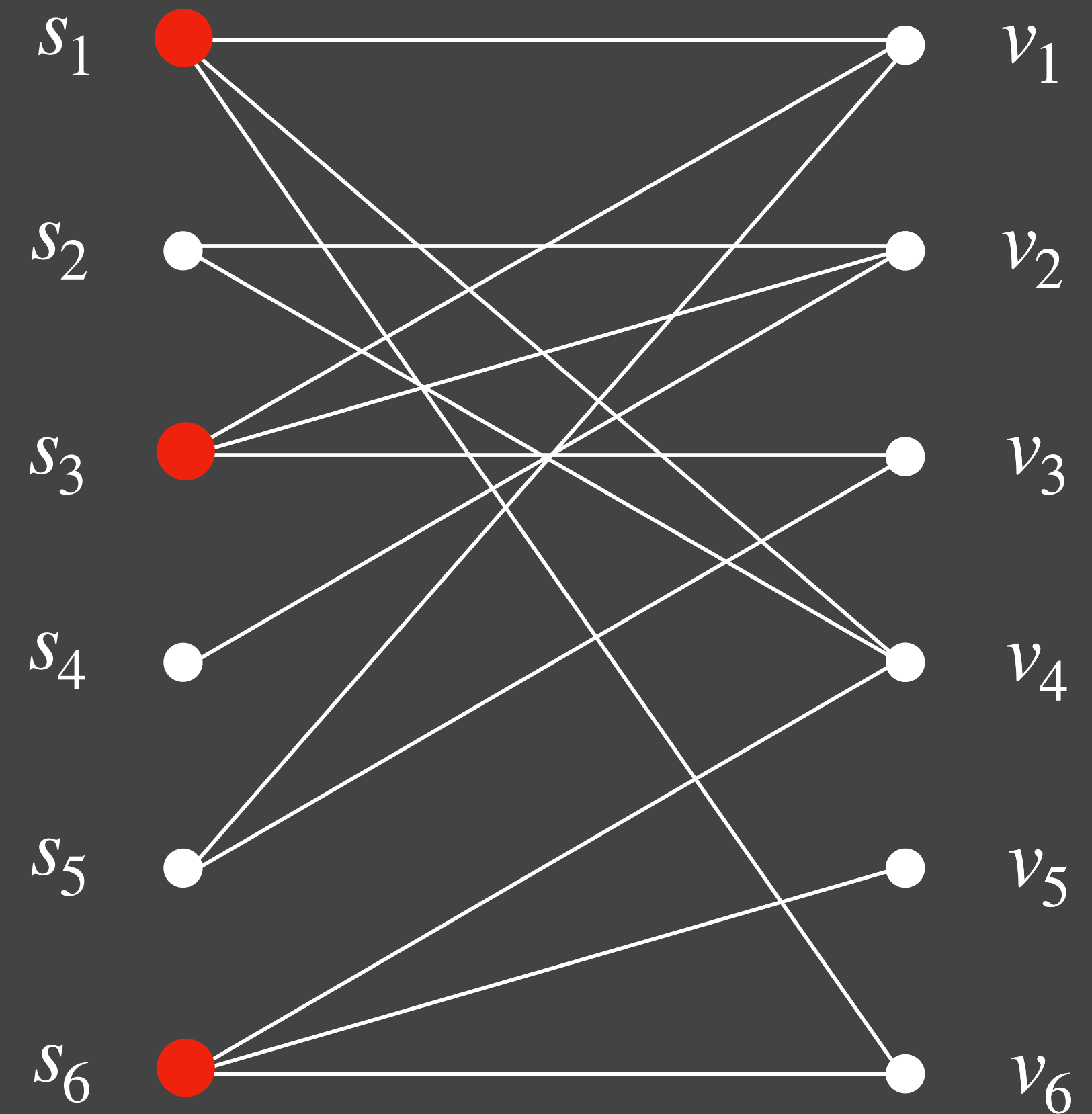
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- Cost: $c(S)$



Abstracting the Problem

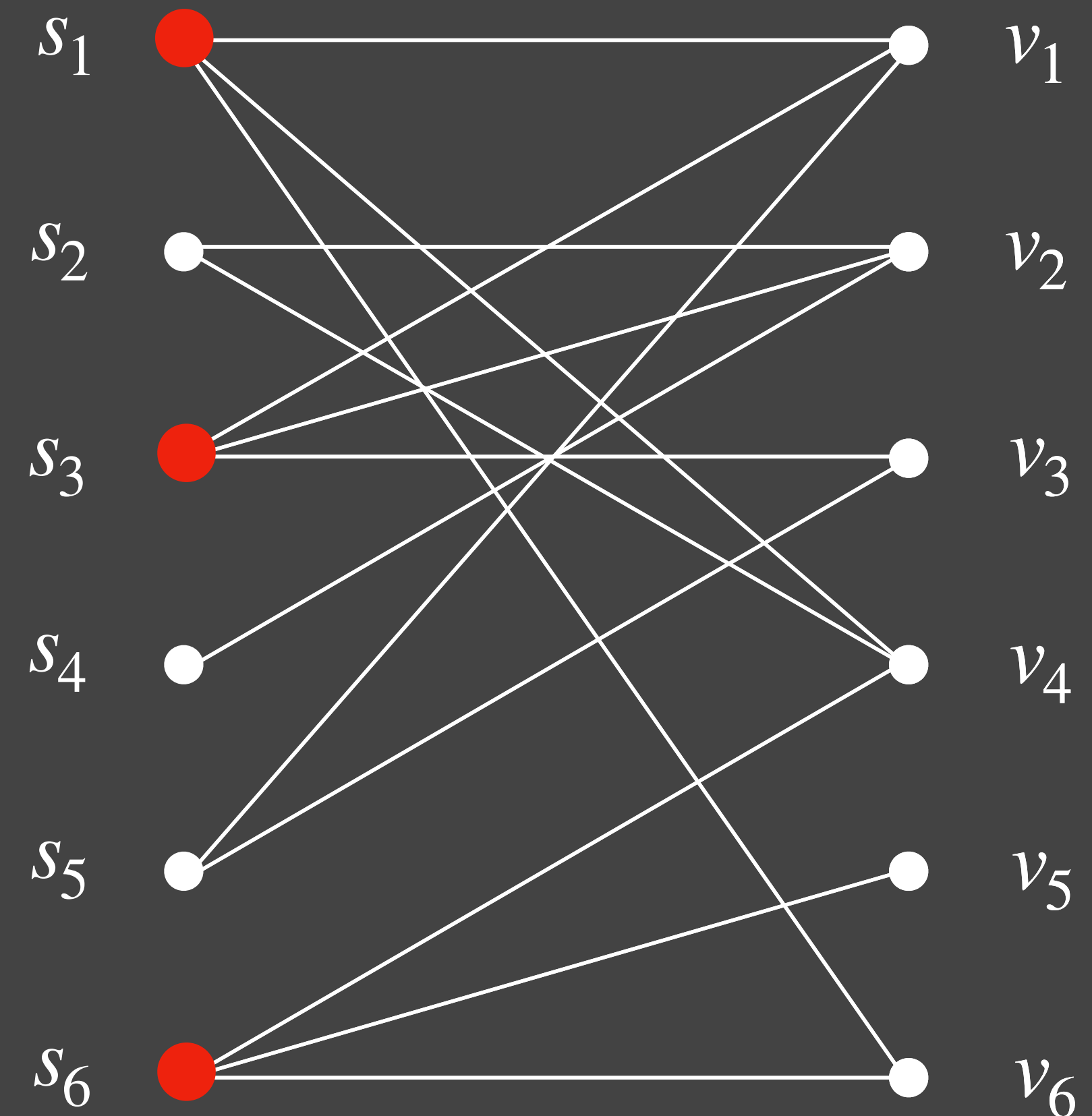
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Want **min cost** solution with **max coverage**!



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$$f(S) \geq n$$

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$f: 2^{\mathcal{N}} \rightarrow \mathbb{R}$ is **monotone**, **nonnegative** and **submodular**.

Abstracting the Problem

a.k.a. **Submodular Cover** [Wolsey 82]

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We will port this [online](#)!

$f: 2^{\mathcal{N}} \rightarrow \mathbb{R}$ is **monotone**, **nonnegative** and **submodular**.

Submodularity

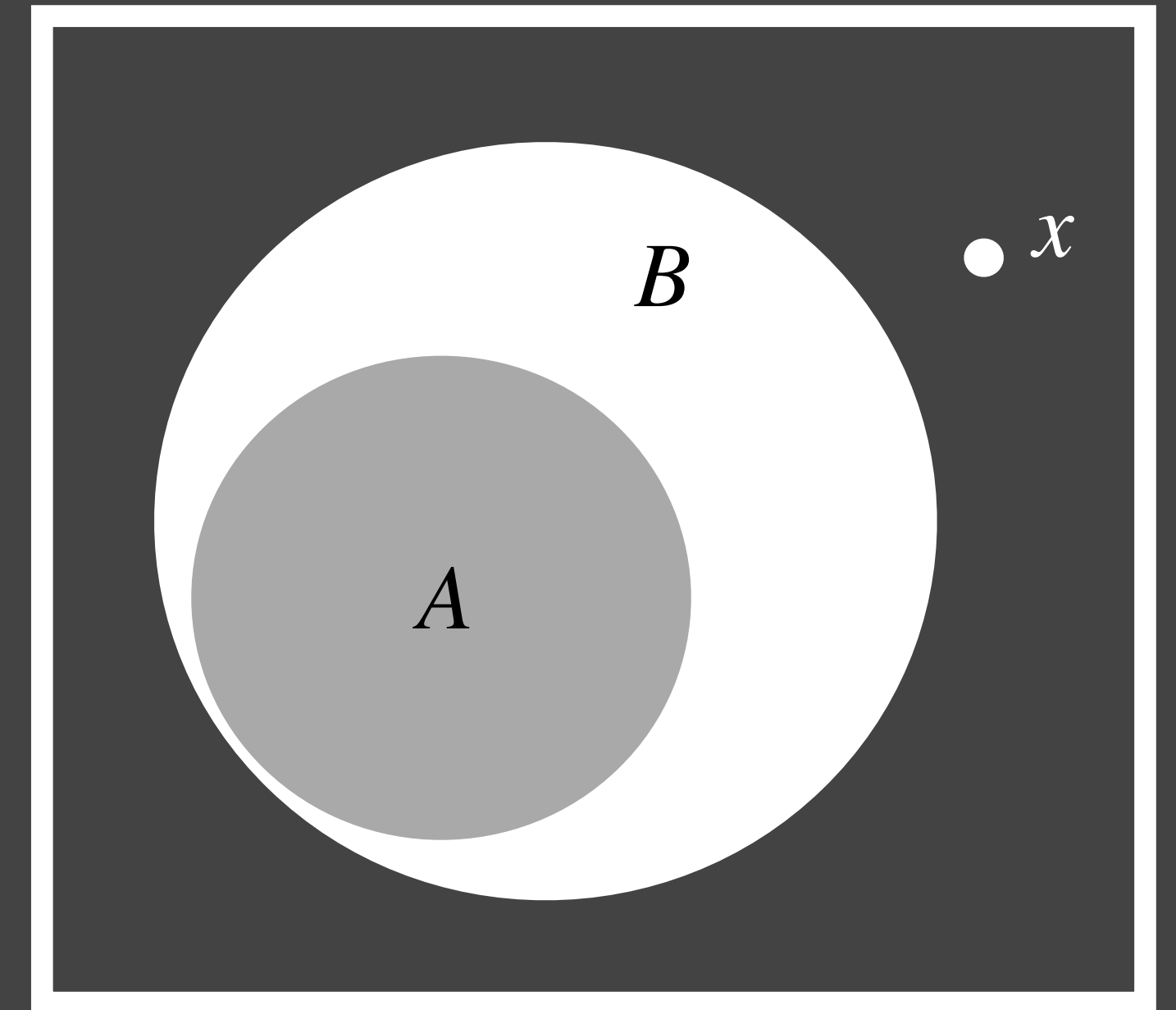
Submodularity

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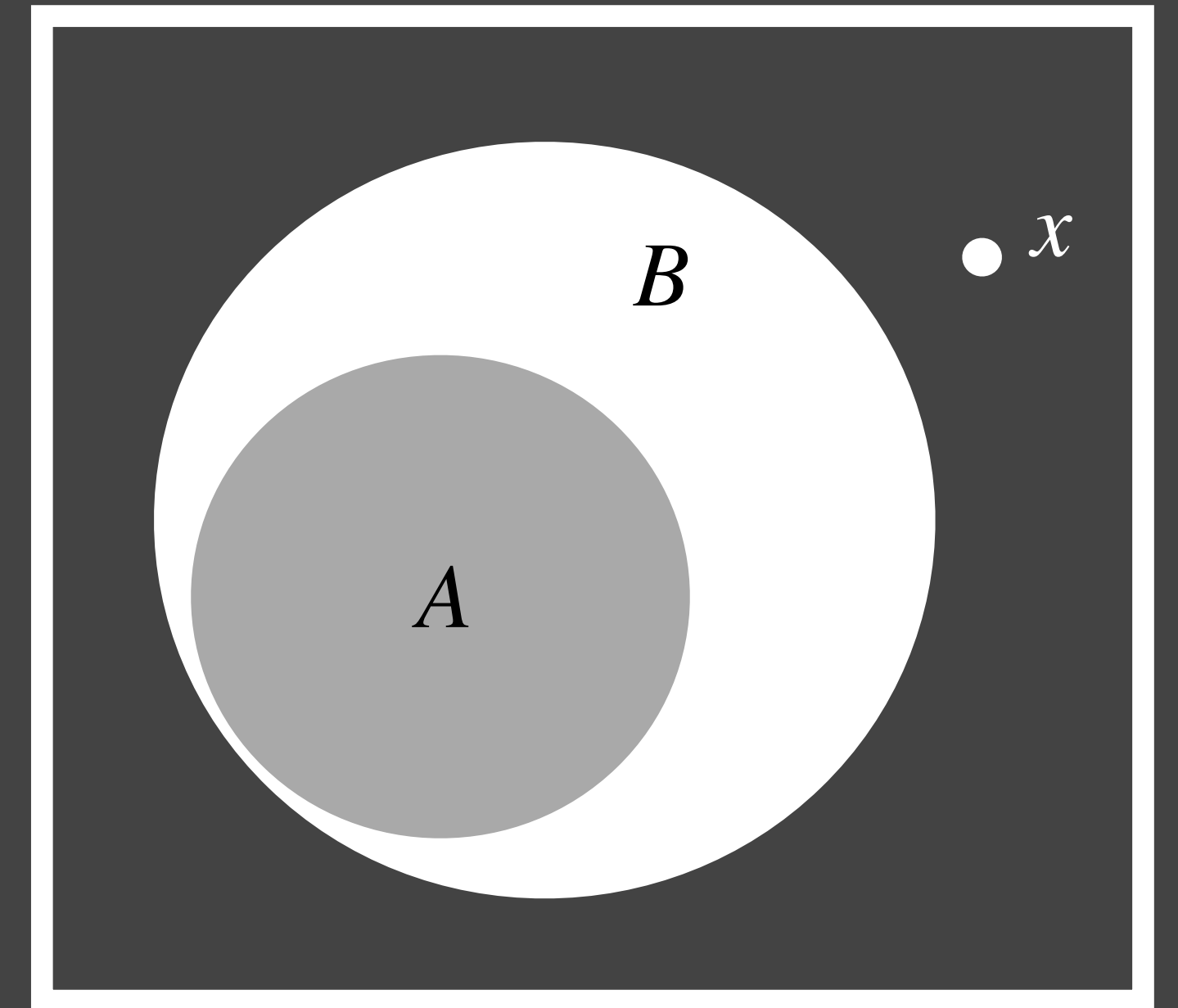


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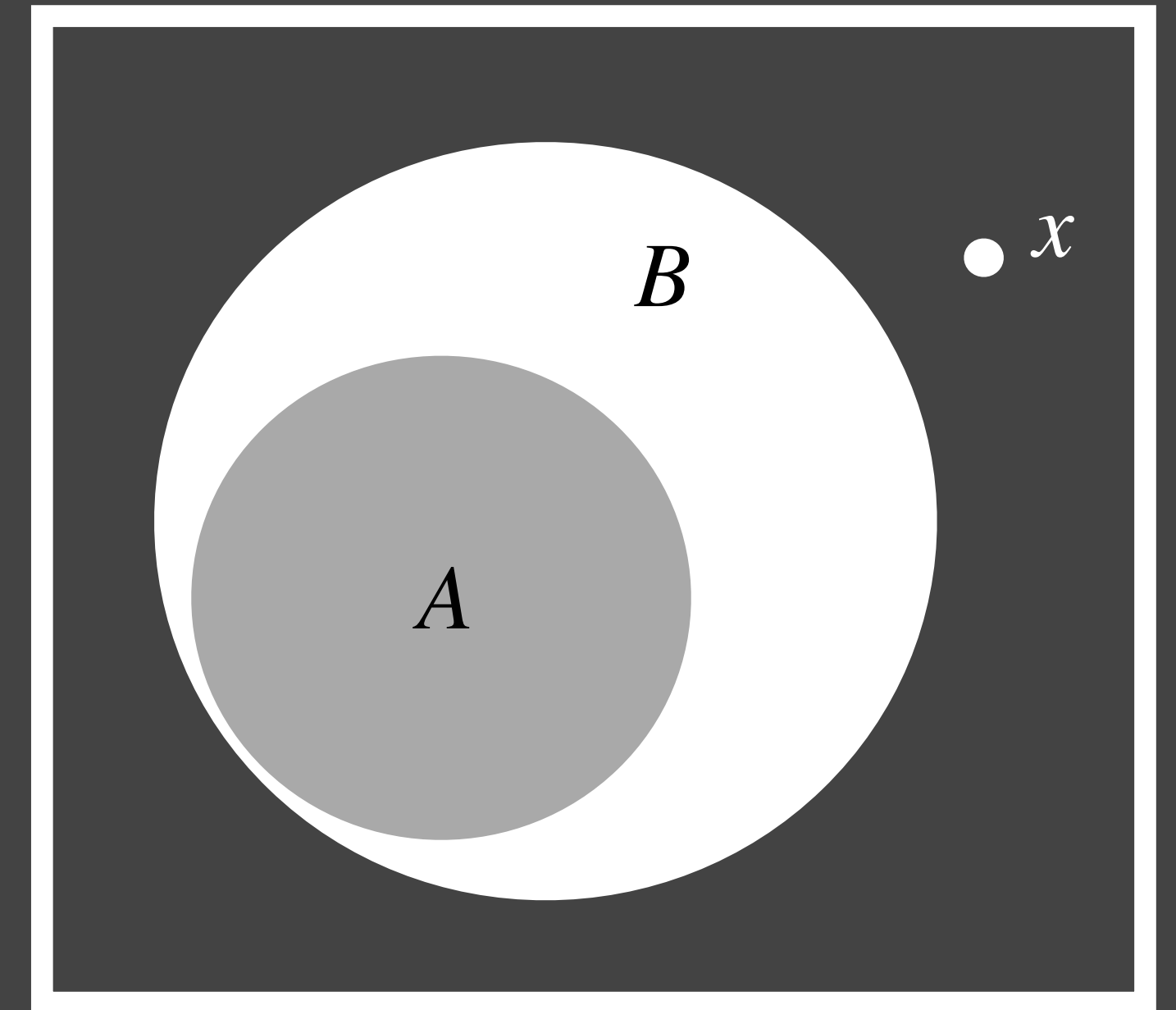
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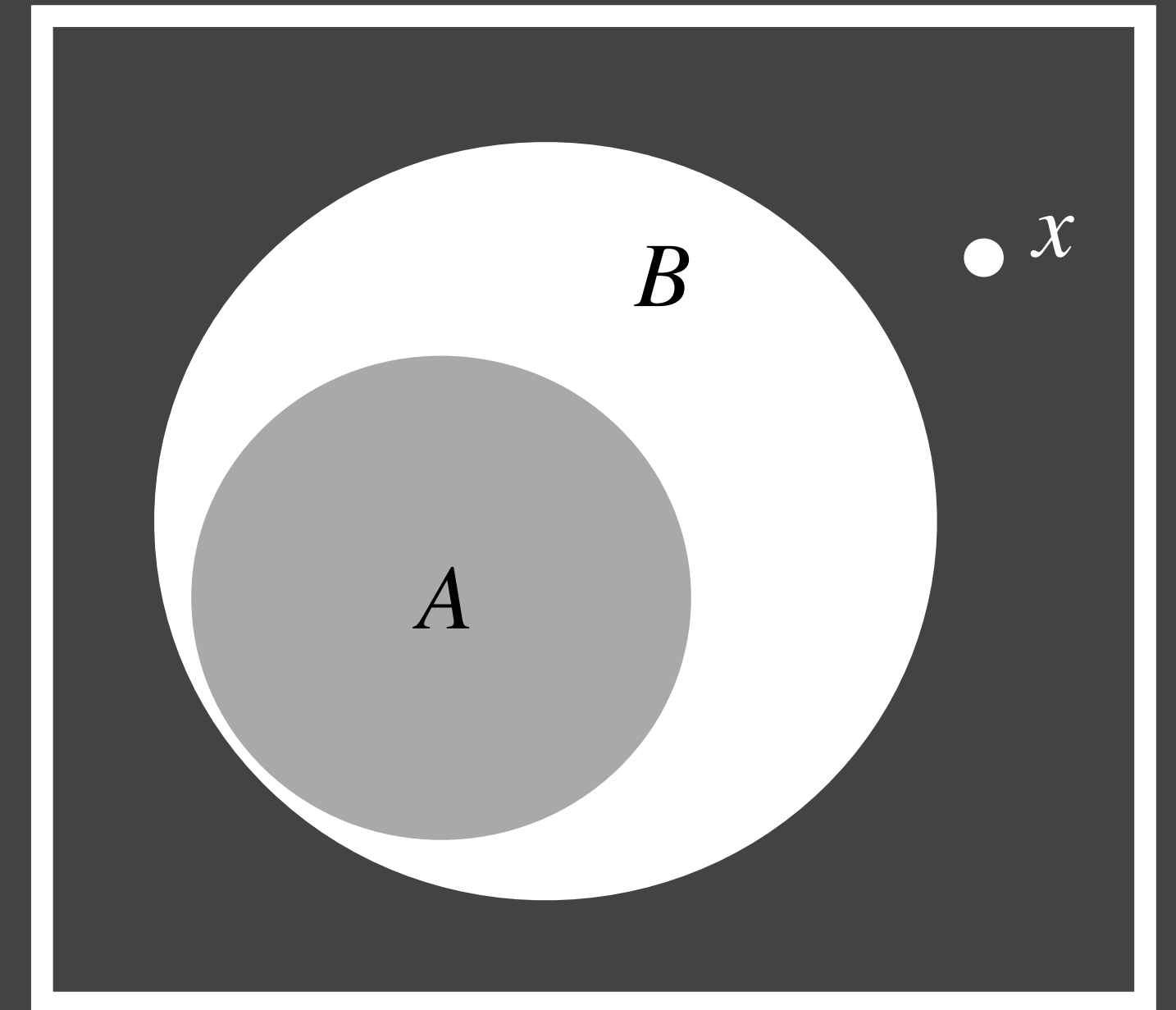
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$$f(\text{🍕} \mid \text{🥕}) \geq f(\text{🍕} \mid \text{🥕}, \text{🍩})$$

Why care about Submodular Cover?

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1. **Highly expressive!** Examples of Submodular Cover:

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Robot
Exploration

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Influence
Maximization

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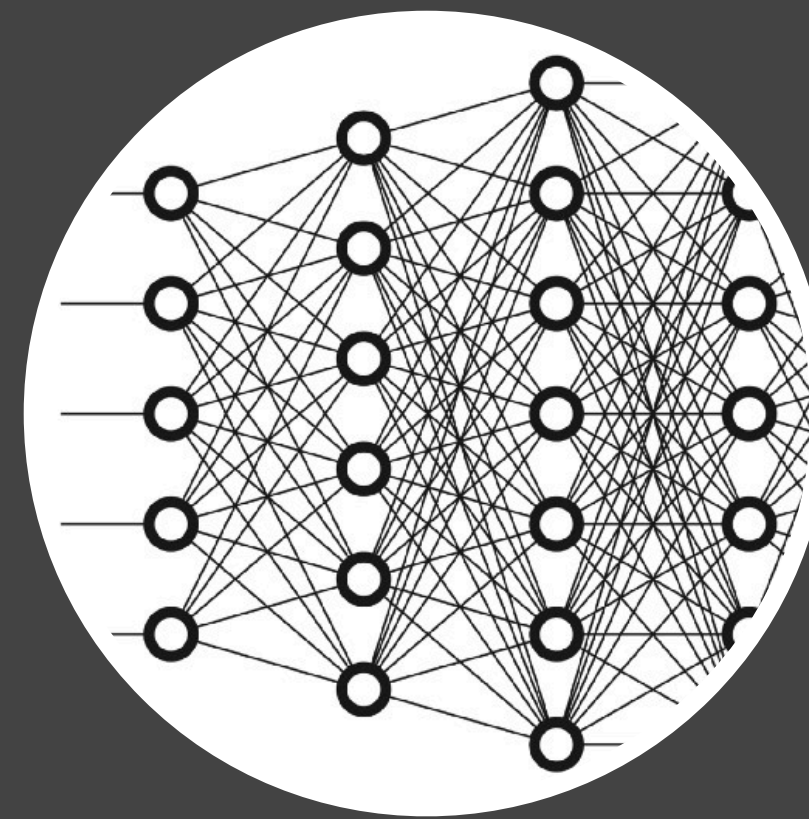
1. **Highly expressive!** Examples of Submodular Cover:



Robot
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Influence
Maximization



Feature
Selection

Why care about Submodular Cover?

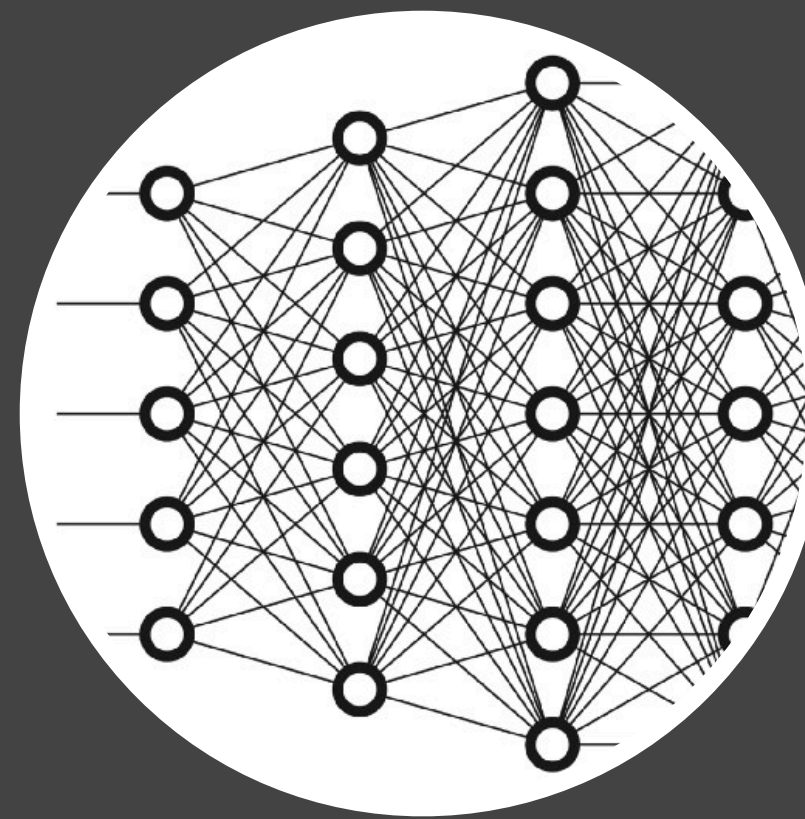
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Robot
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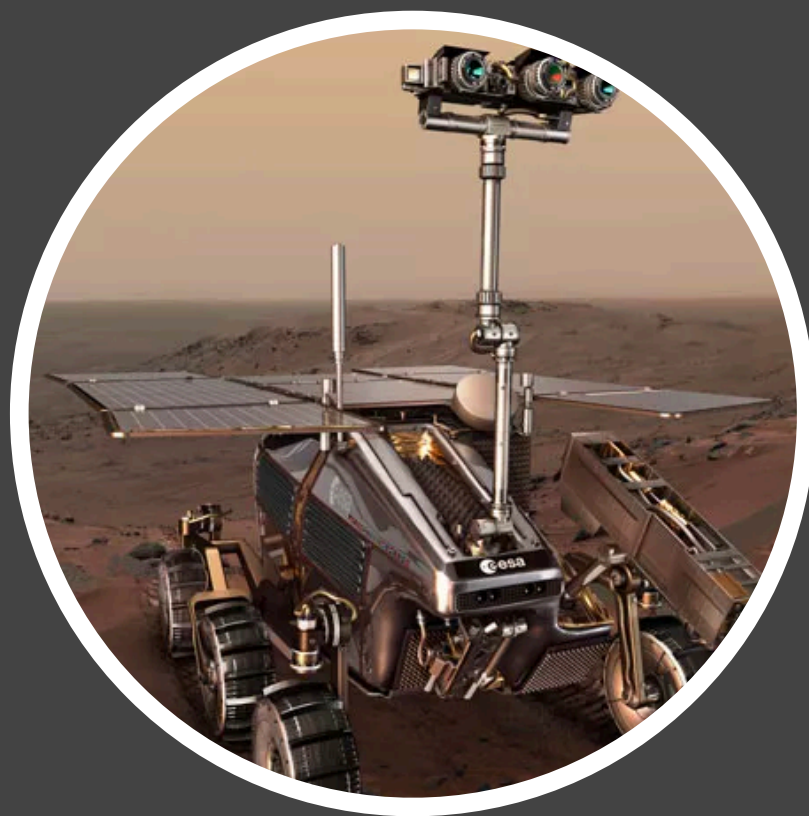
Feature
Selection



Document
Summarization

Why care about Submodular Cover?

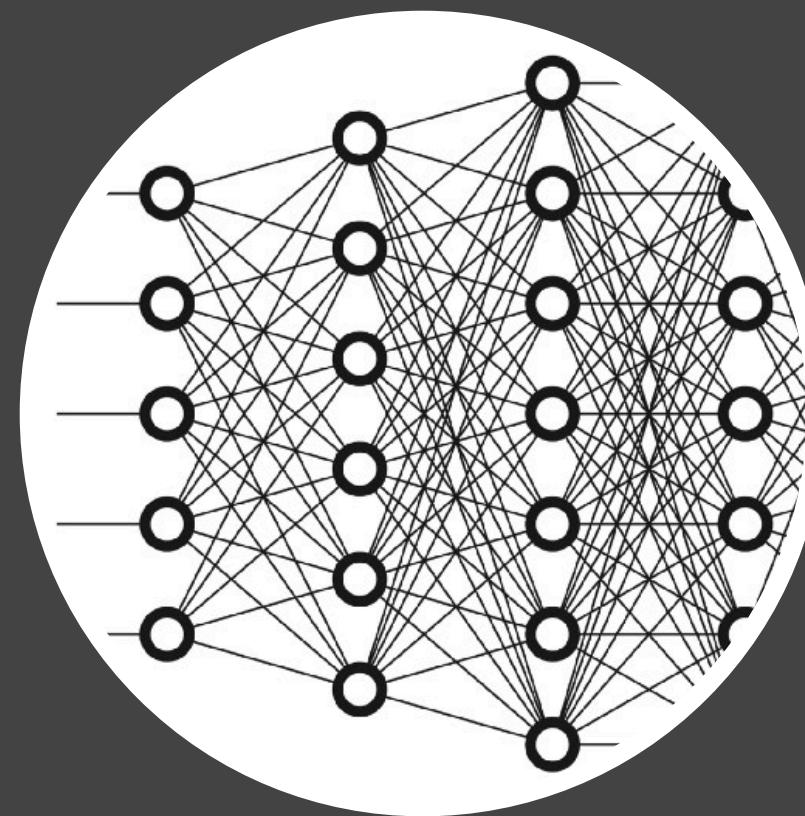
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Resource
allocation

Why care about Submodular Cover?

Popular to reduce to Submodular Cover!

[Goyal+ 13][Loukides Gwadera 16][Zheng+ 17][Andreev+ 09][Lee+ 13]
[Lukovszki+ 18][Poularakis+ 17][Krause+ 08][Kortsarz Nutov 15][Jorgensen+
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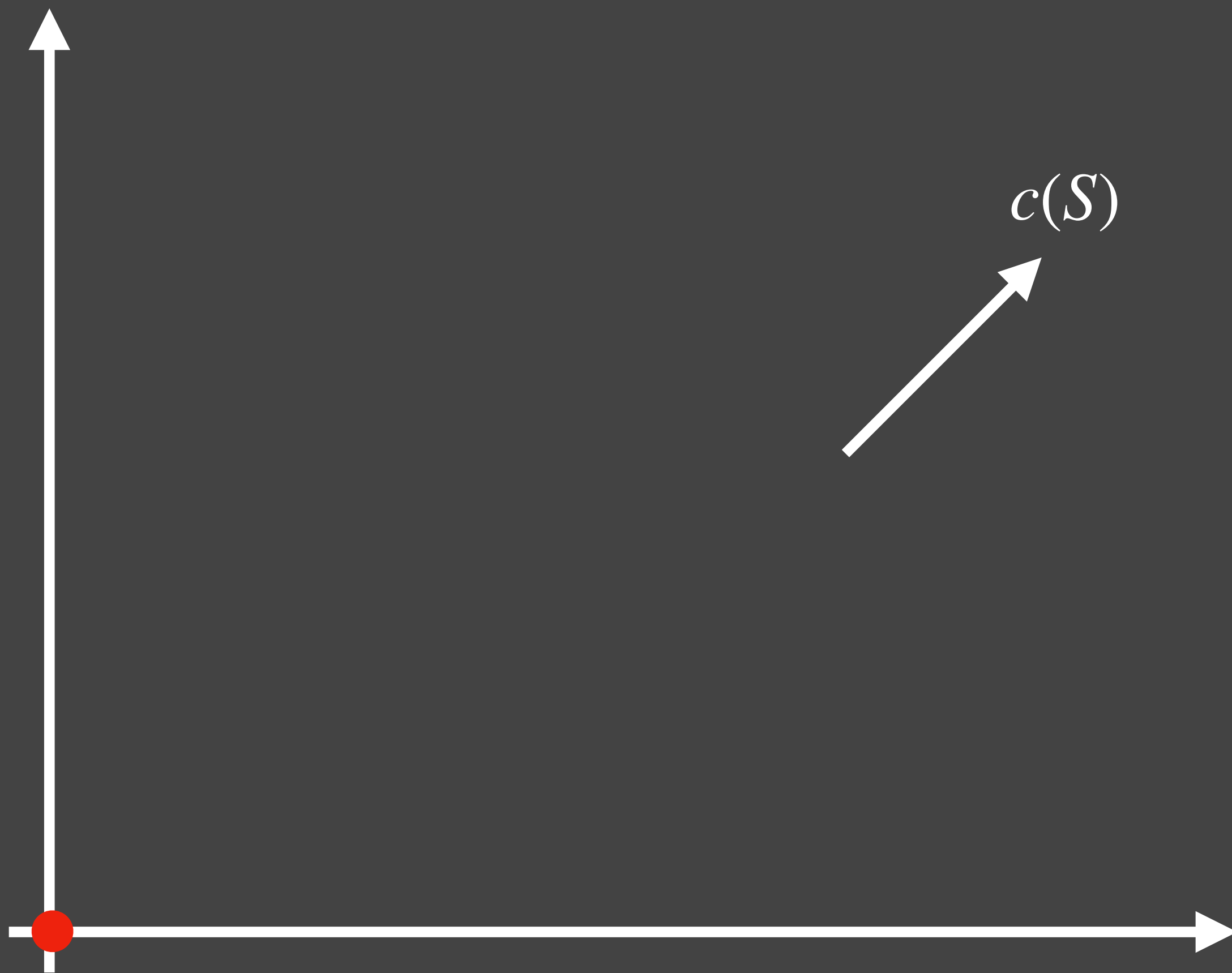
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Punchline: Sweet spot between **generality** and **tractability**!

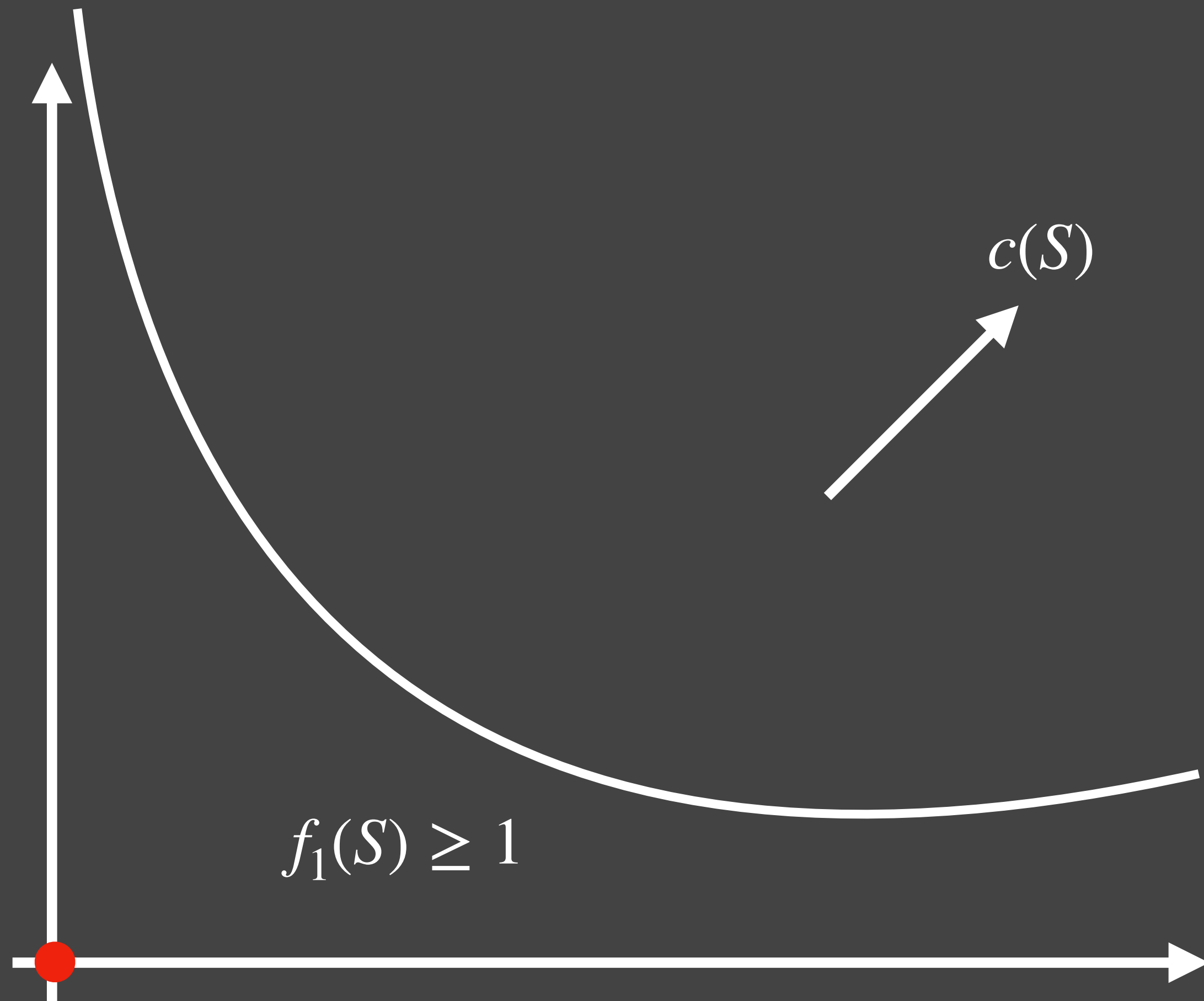
Online Submodular Cover

[Gupta L. SODA 20]



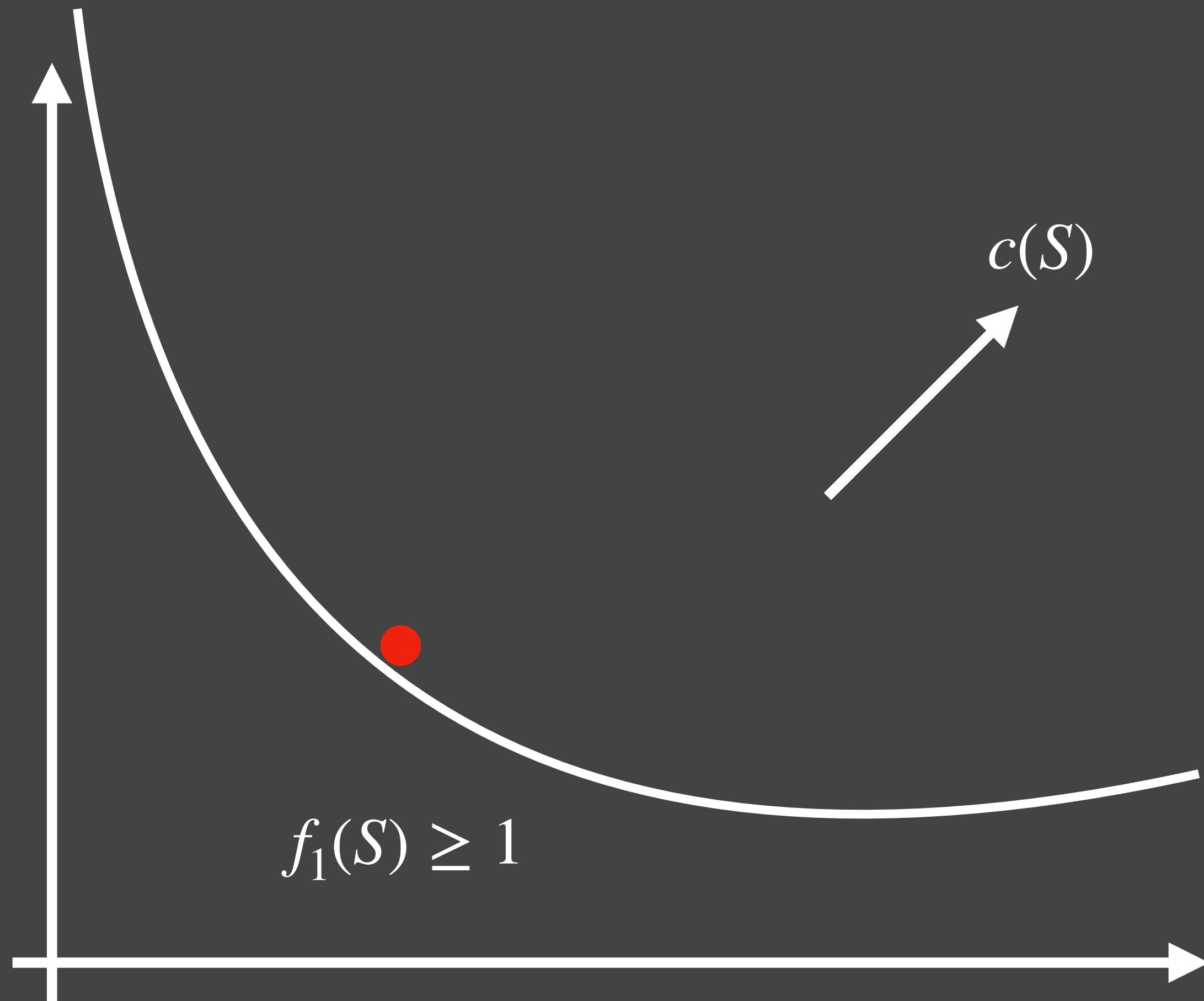
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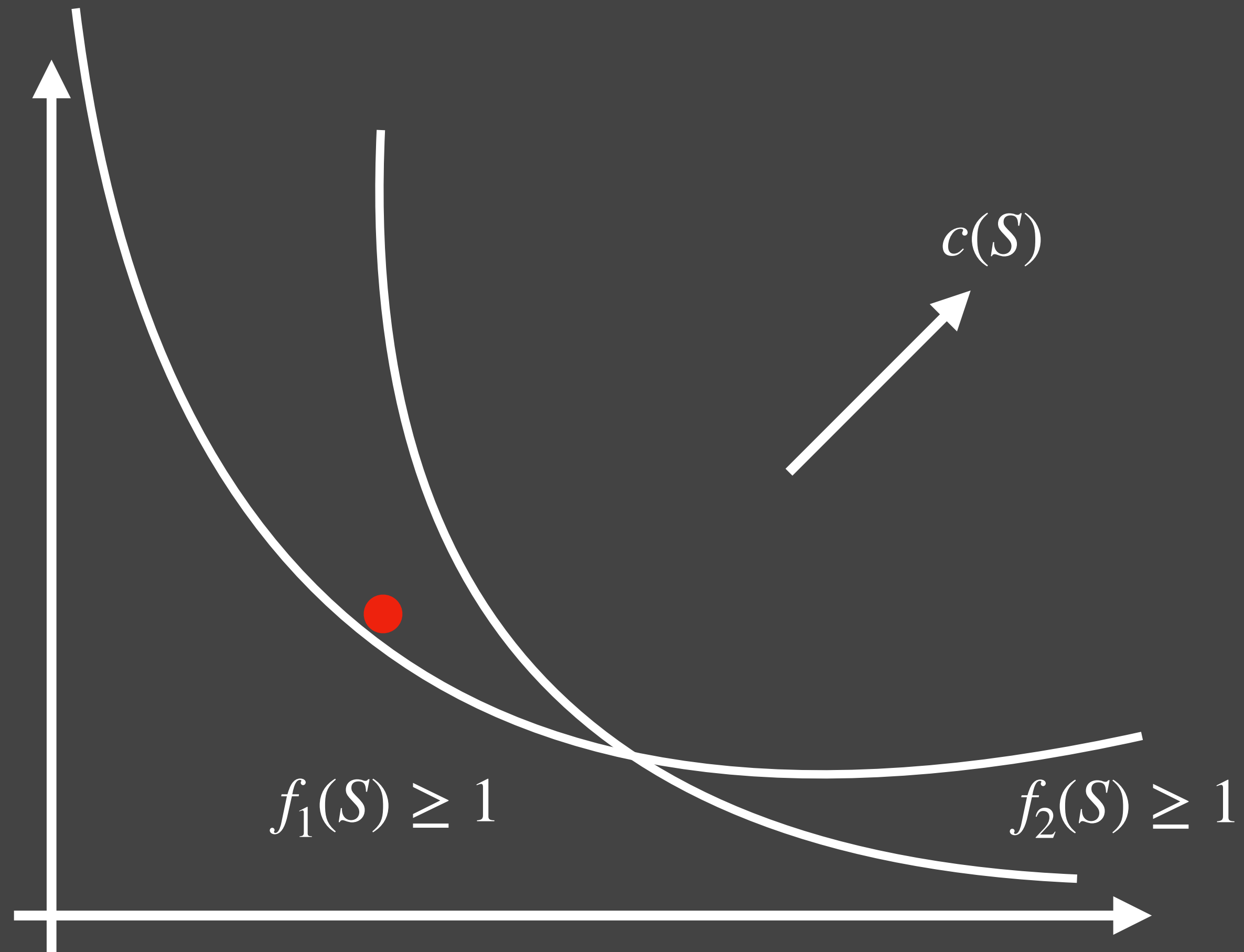
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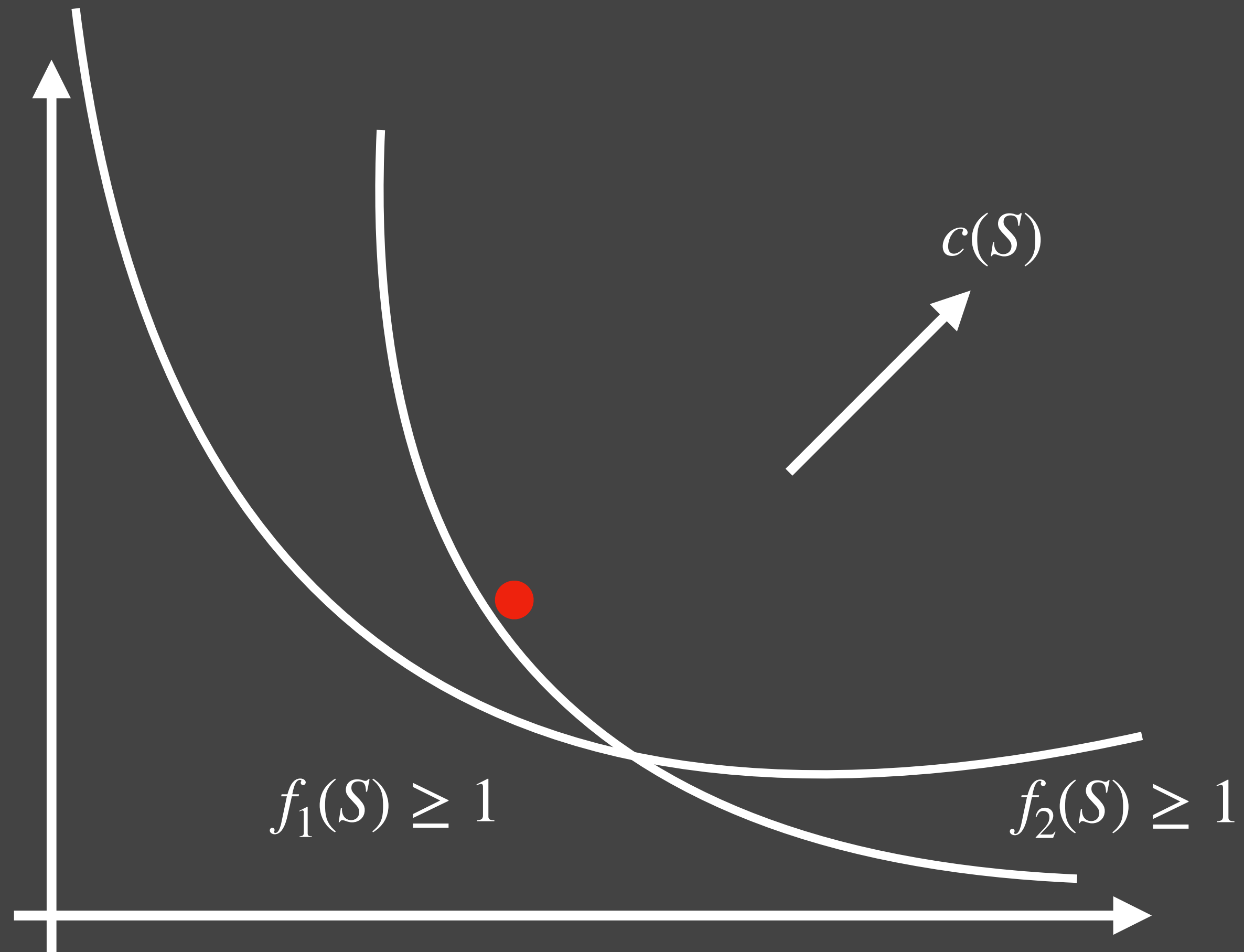
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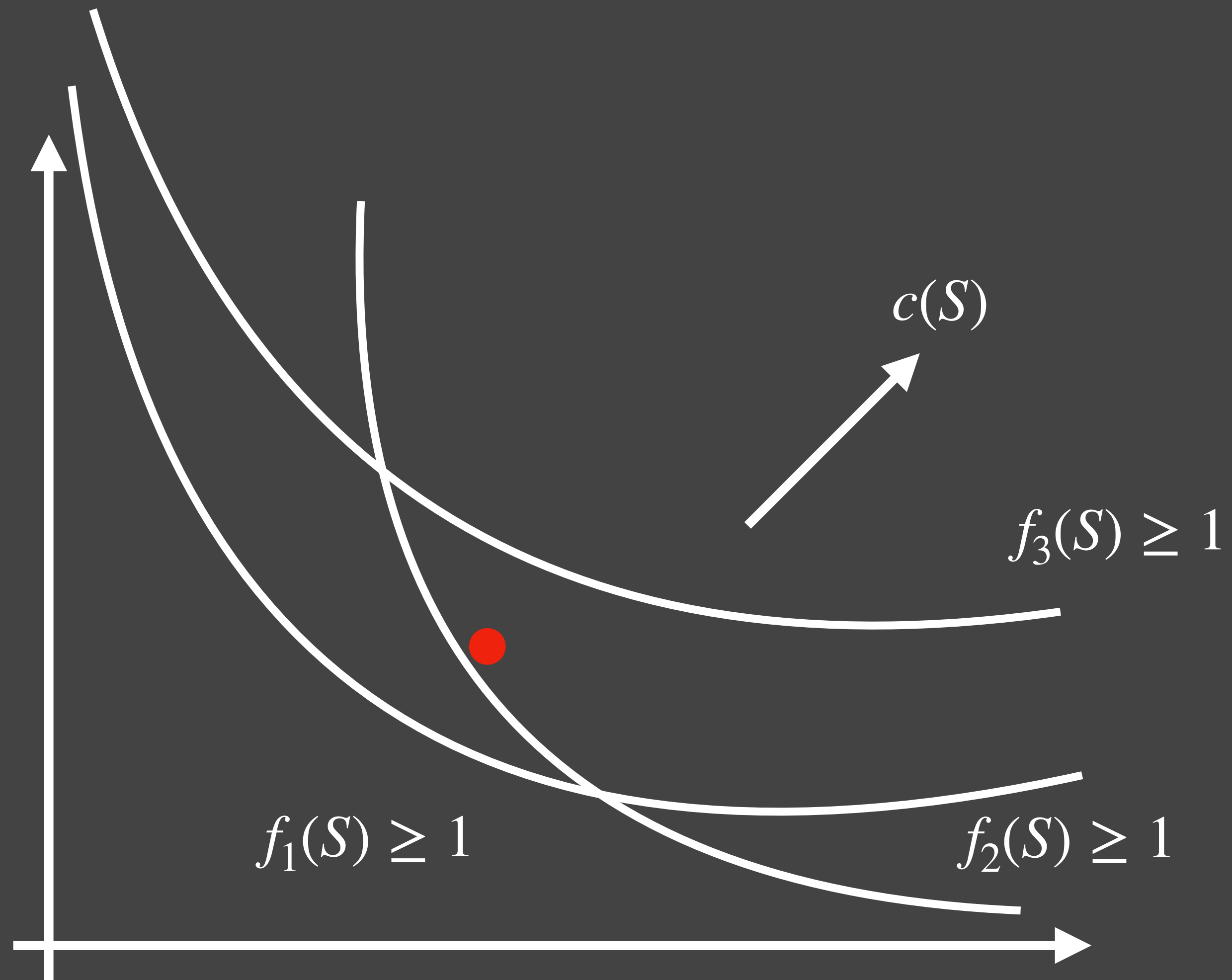
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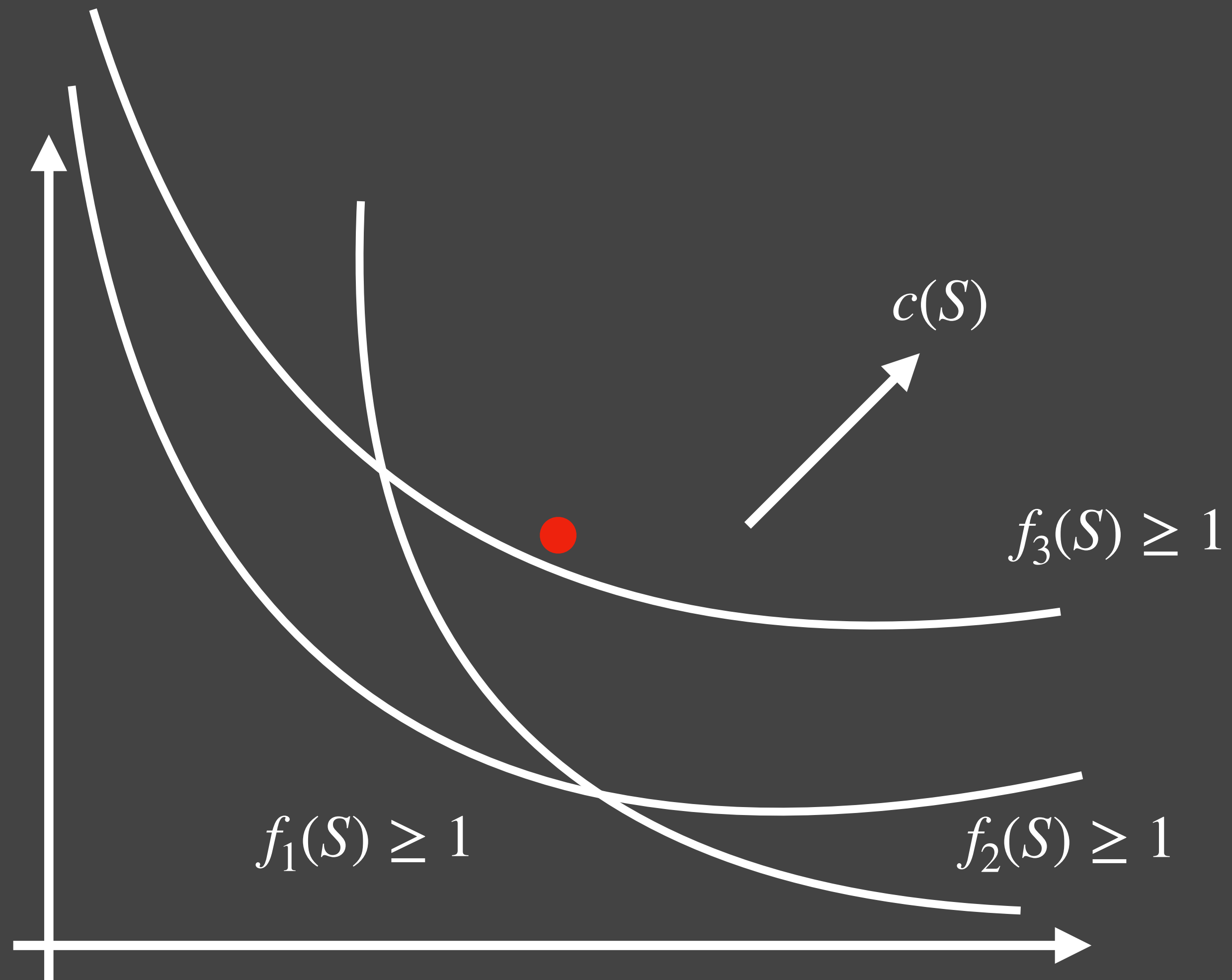
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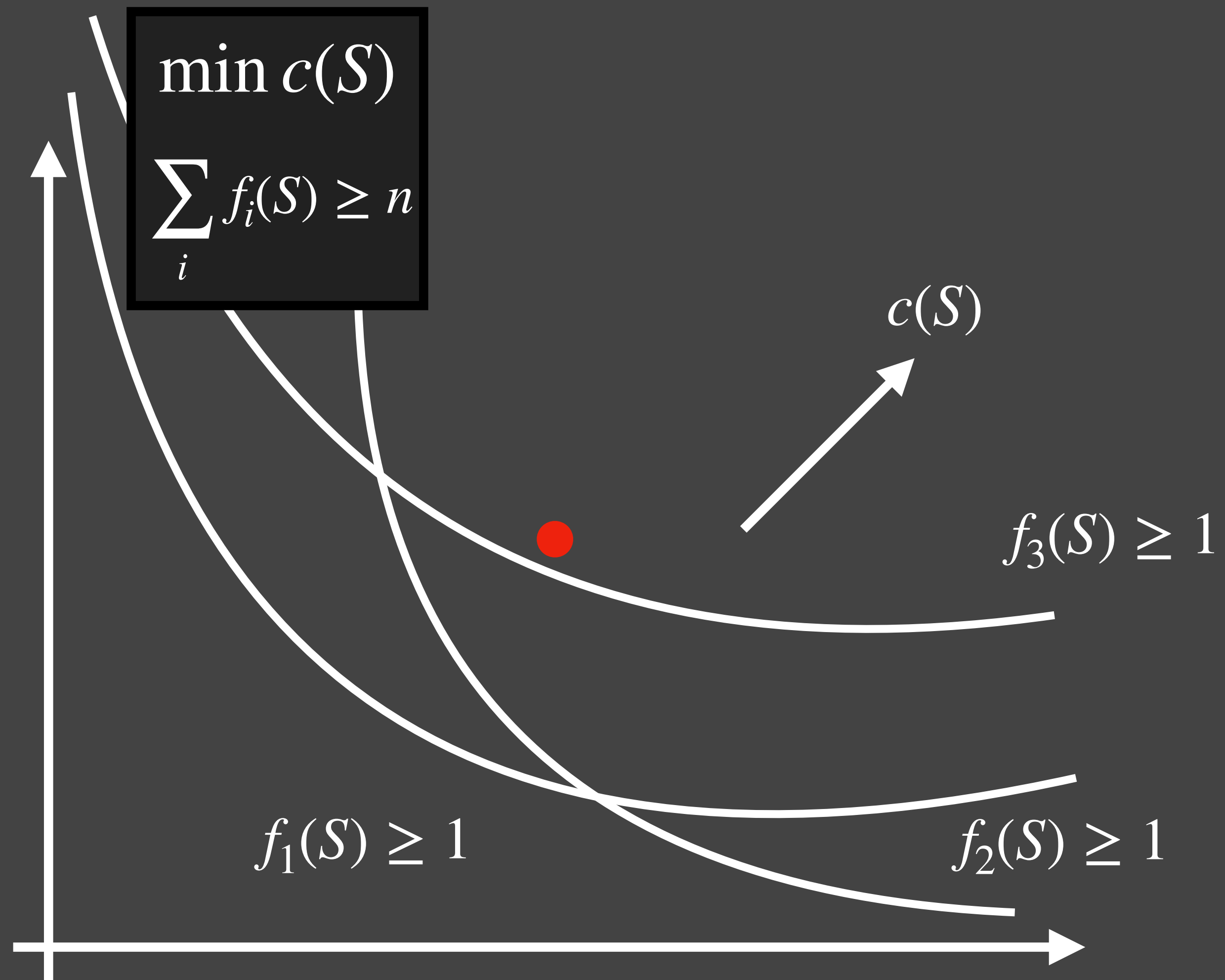
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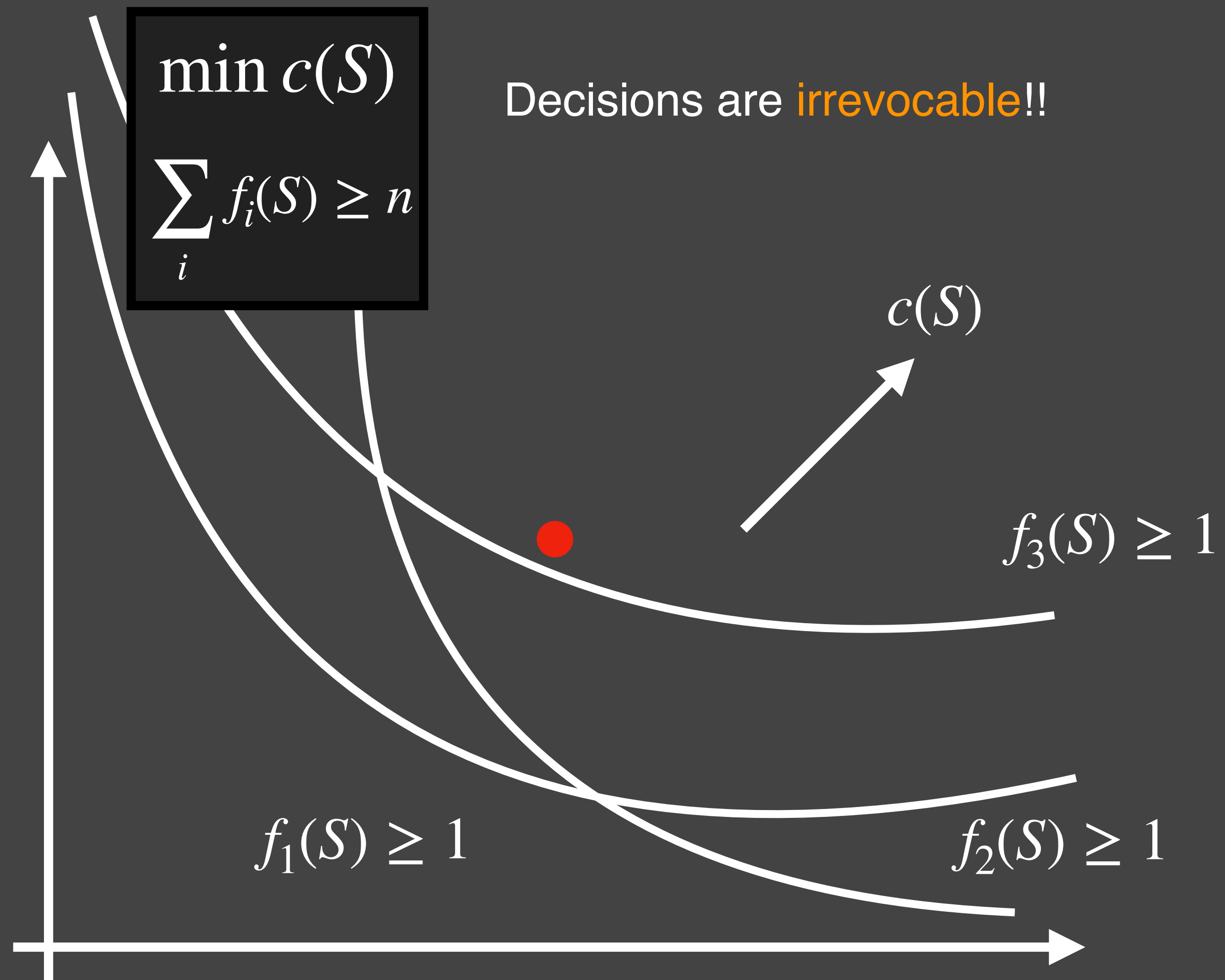
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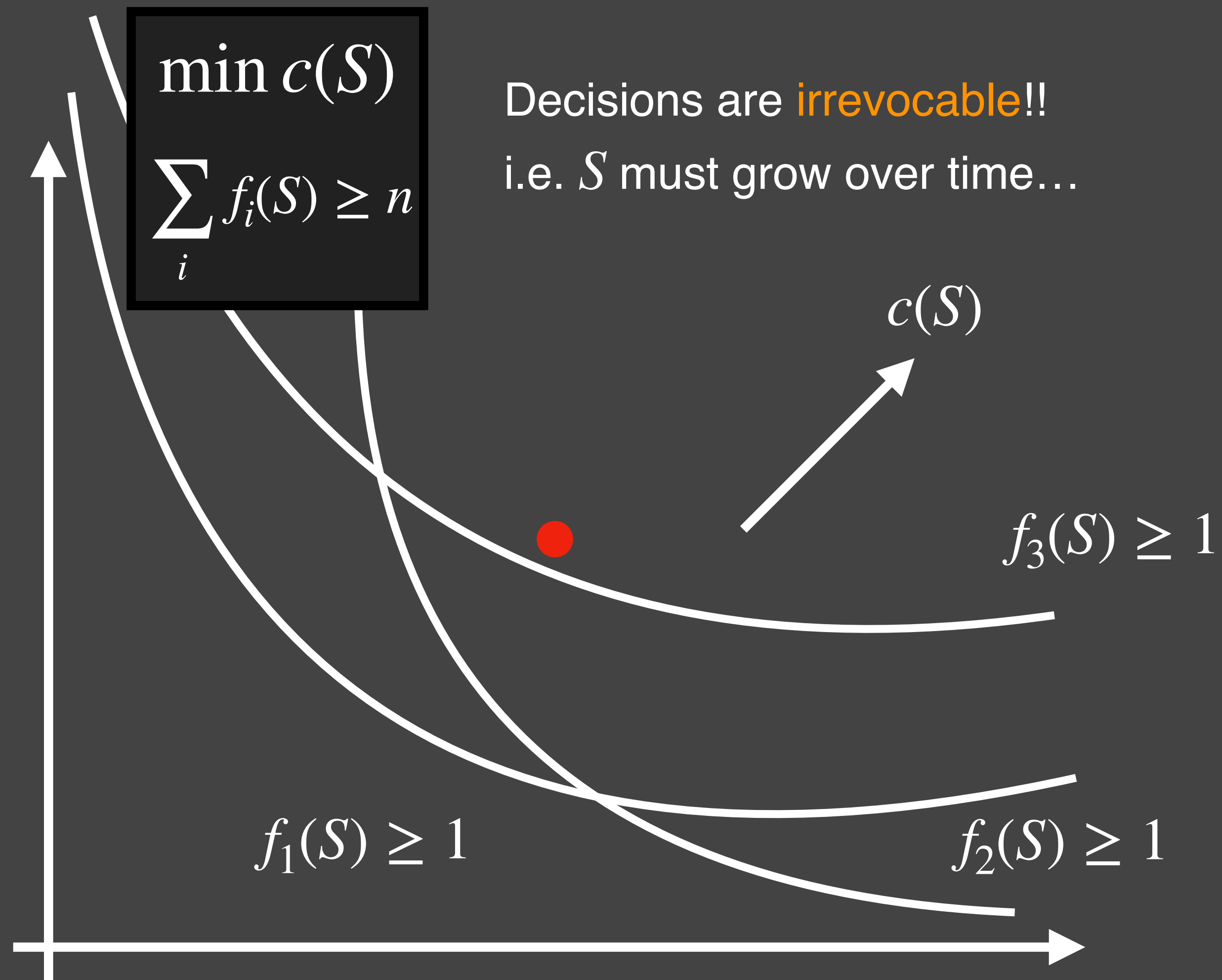
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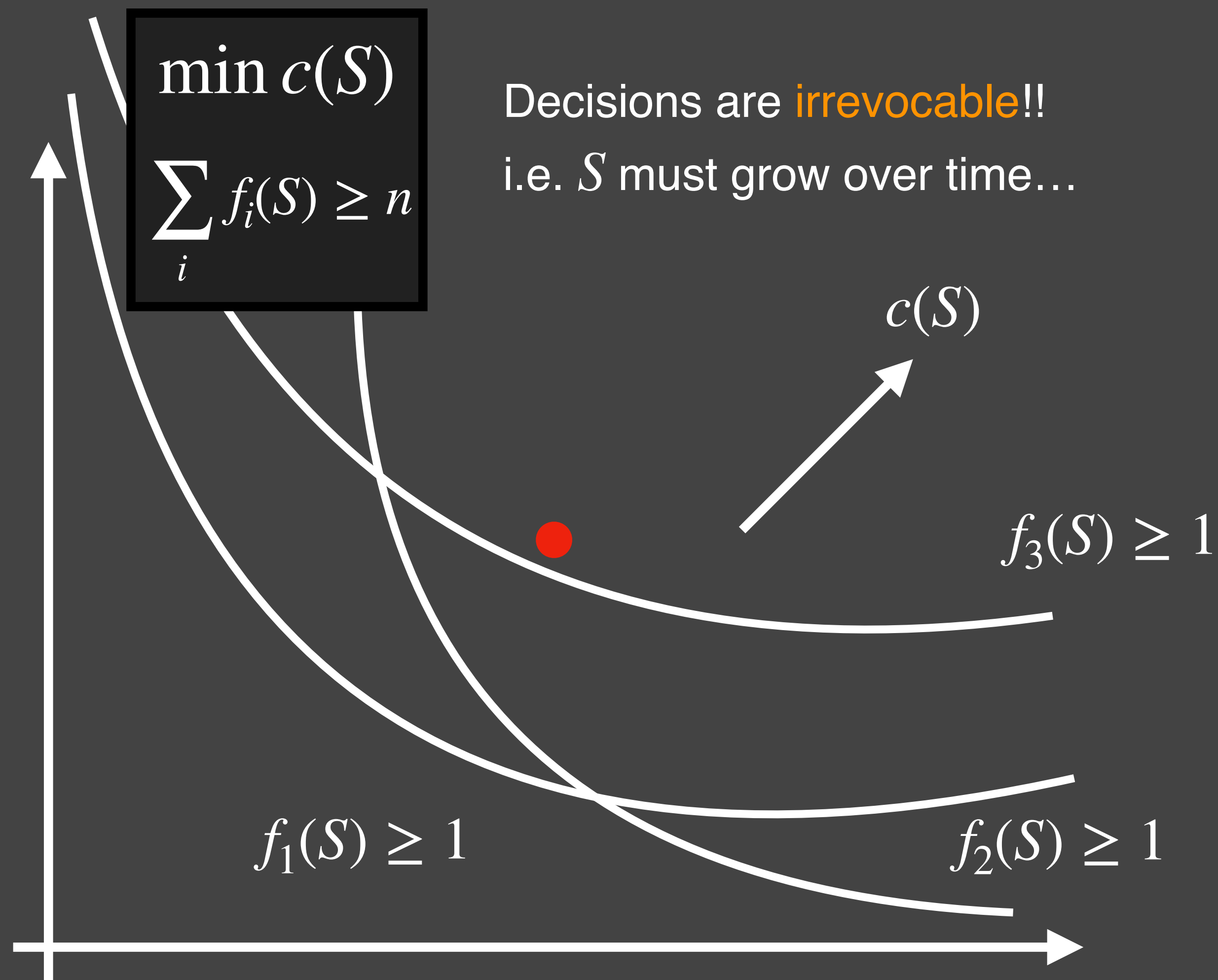
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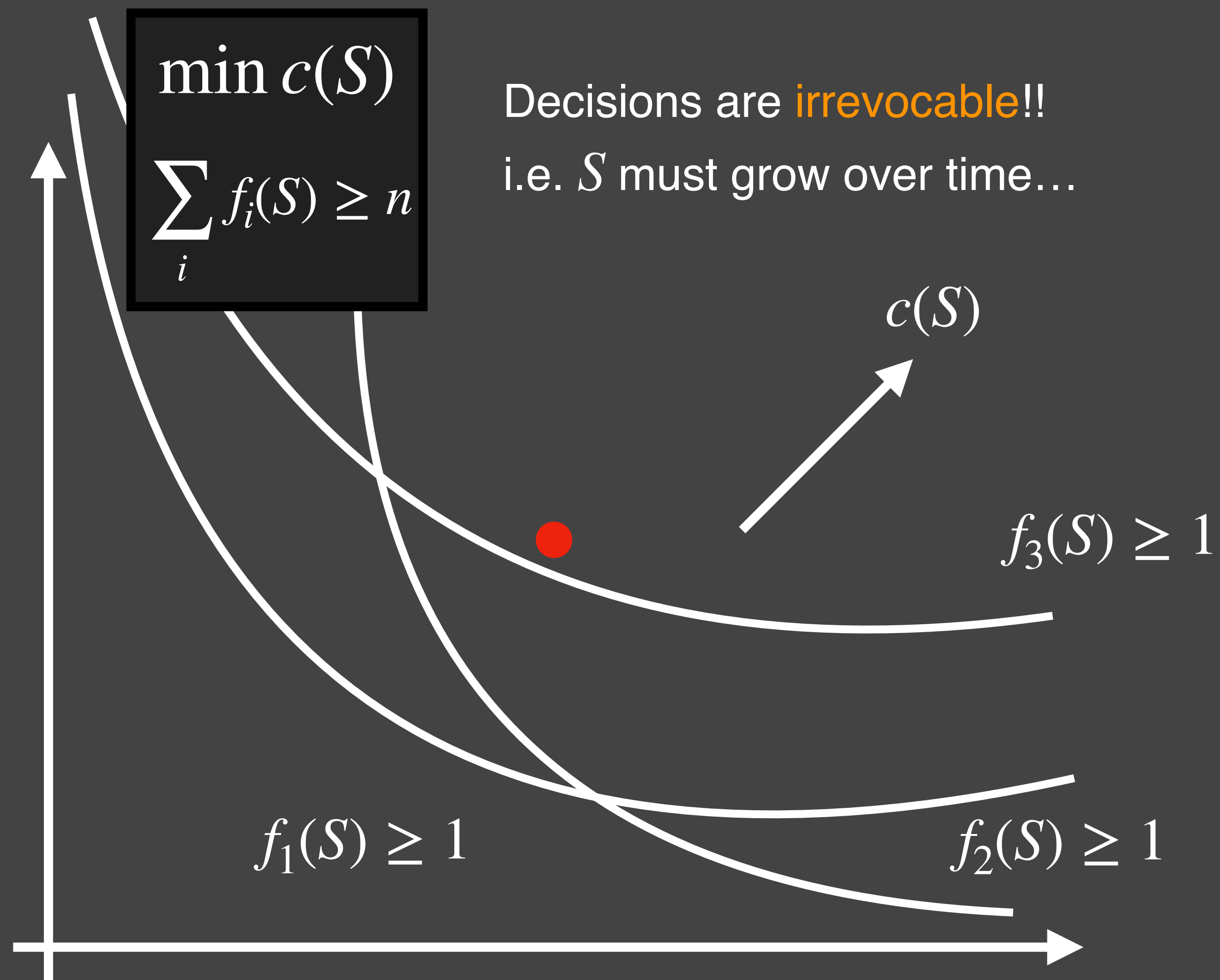


Theorem [Gupta L. SODA 20]:

Polynomial time algo for
Online Submod Cover with
approximation $O(\log^2 n)$.

Online Submodular Cover

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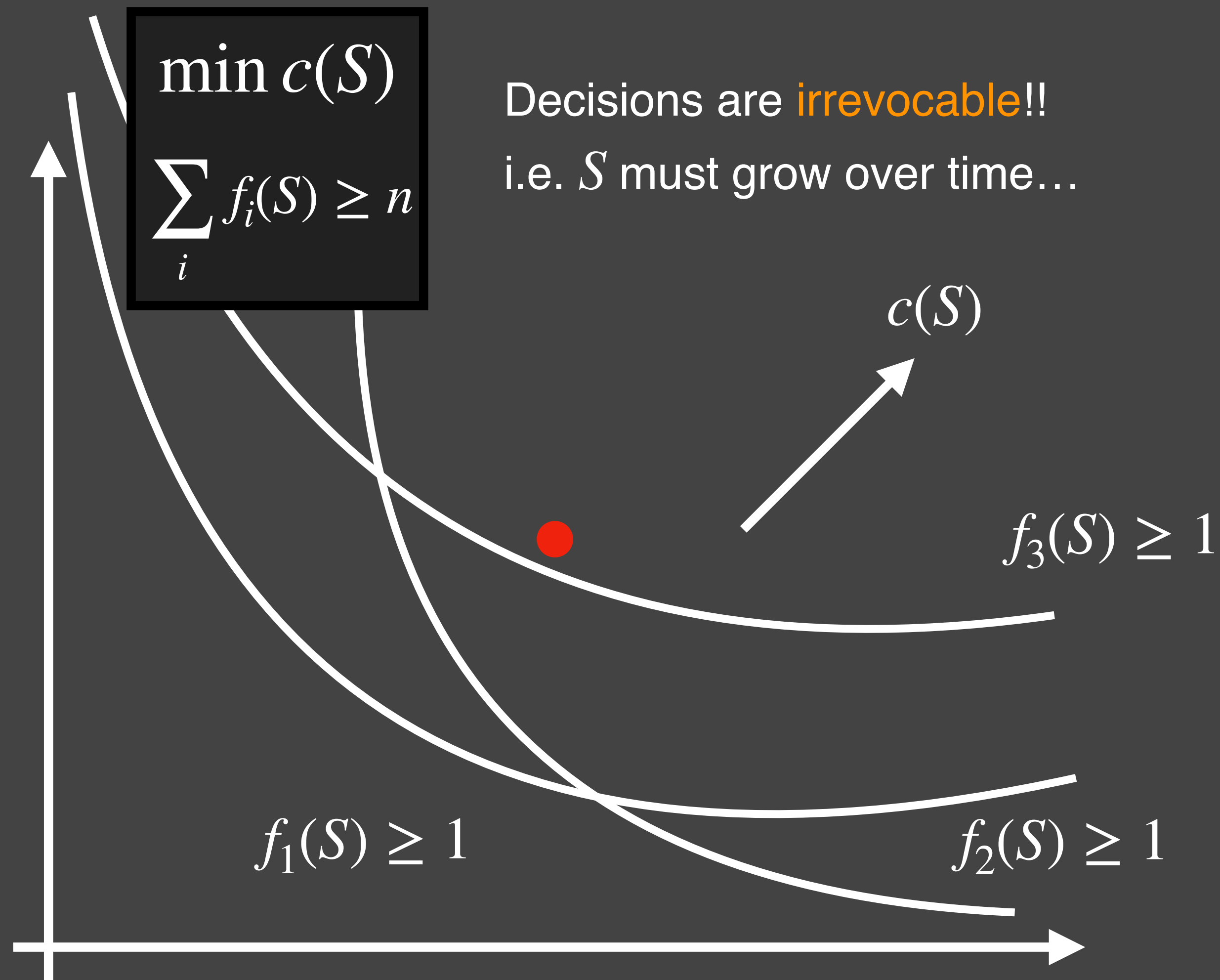
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Optimal!

Online Submodular Cover

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Polynomial time algo for
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Optimal!

Technical Ingredient:

RoundOrSeparate for LP relaxation
of Submodular Cover &
generalization of **Mutual Information!**

Online Submodular Cover

[Gupta L. SODA 20]

Online Set Cover
 $O(\log^2 n)$

Submodular Cover
 $O(\log n)$

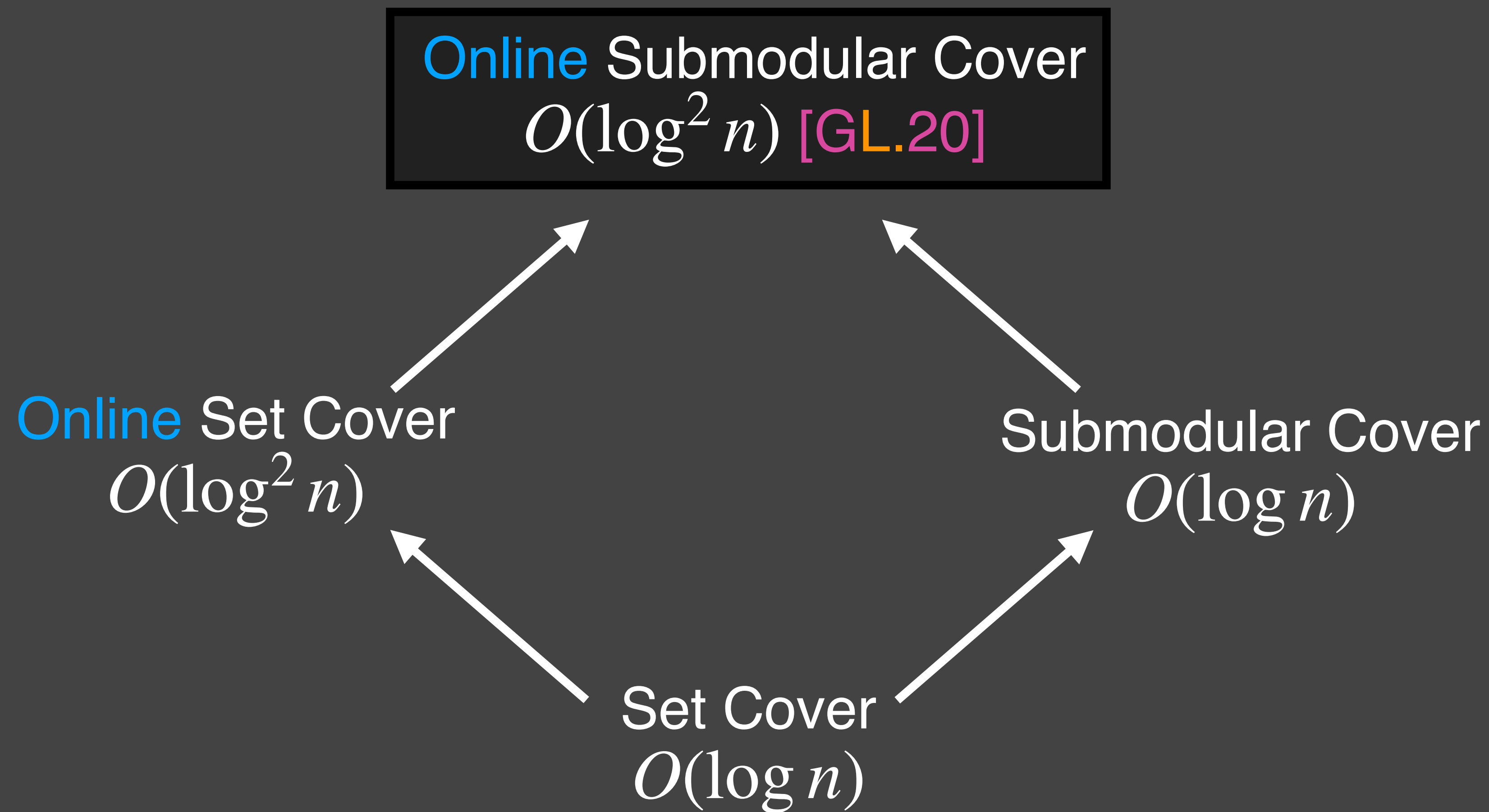
Set Cover
 $O(\log n)$

```
graph TD; SC["Set Cover  
O(log n)"] --> OSC["Online Set Cover  
O(log^2 n)"]; SC --> SUBC["Submodular Cover  
O(log n)"]
```

The diagram illustrates the relationship between three covering problems. At the bottom center is 'Set Cover' with complexity $O(\log n)$. Two arrows point upwards from this central node. The left arrow points to 'Online Set Cover' with complexity $O(\log^2 n)$. The right arrow points to 'Submodular Cover' with complexity $O(\log n)$.

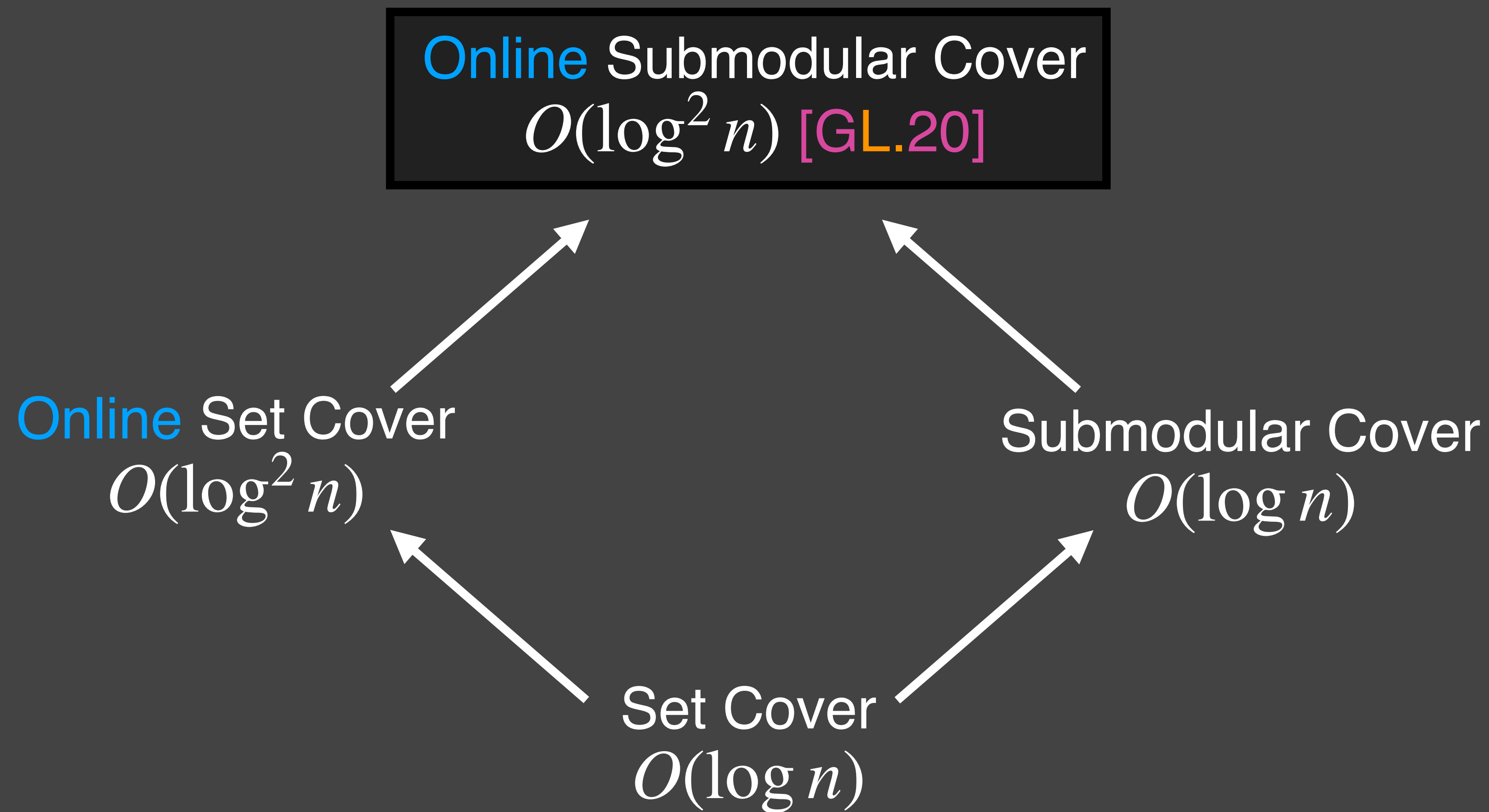
Online Submodular Cover

[Gupta L. SODA 20]



Online Submodular Cover

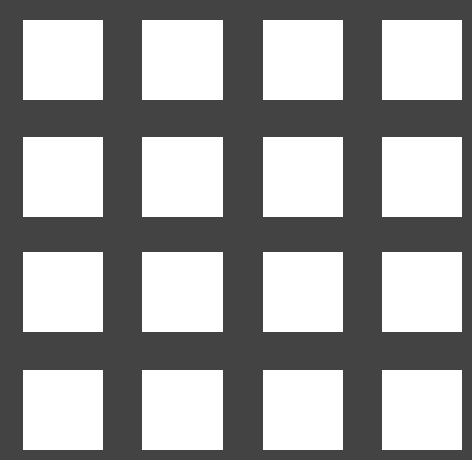
[Gupta L. SODA 20]



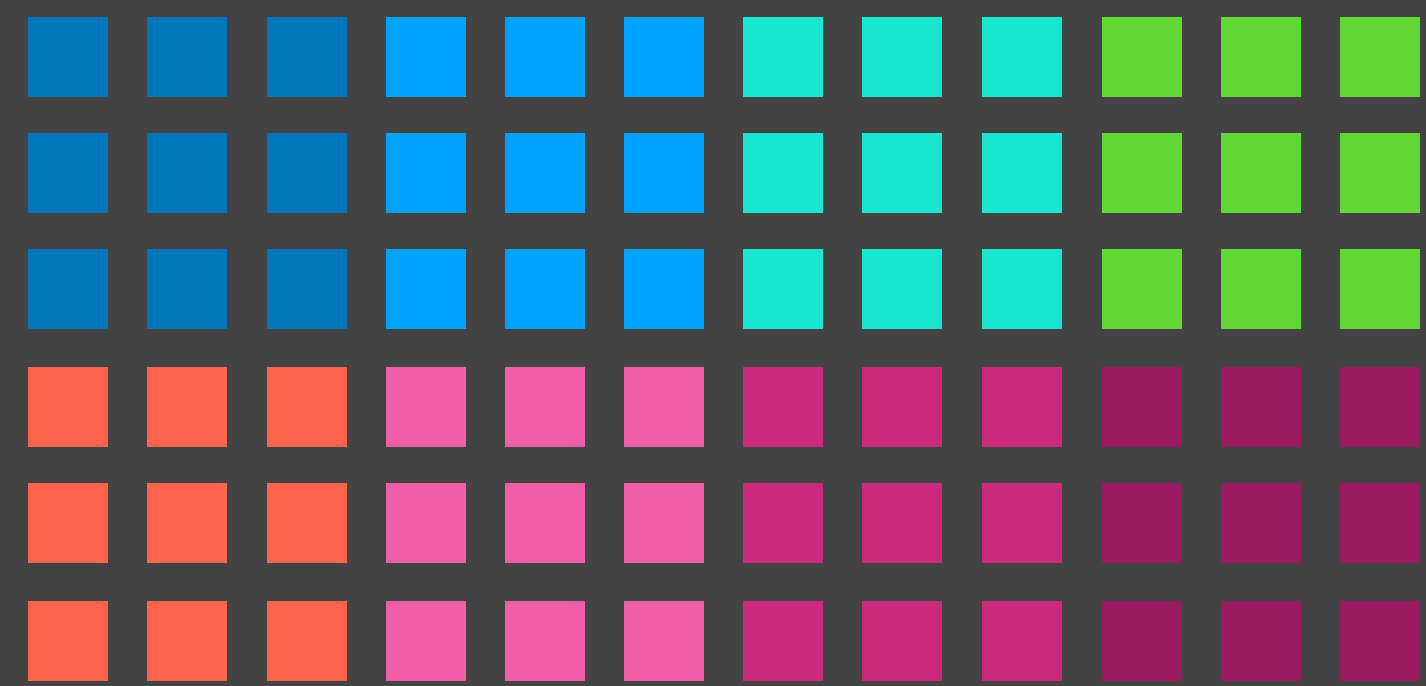
Best of both worlds: modeling power of Submodular Cover + Online.

Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

Cache of size k

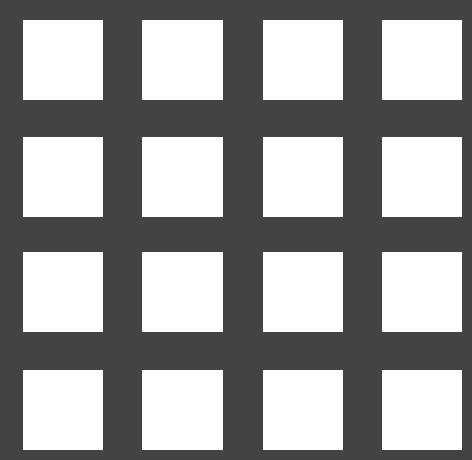


n total pages, **divided into blocks**

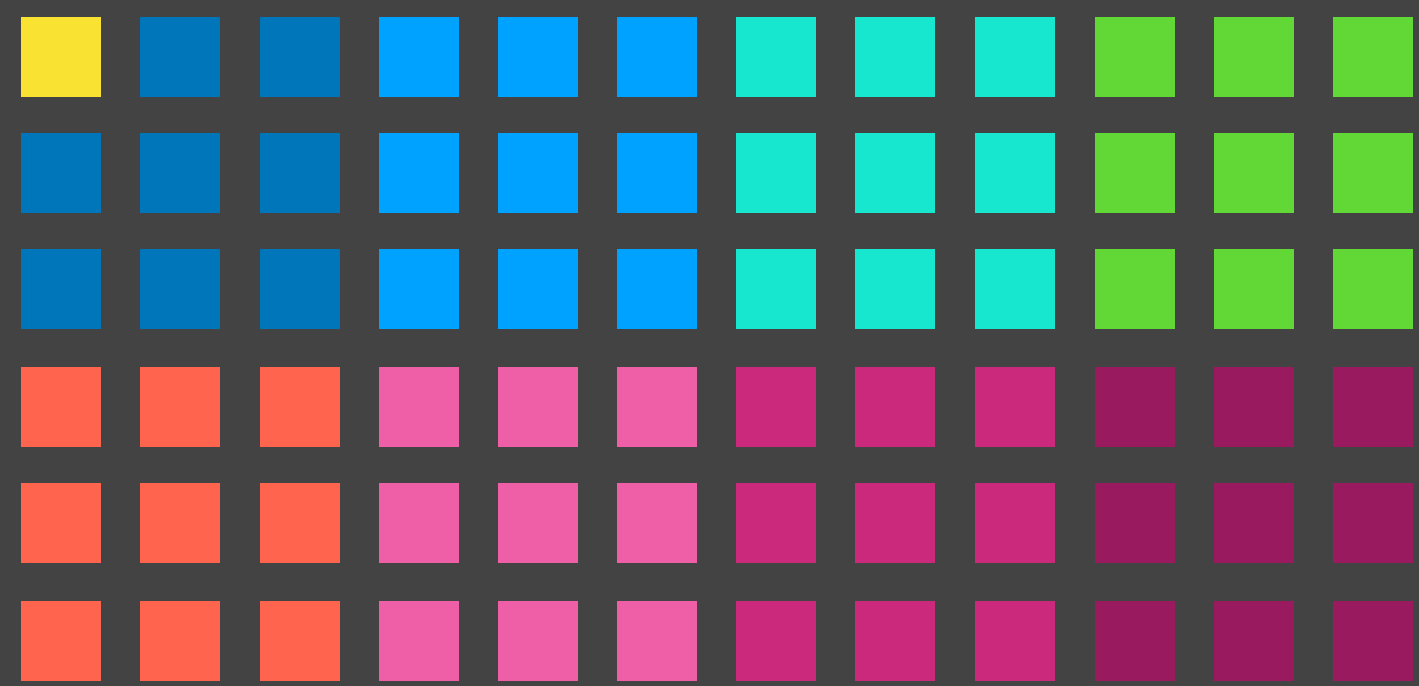


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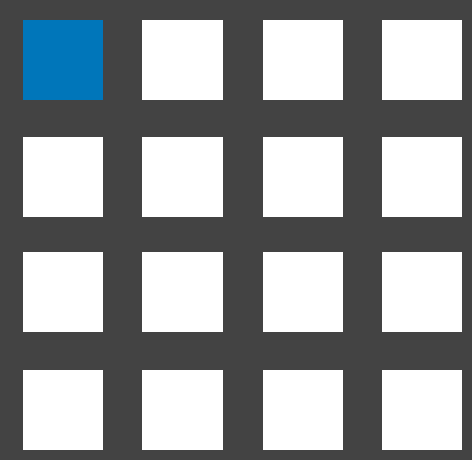


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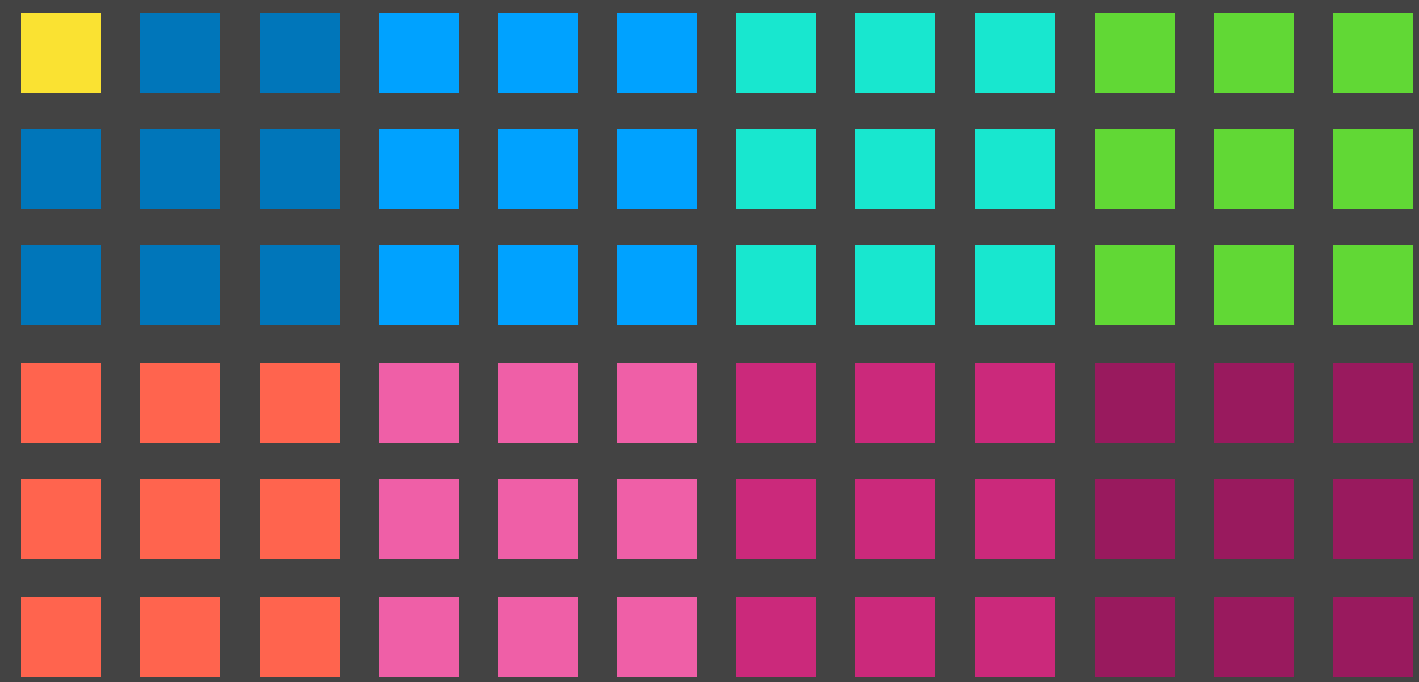


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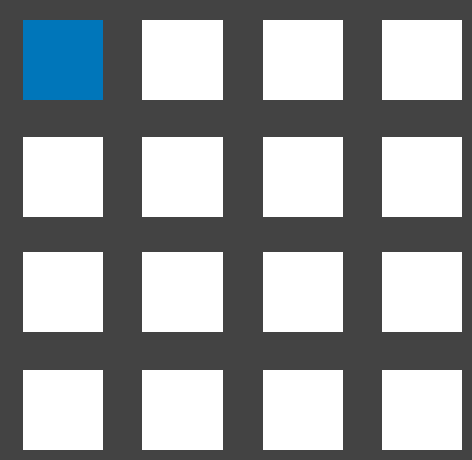


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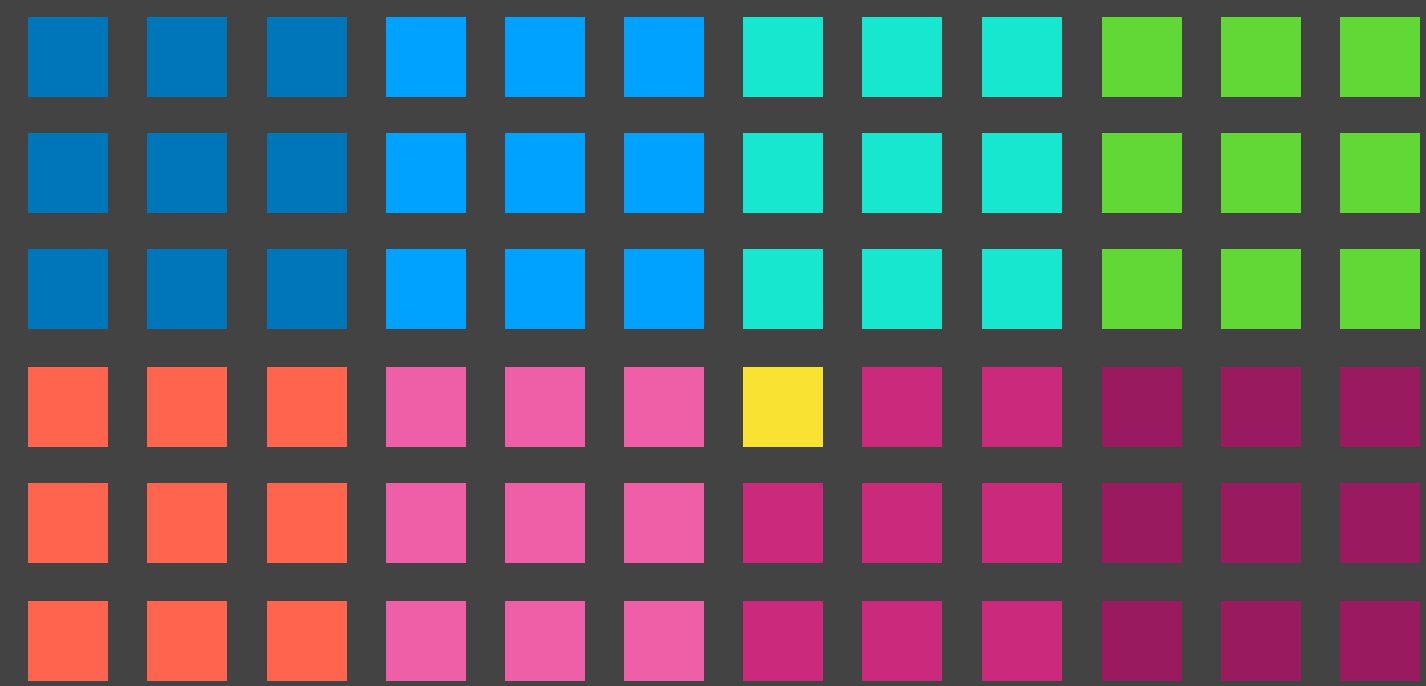


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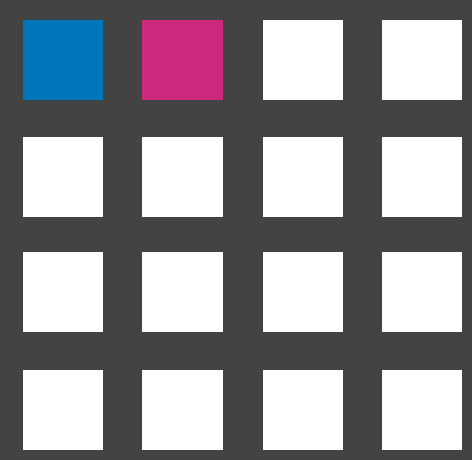


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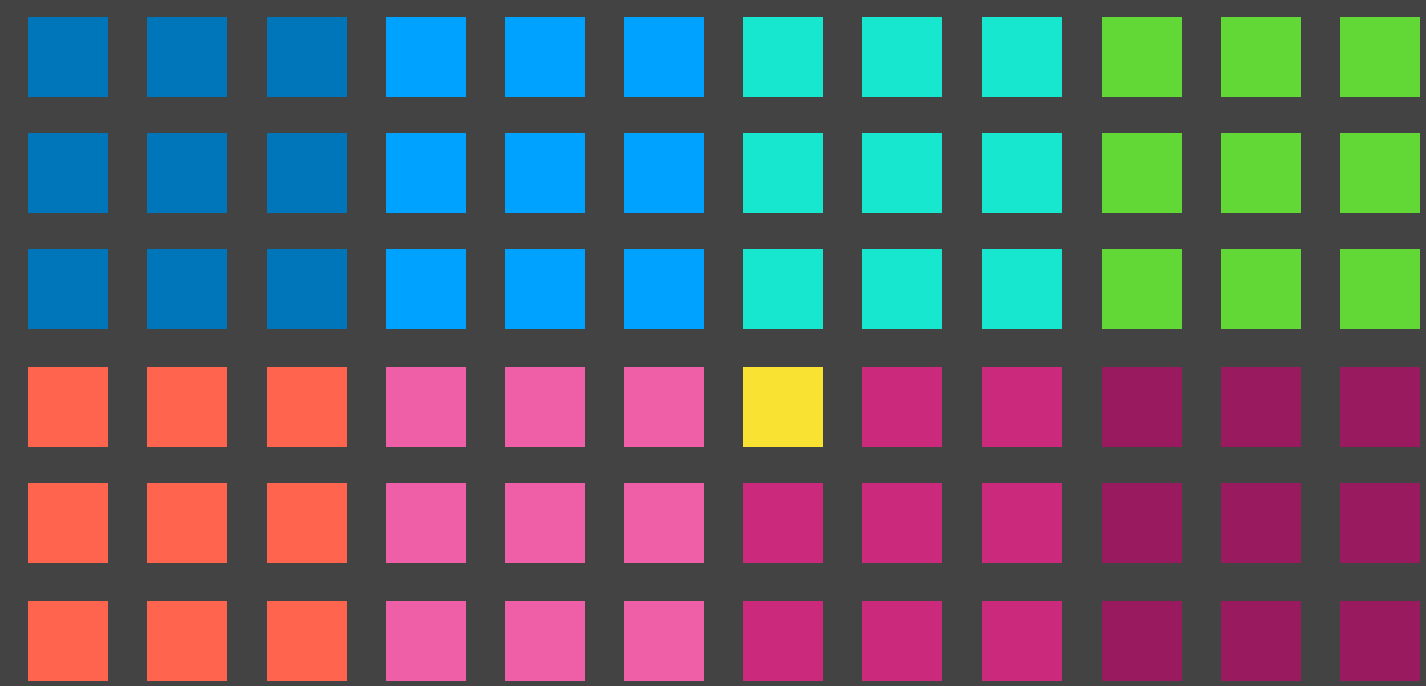


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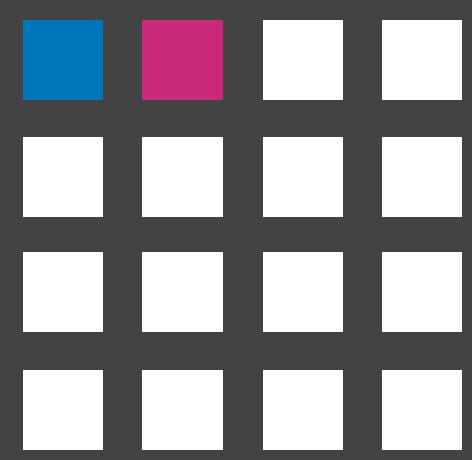


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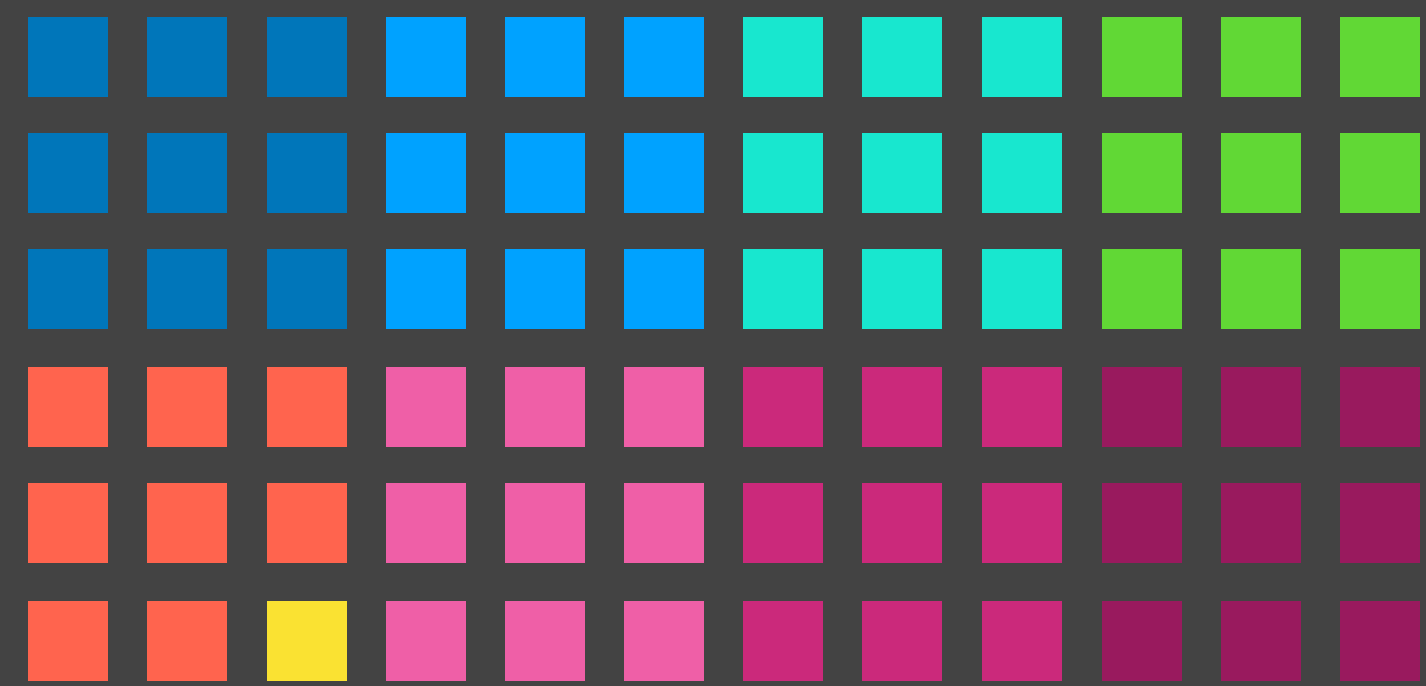


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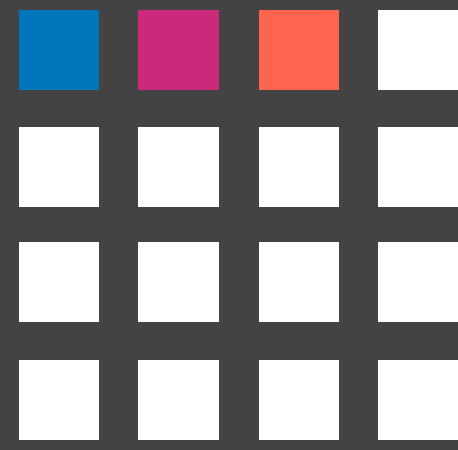
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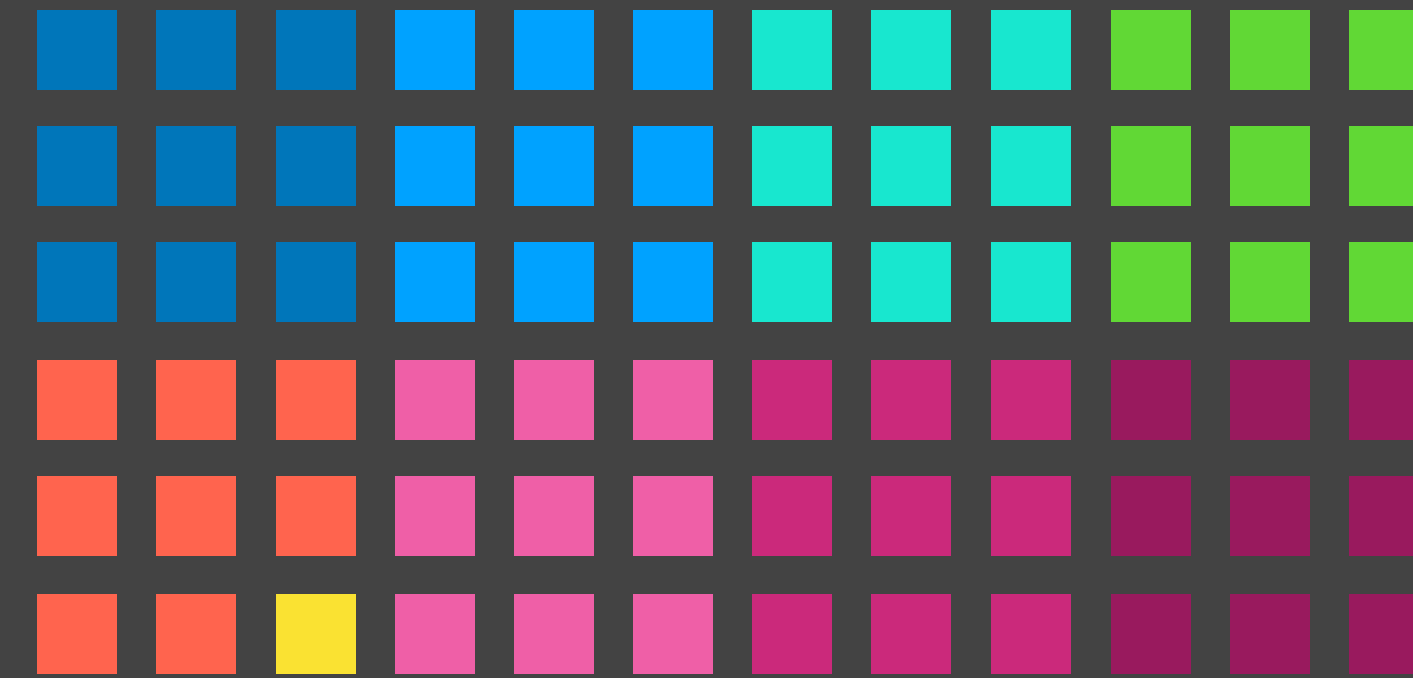
Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

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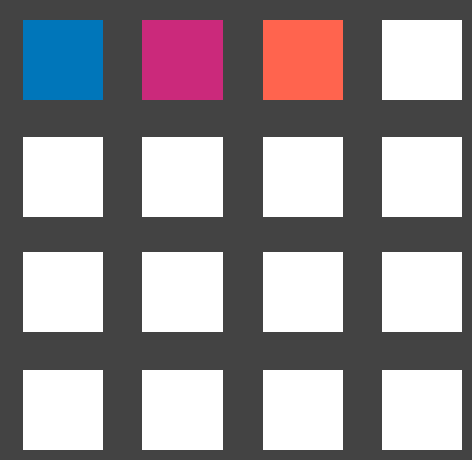


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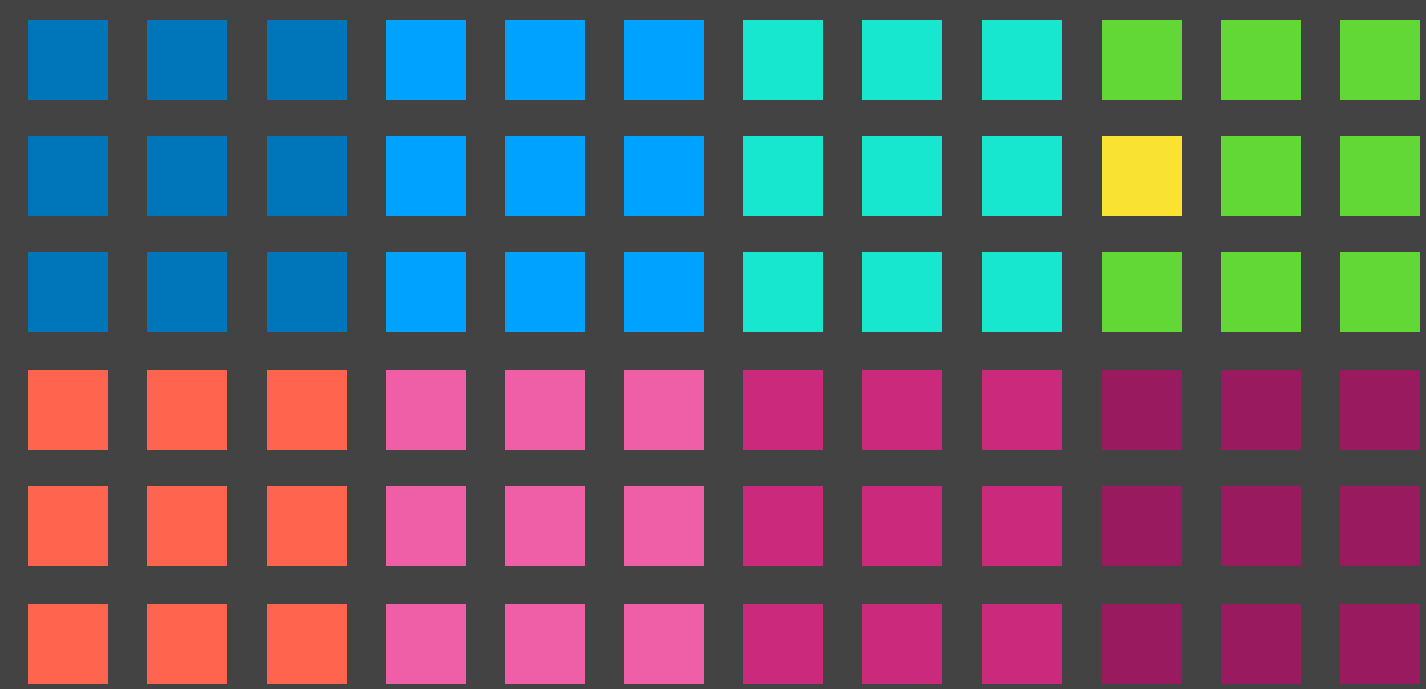


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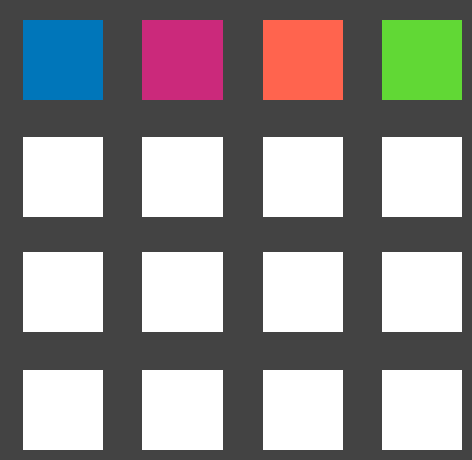


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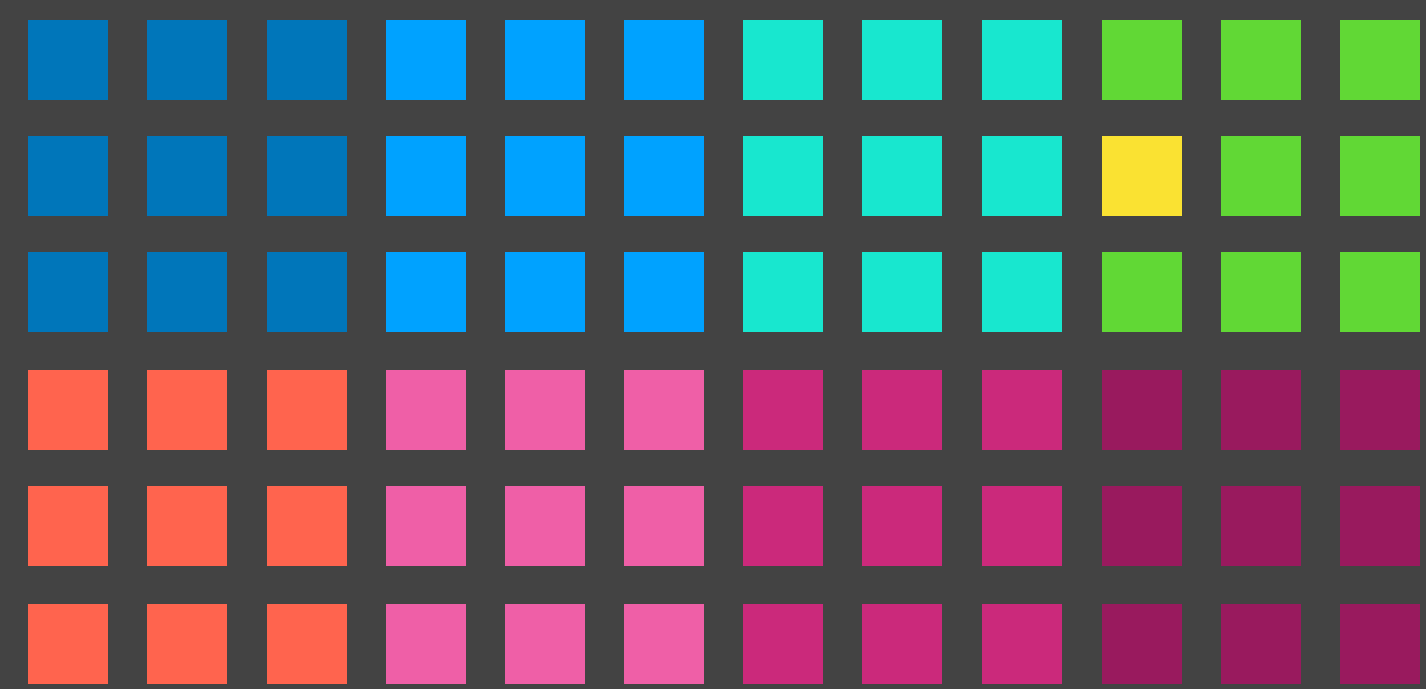


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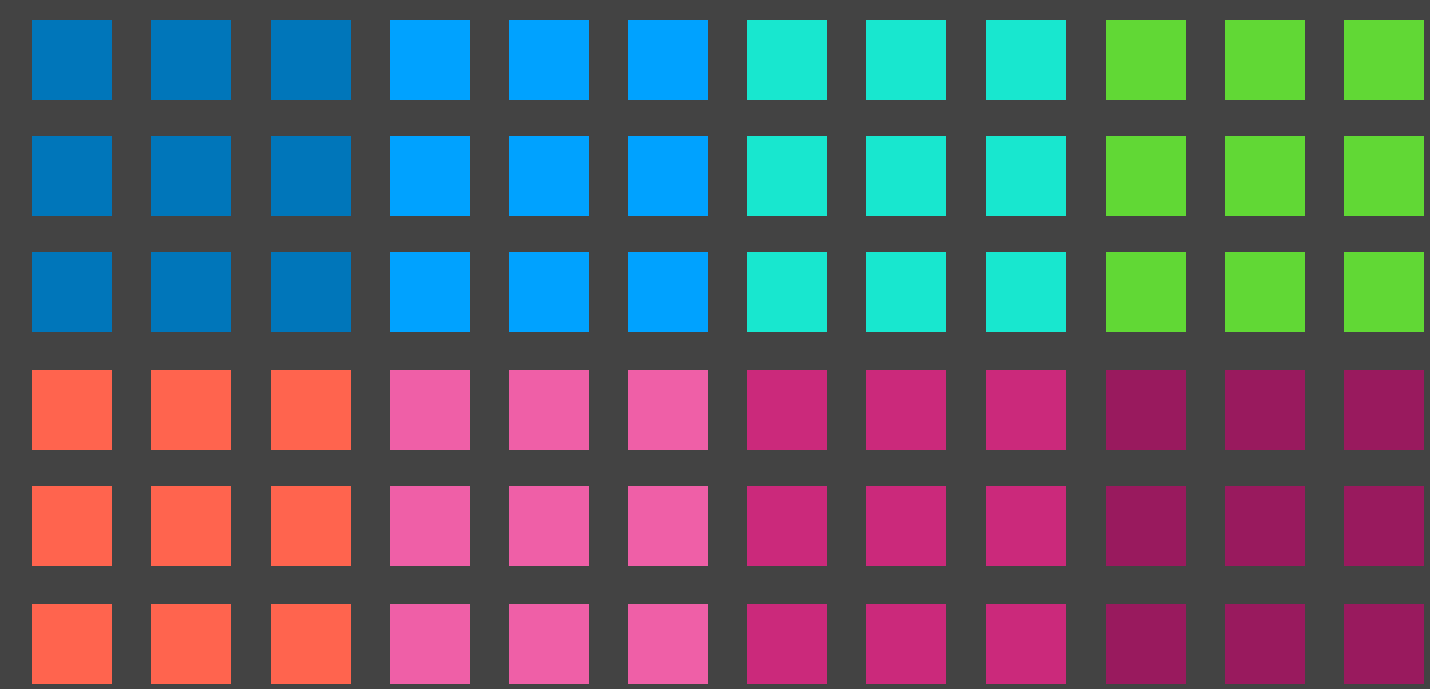


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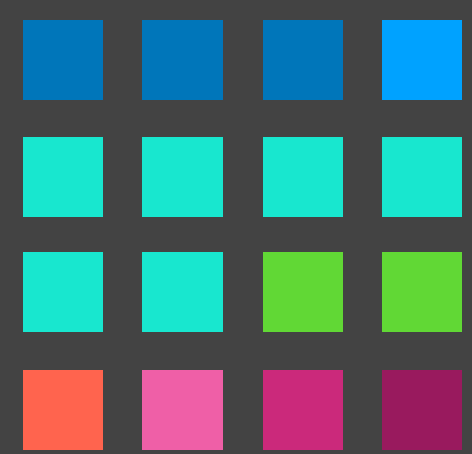


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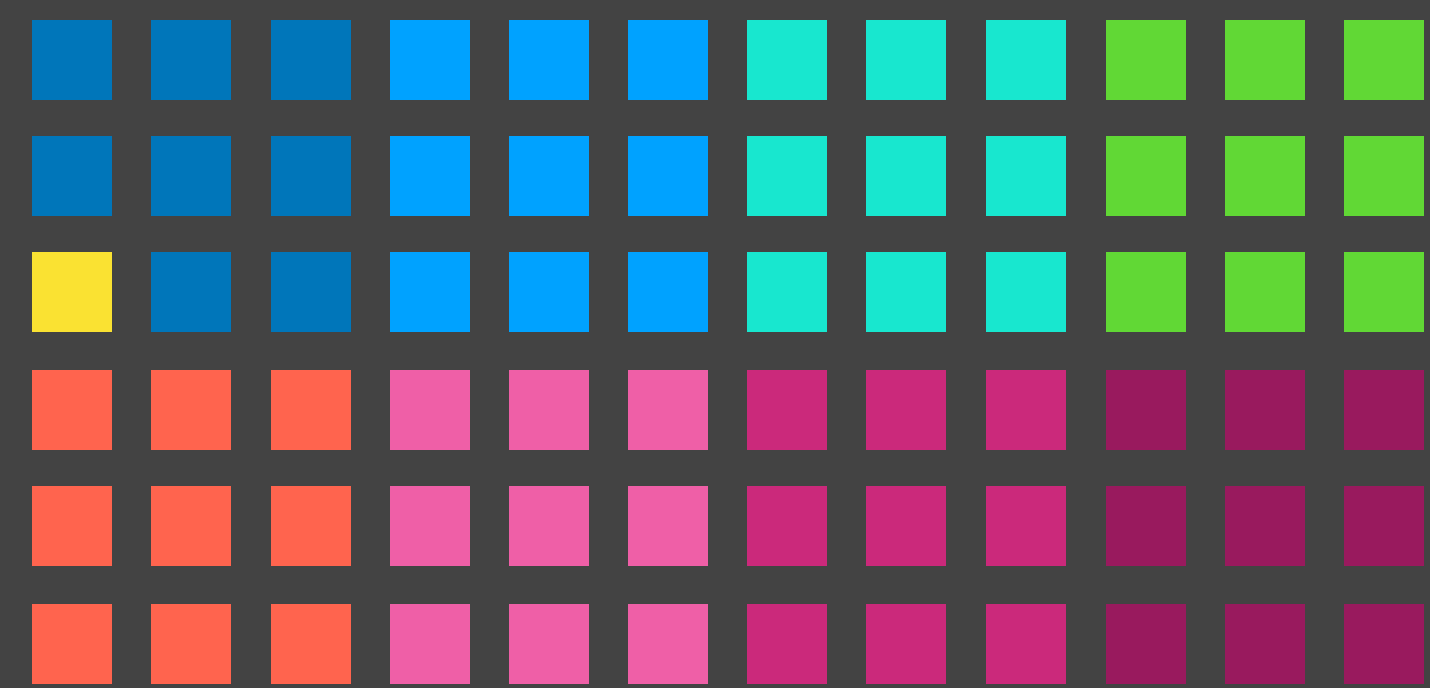


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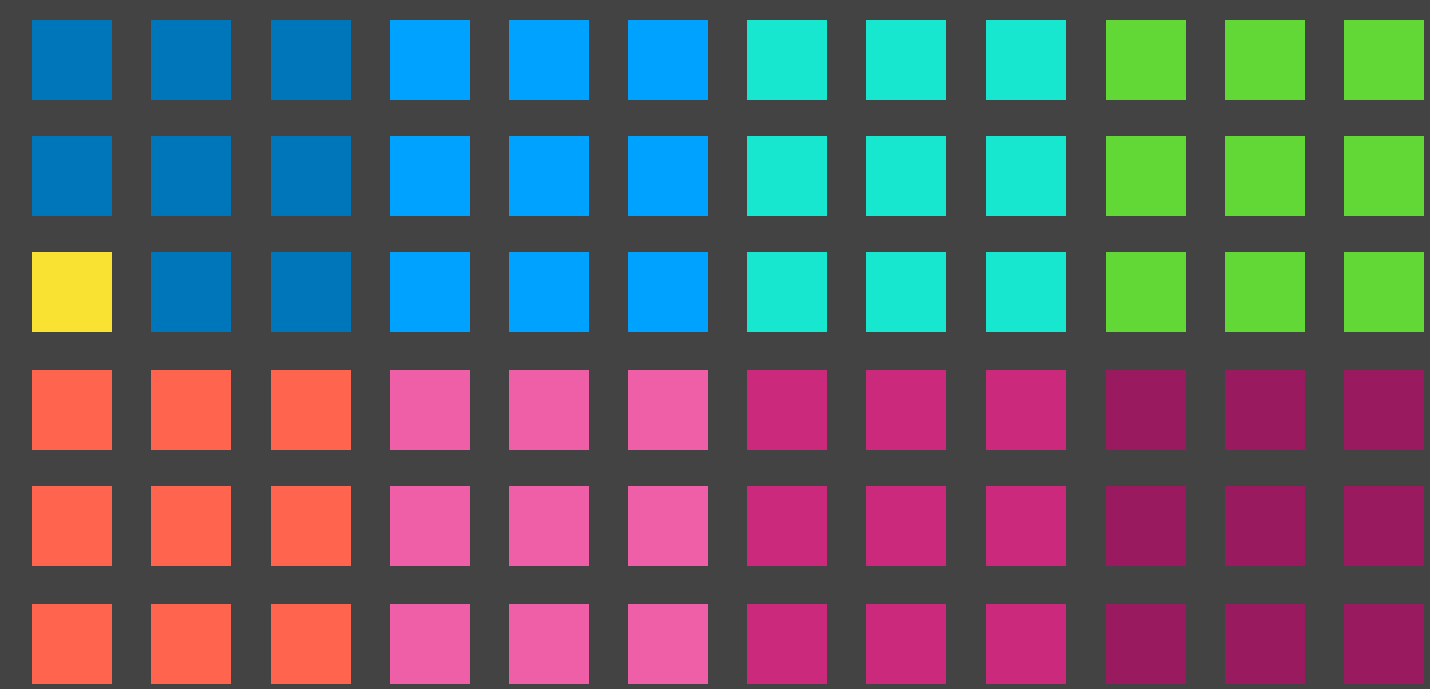


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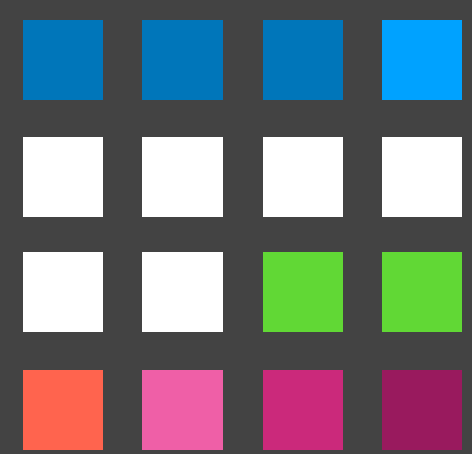


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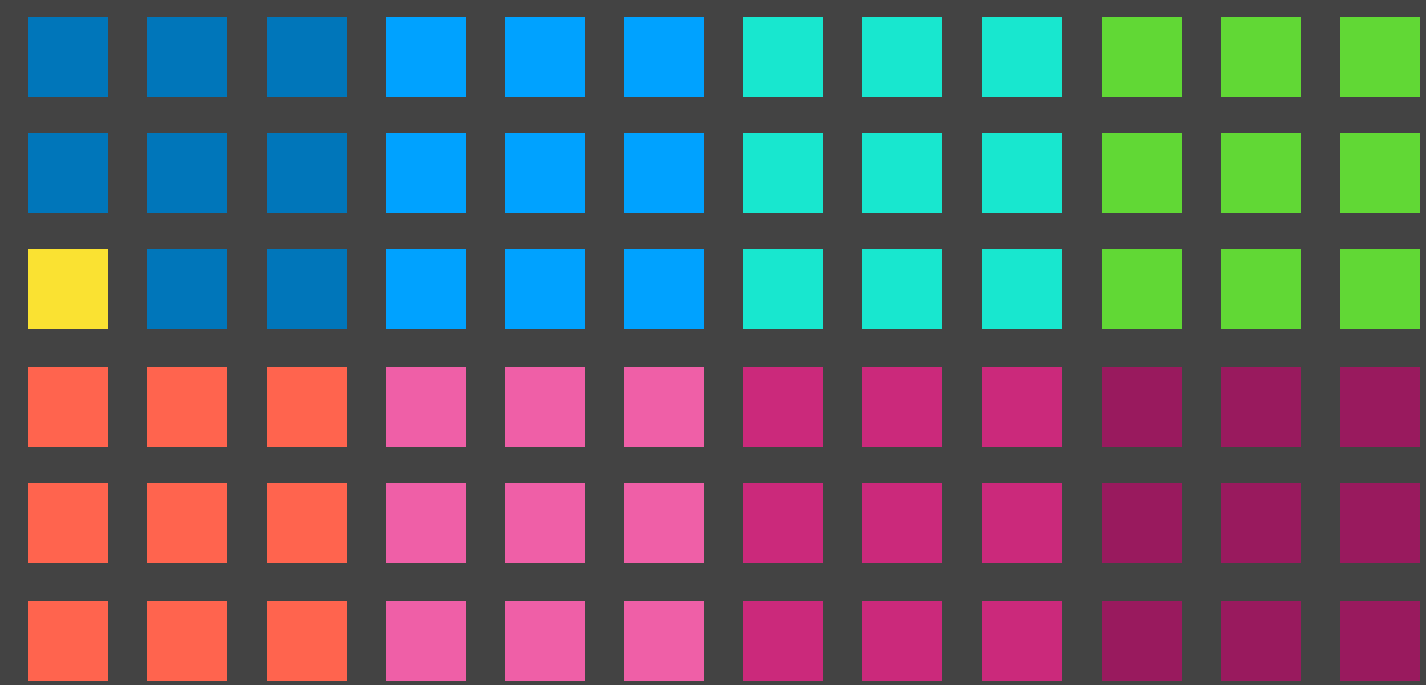


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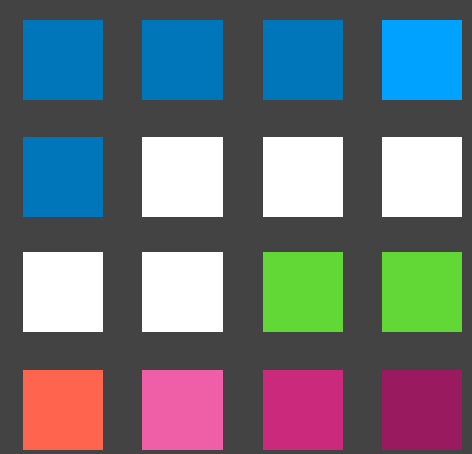


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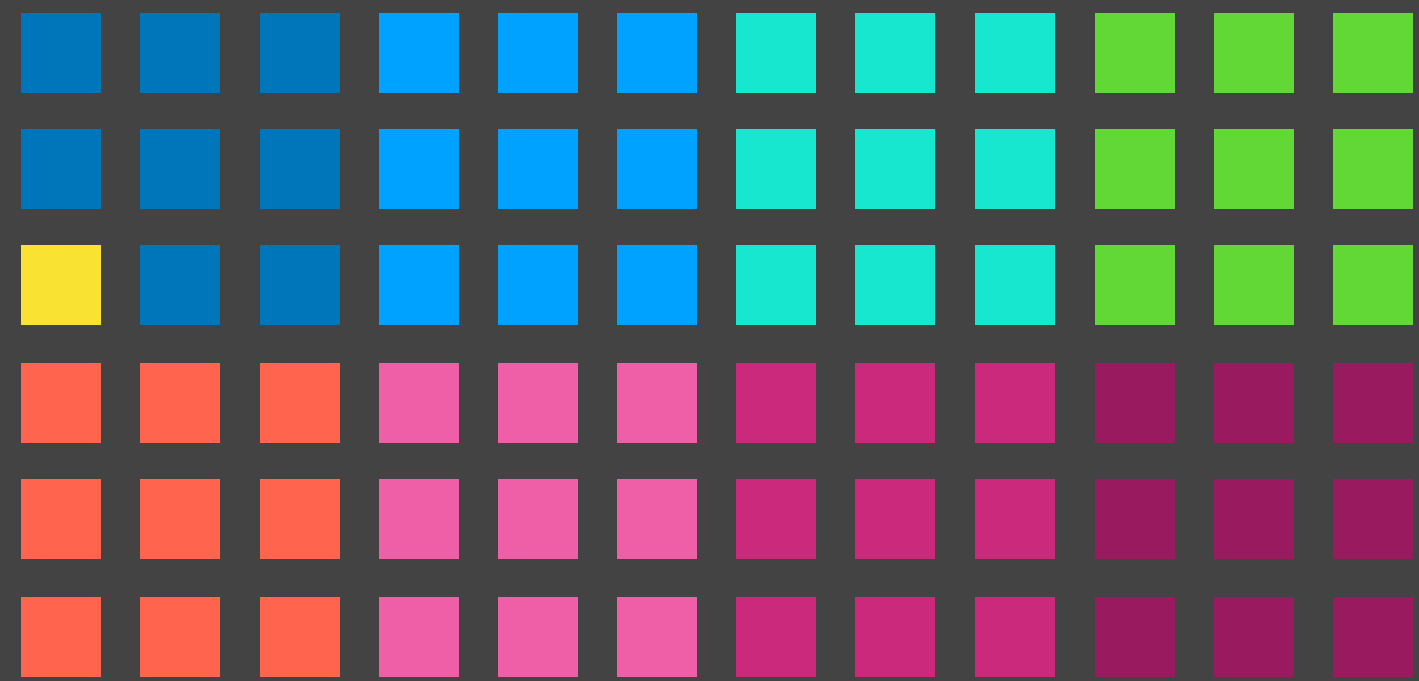


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Cache of size k

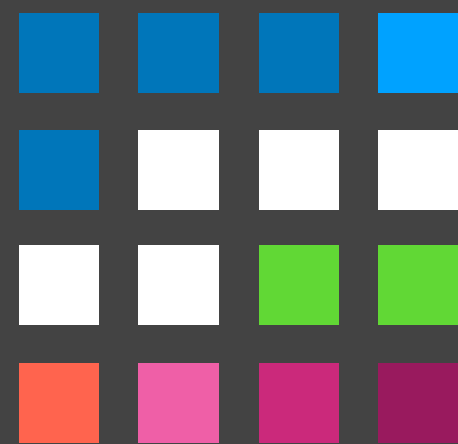


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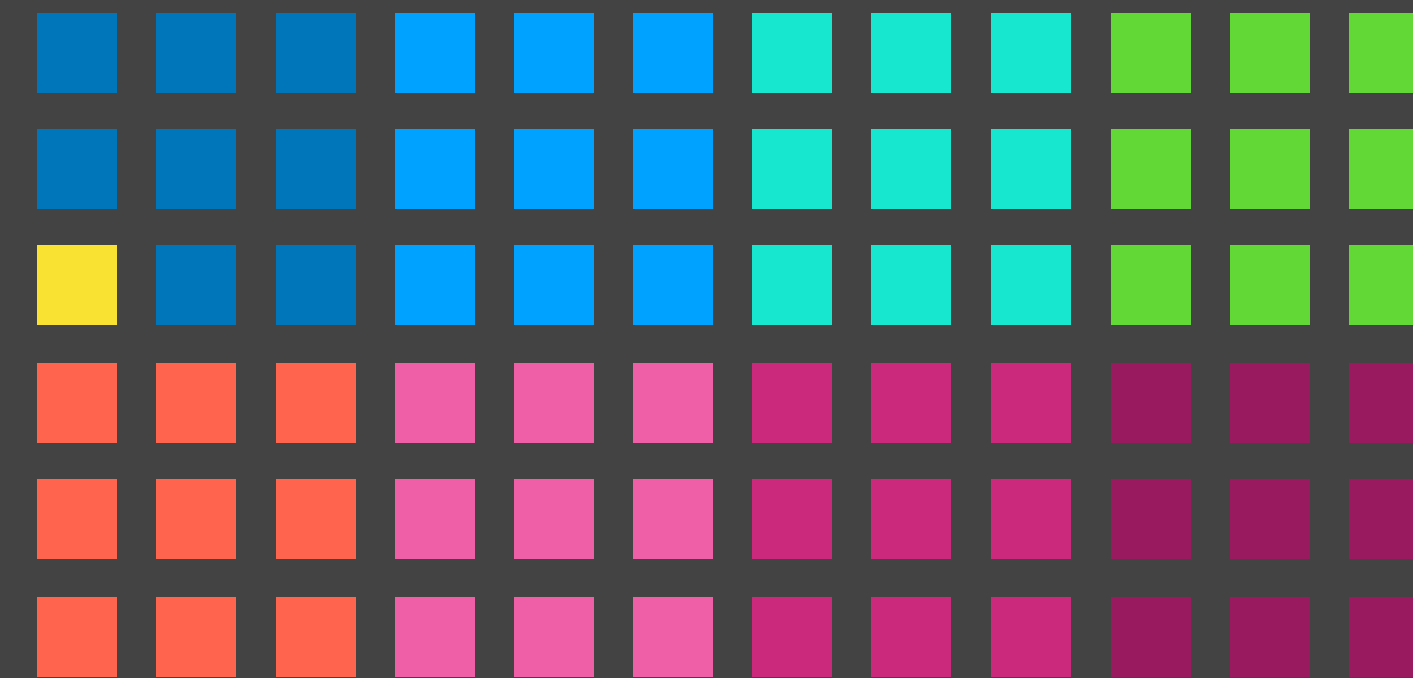


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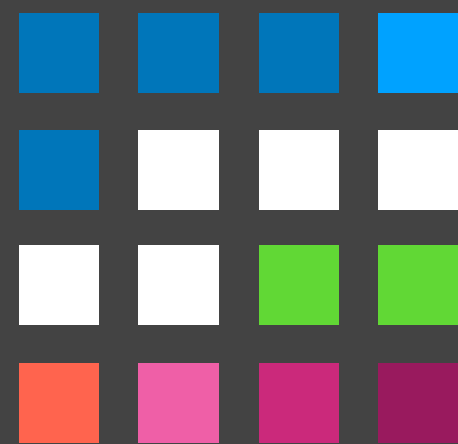
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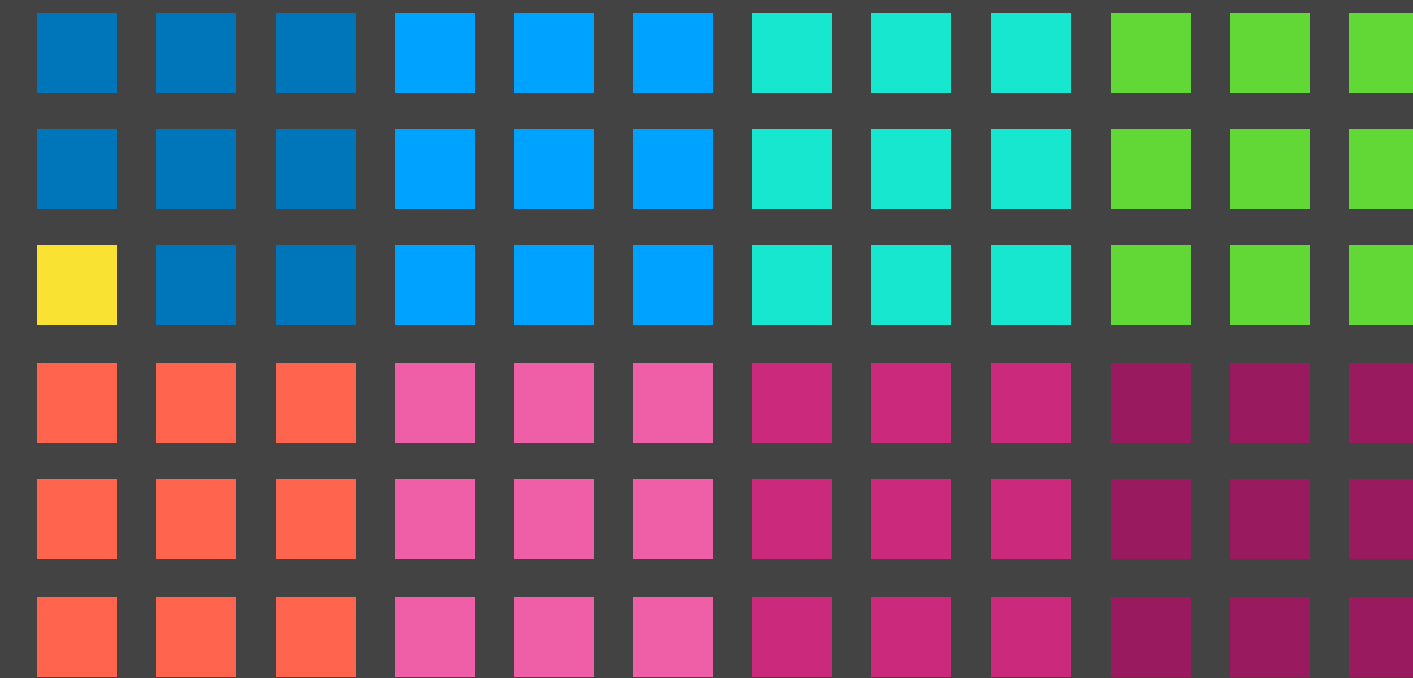
Goal is to minimize number of **blocks** fetched/evicted!

Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

Cache of size k



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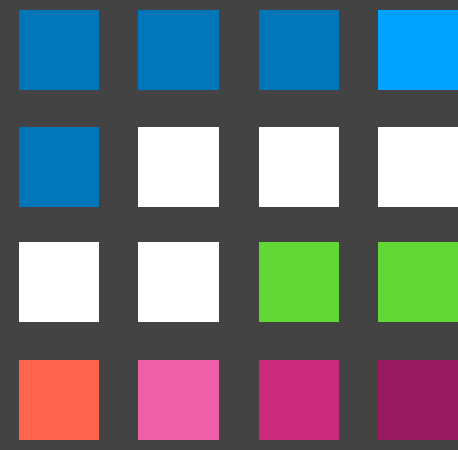


Goal is to minimize number of **blocks** fetched/evicted!

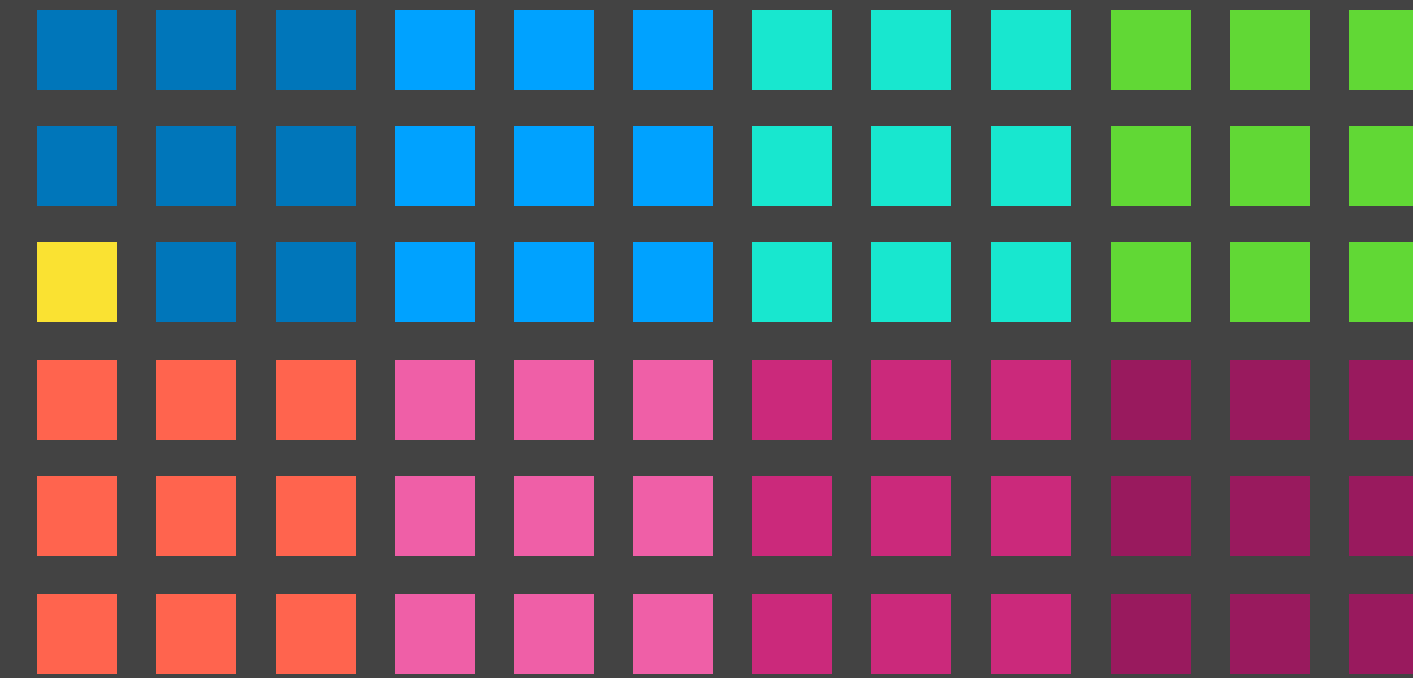
[Beckmann
Gibbons
McGuffey
SPAA 21]

Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

Cache of size k

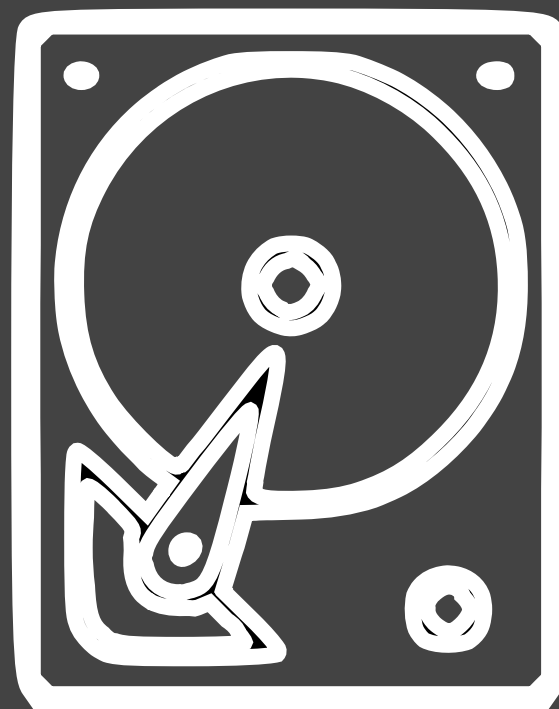


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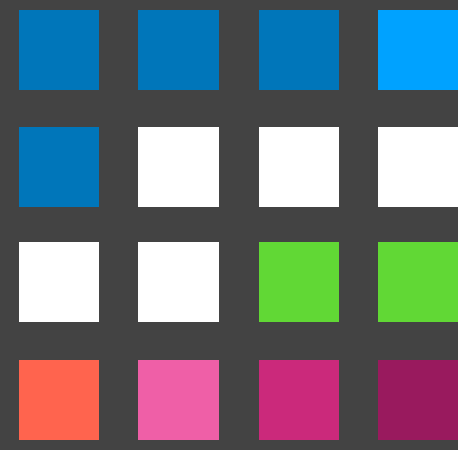
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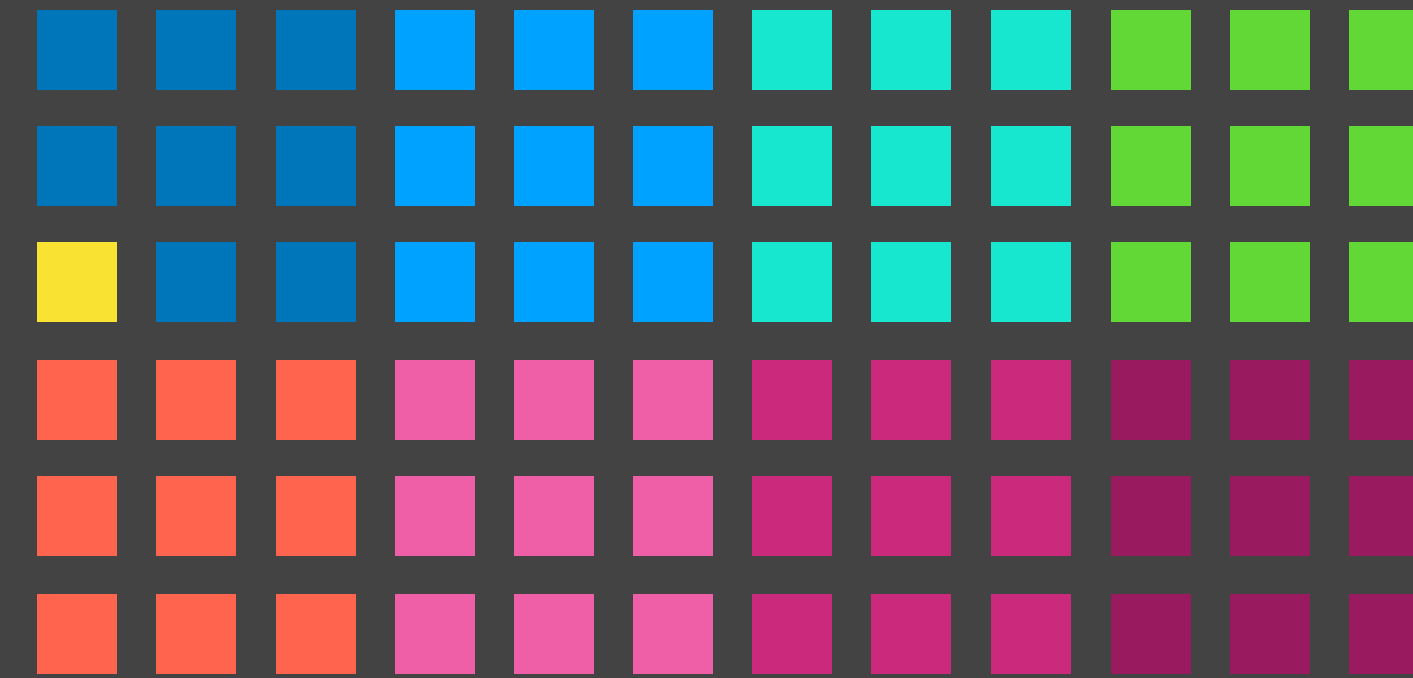


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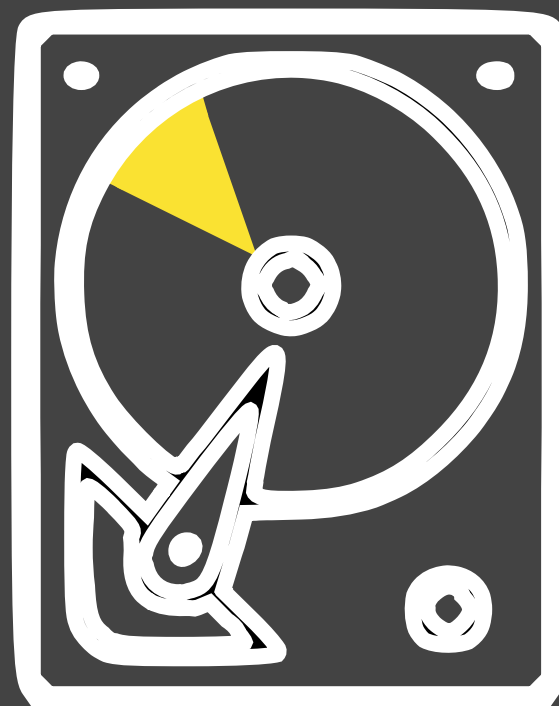


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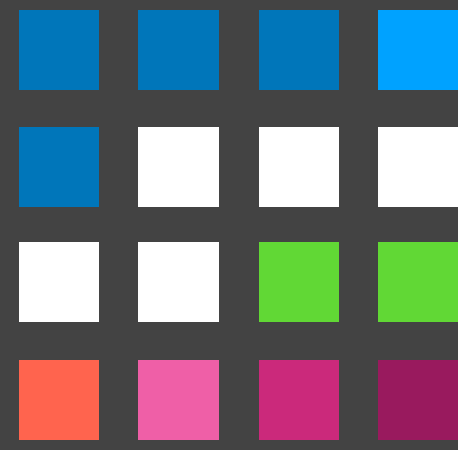
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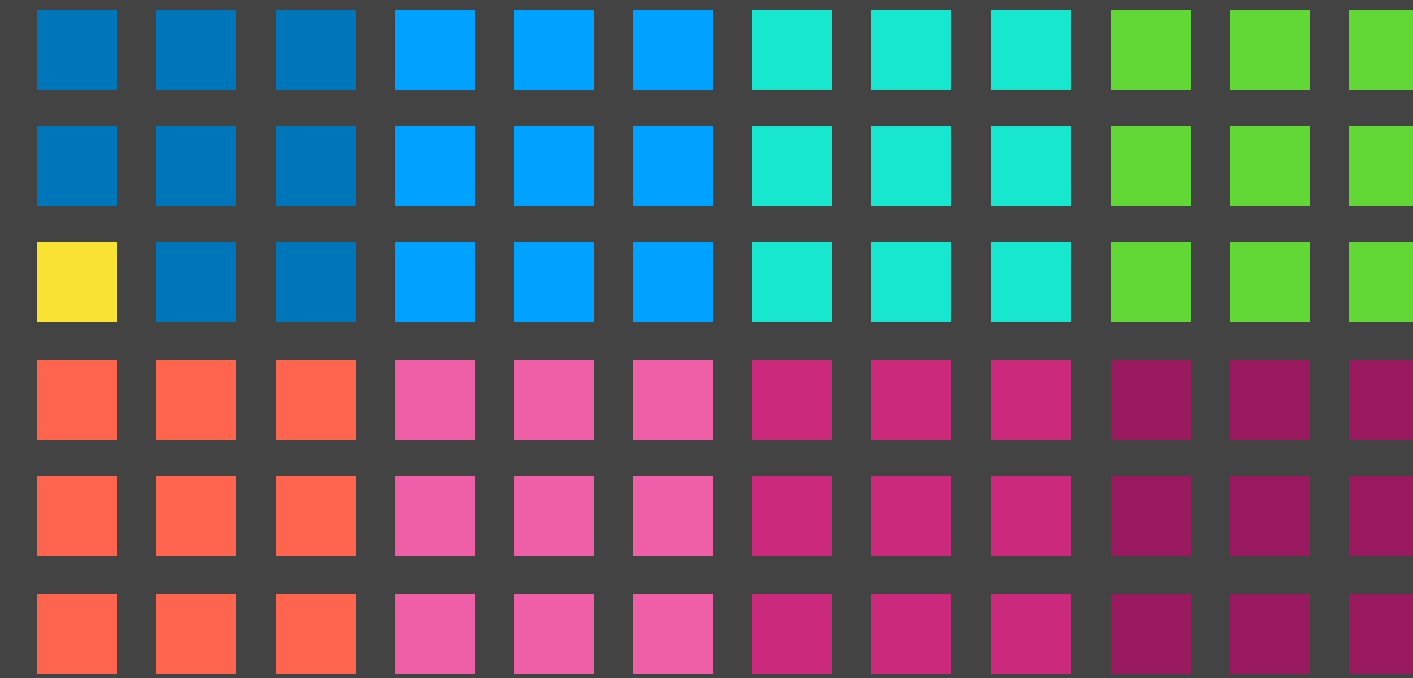


Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

Cache of size k

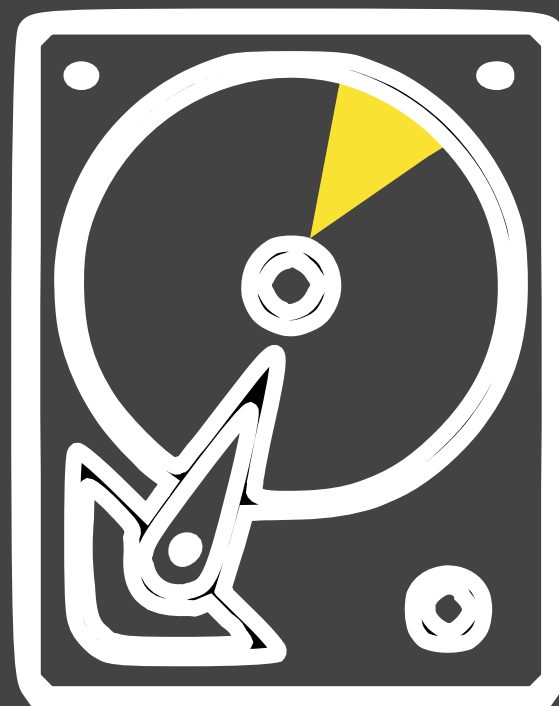


n total pages, **divided into blocks**



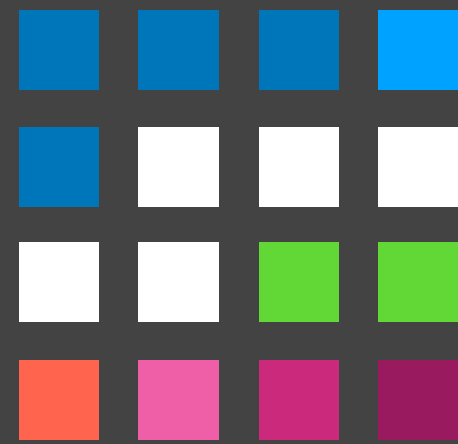
Goal is to minimize number of **blocks** fetched/evicted!

[Beckmann
Gibbons
McGuffey
SPAA 21]

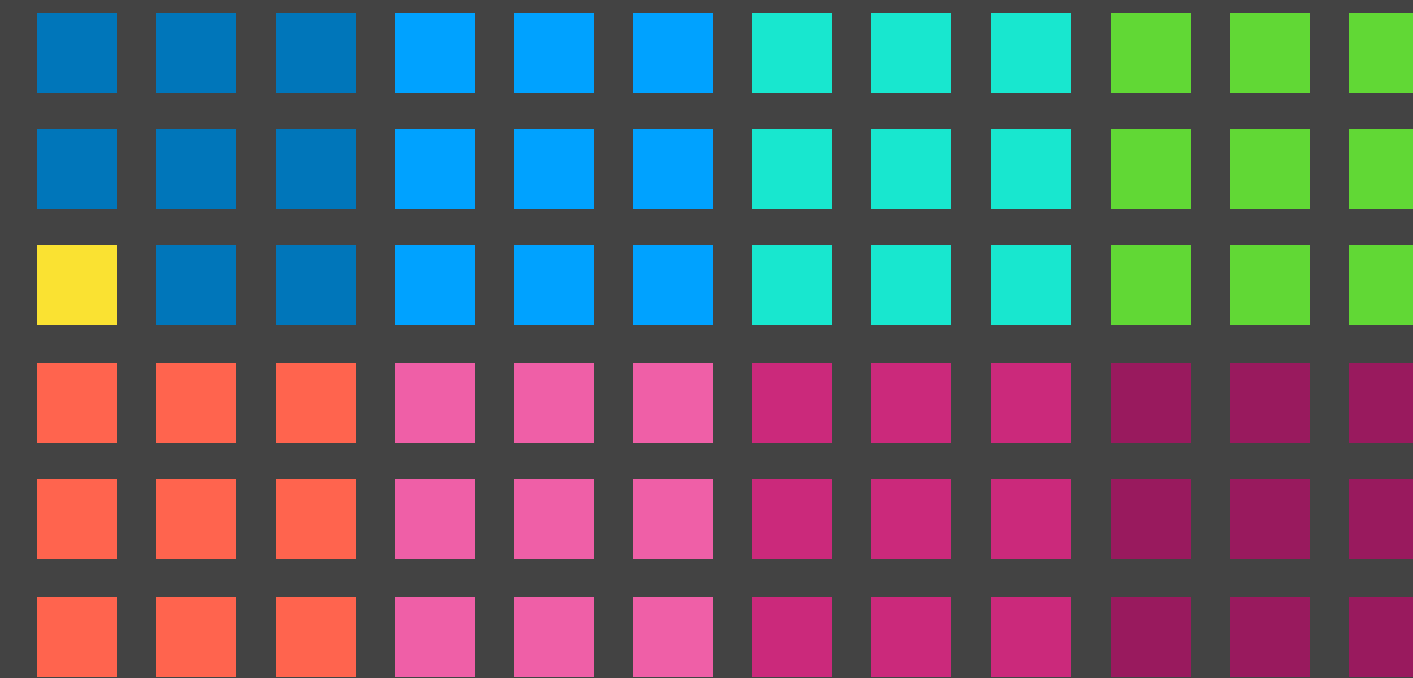


Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

Cache of size k

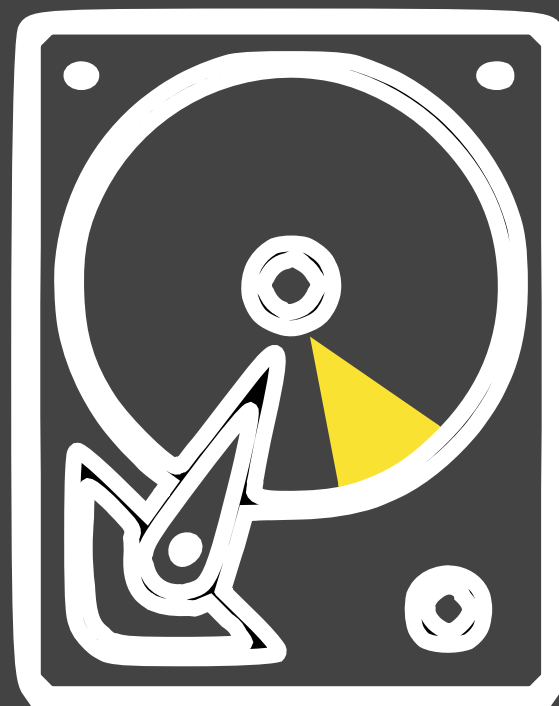


n total pages, **divided into blocks**



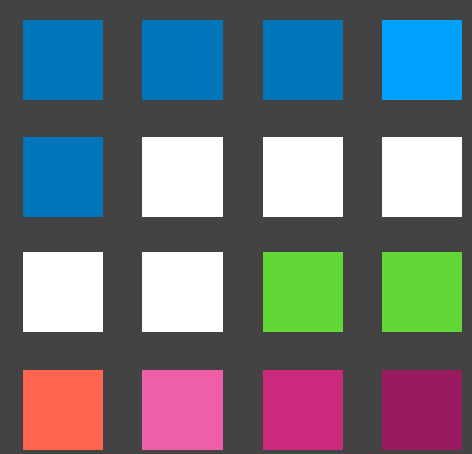
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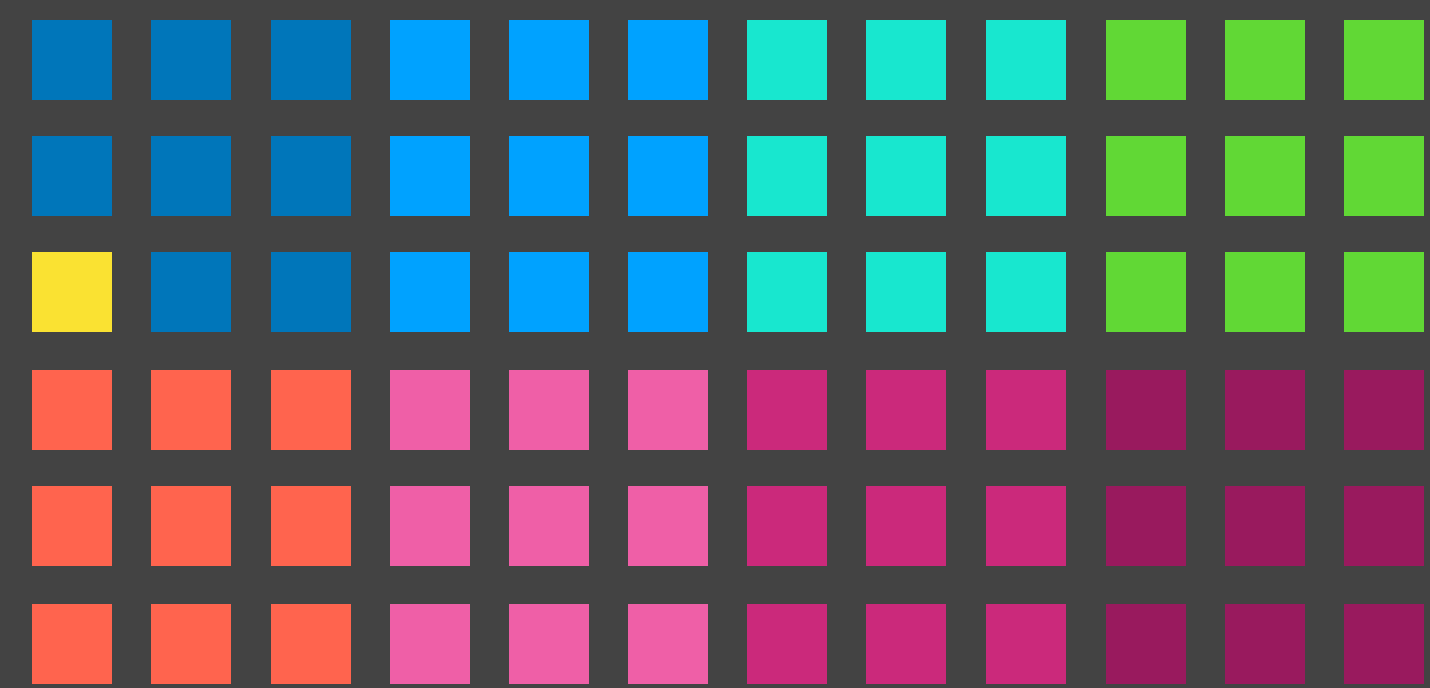


Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

Cache of size k



n total pages, **divided into blocks**



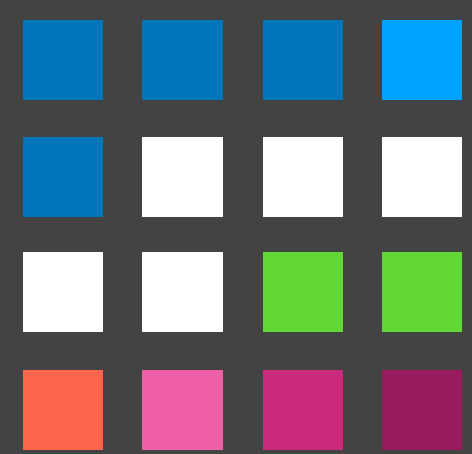
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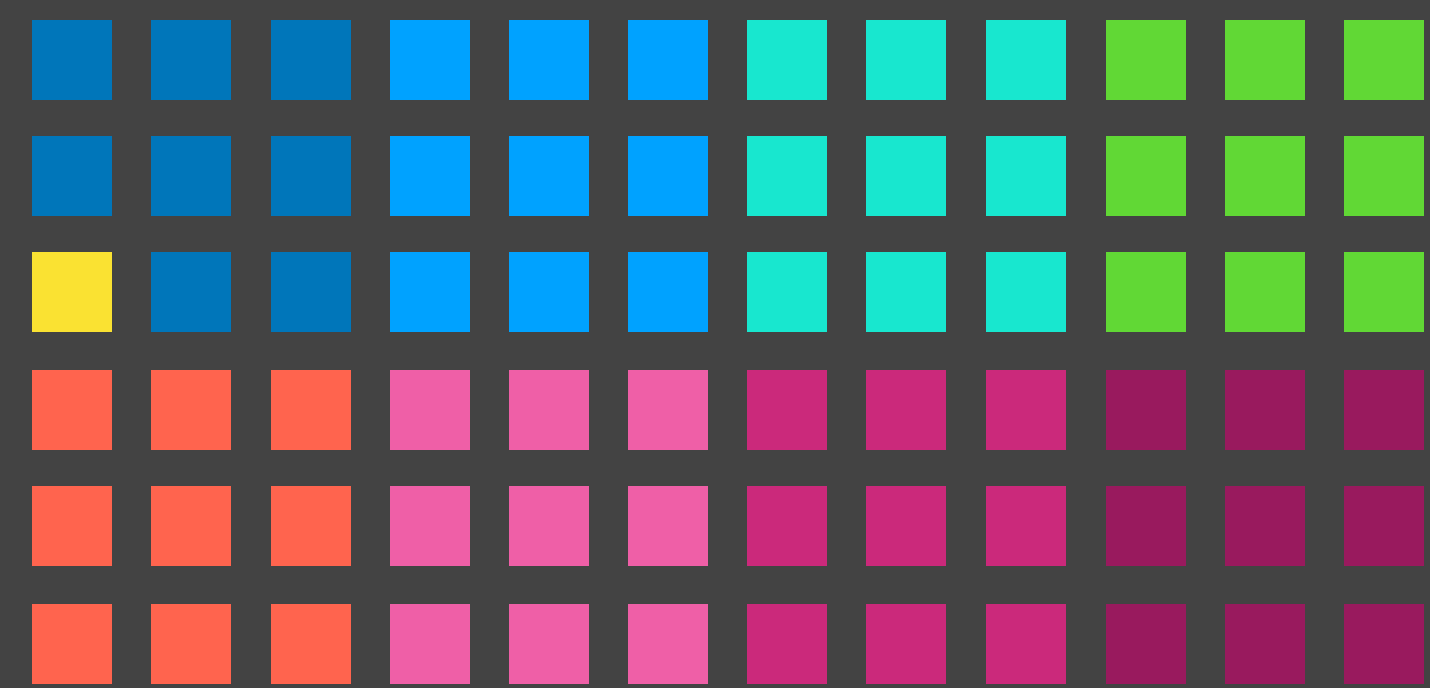


Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

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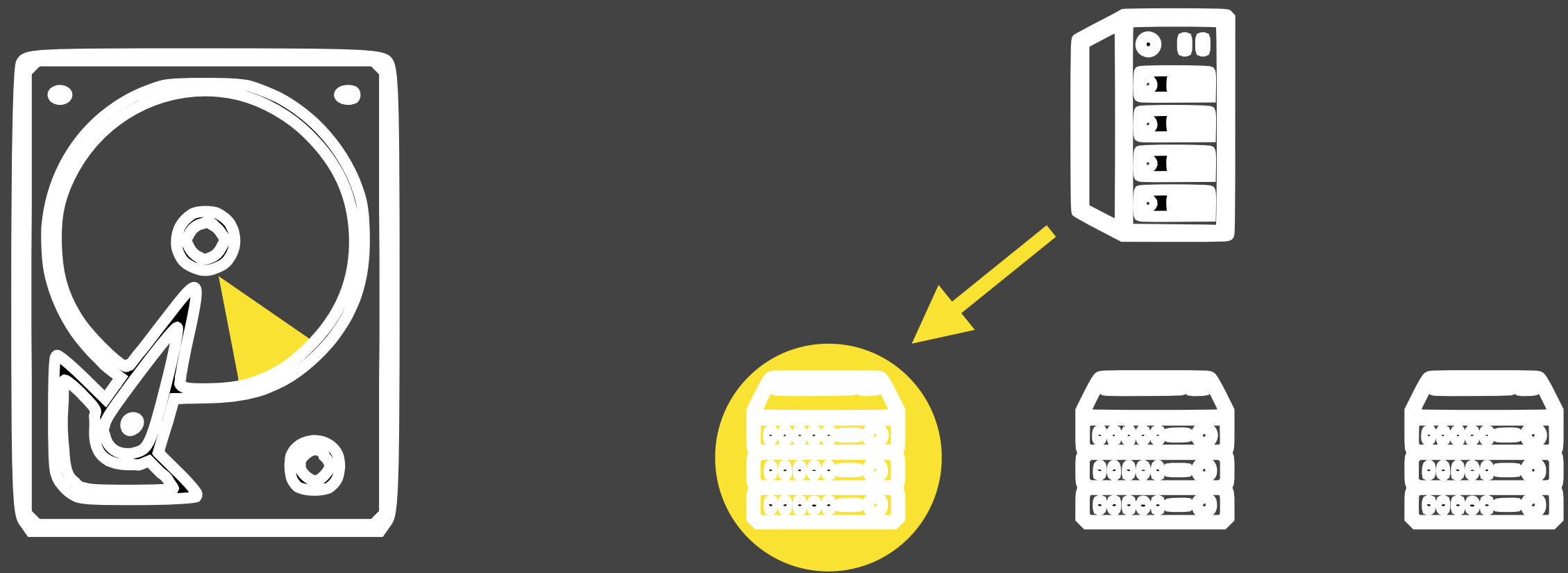


n total pages, **divided into blocks**



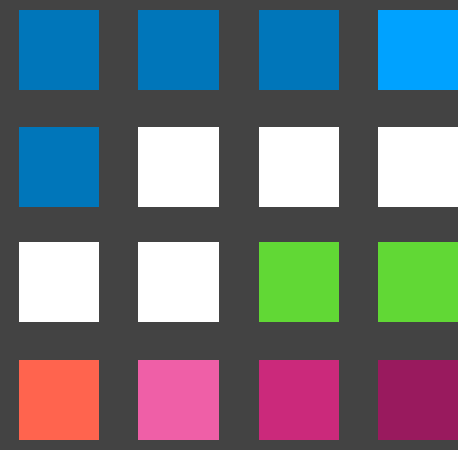
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Gibbons
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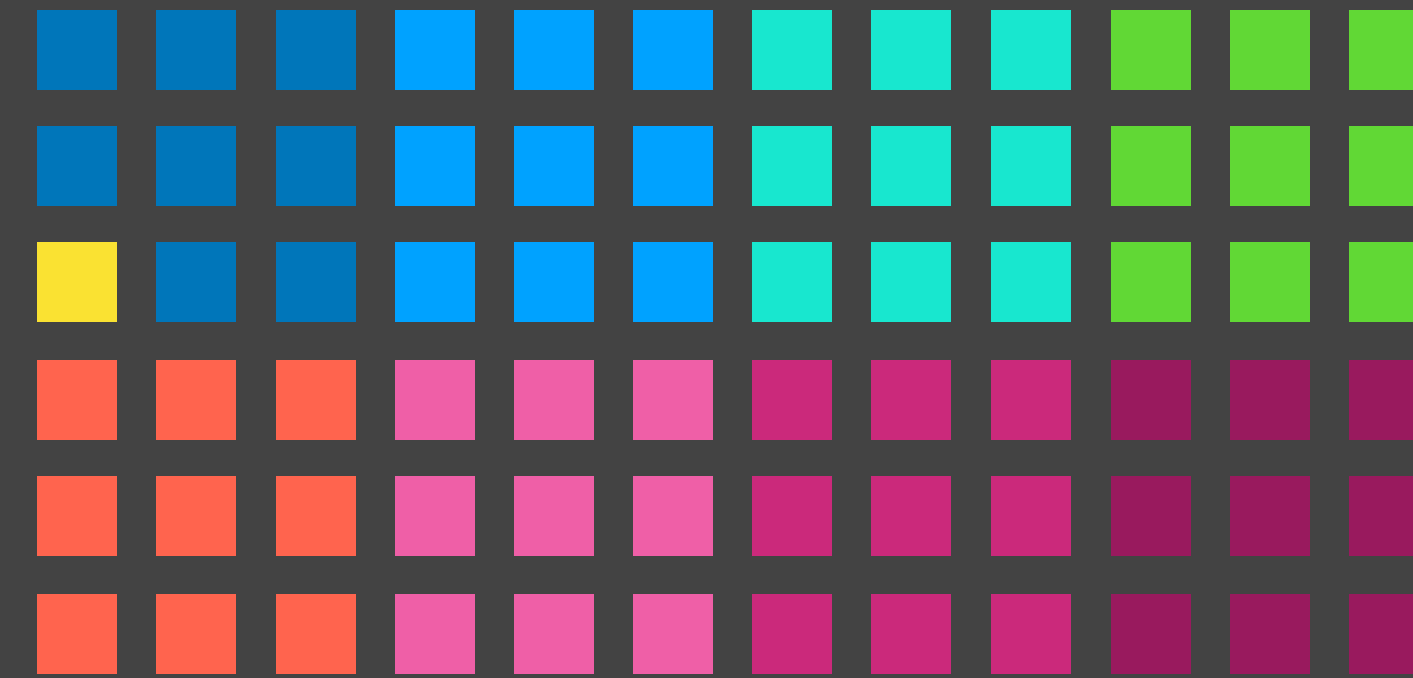


Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

Cache of size k

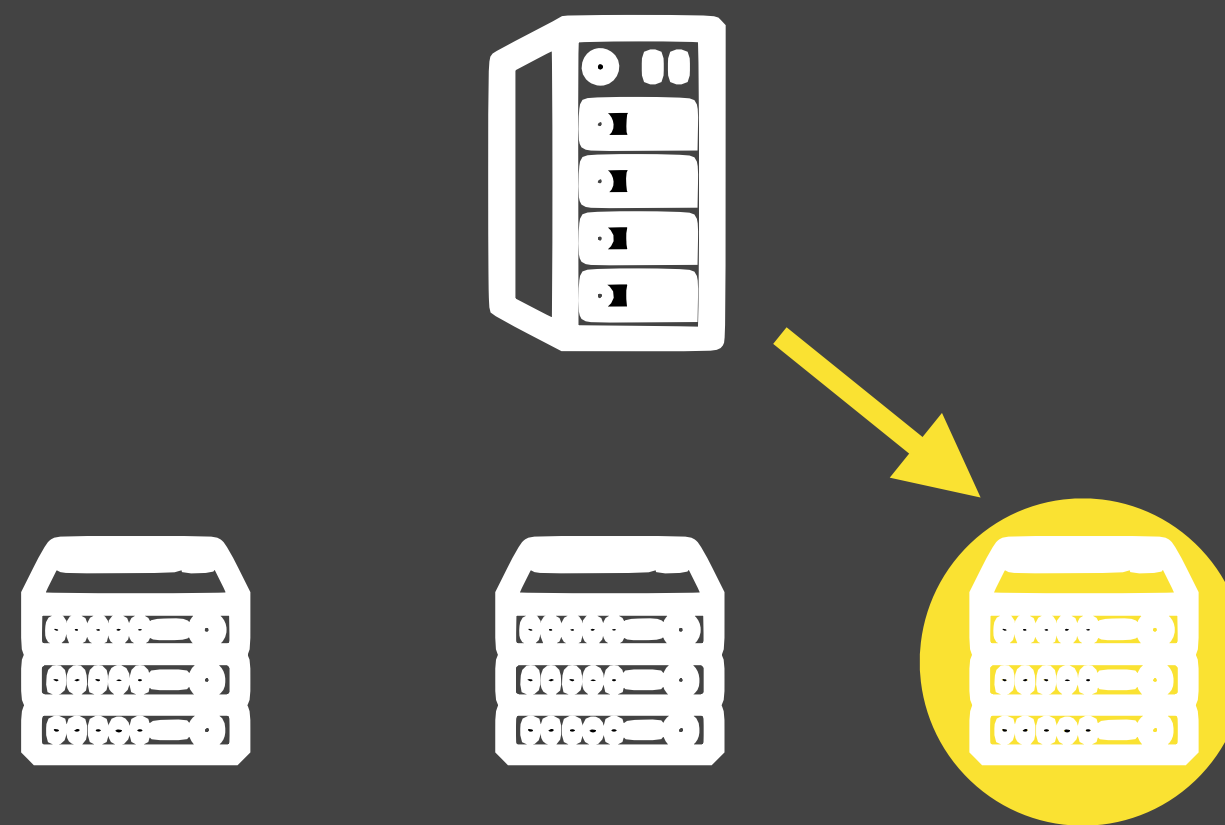
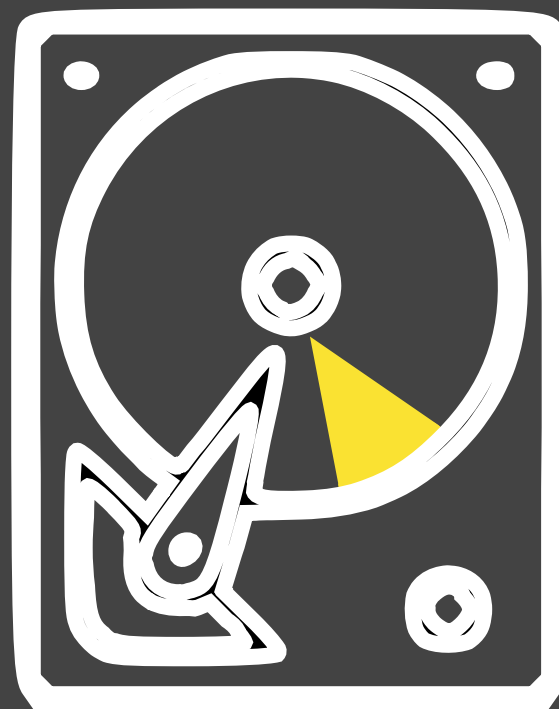


n total pages, **divided into blocks**



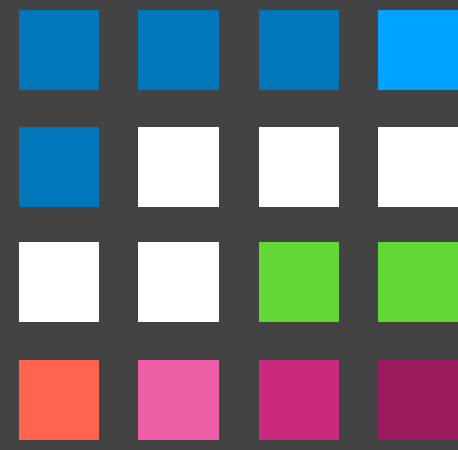
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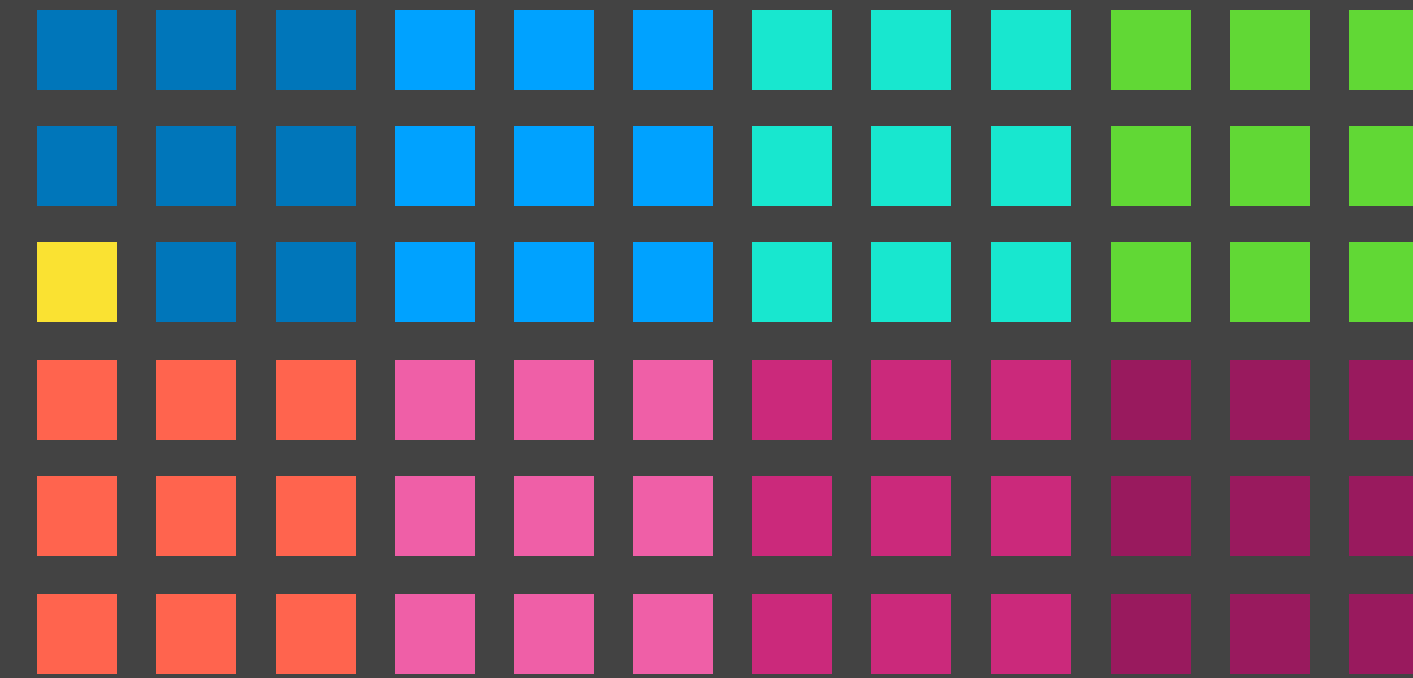


Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

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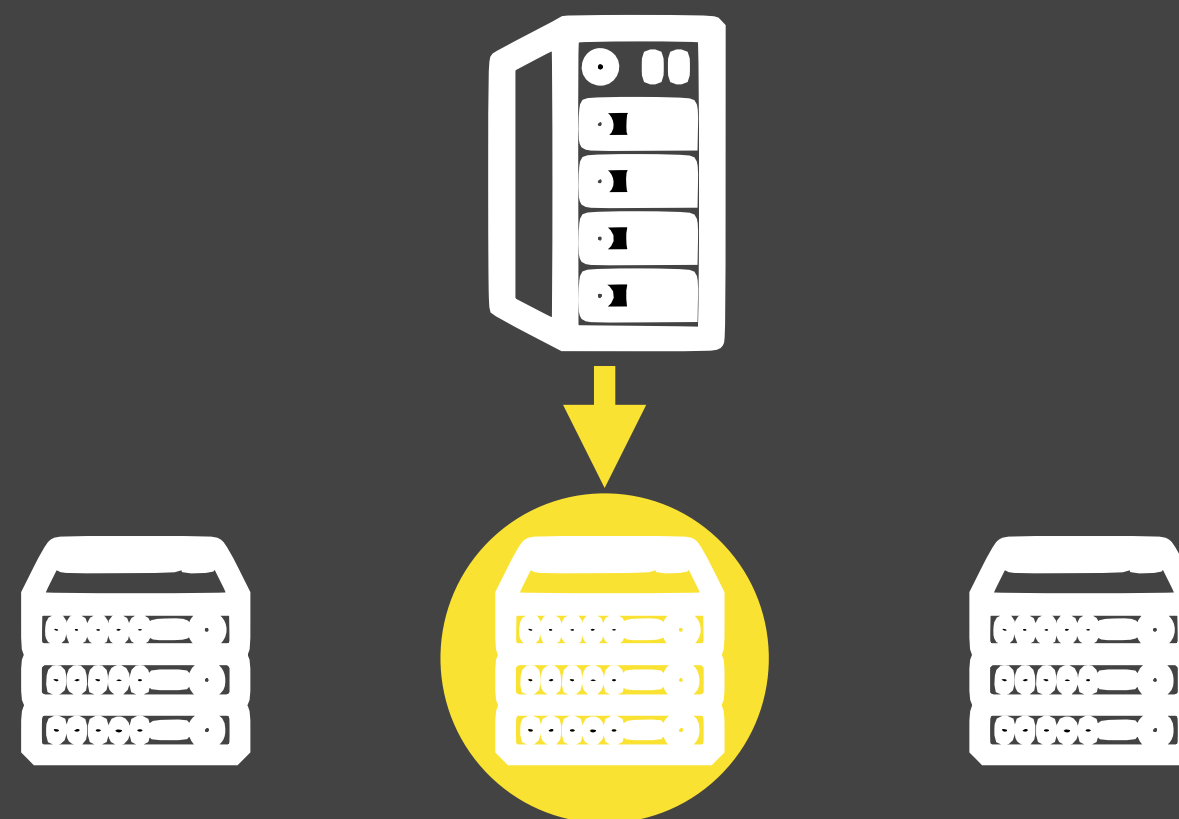
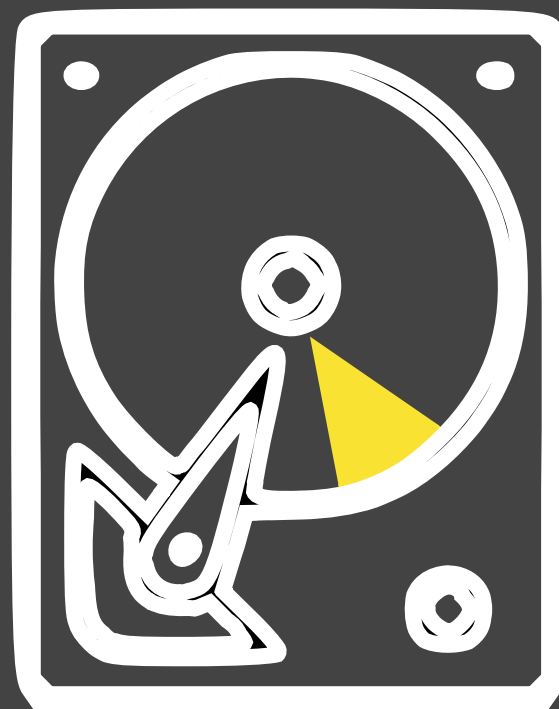


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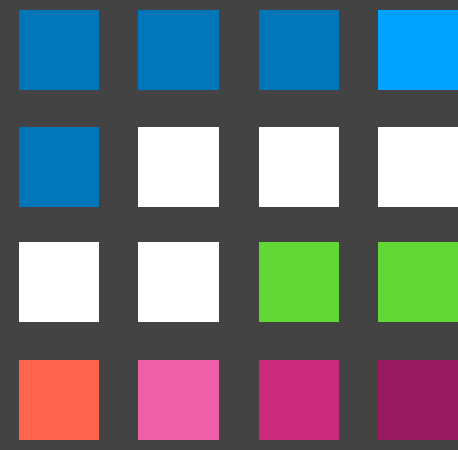
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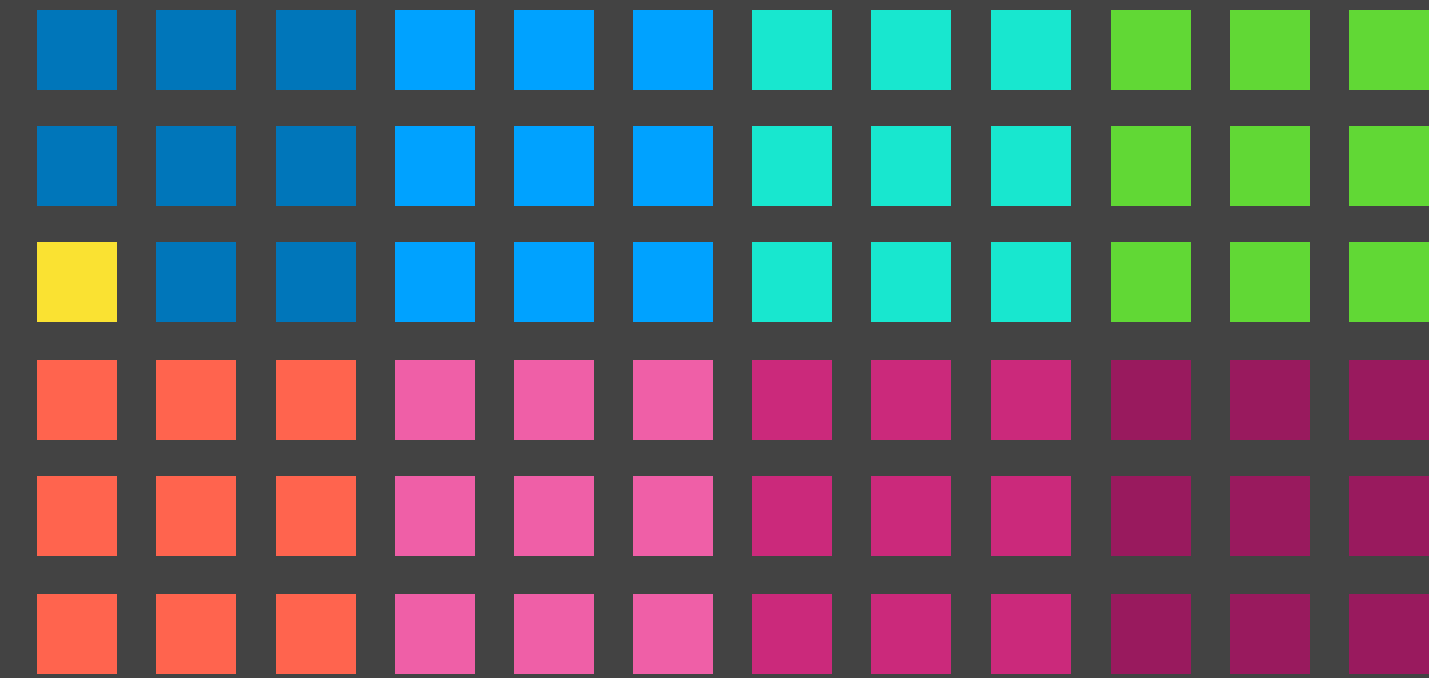


Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

Cache of size k

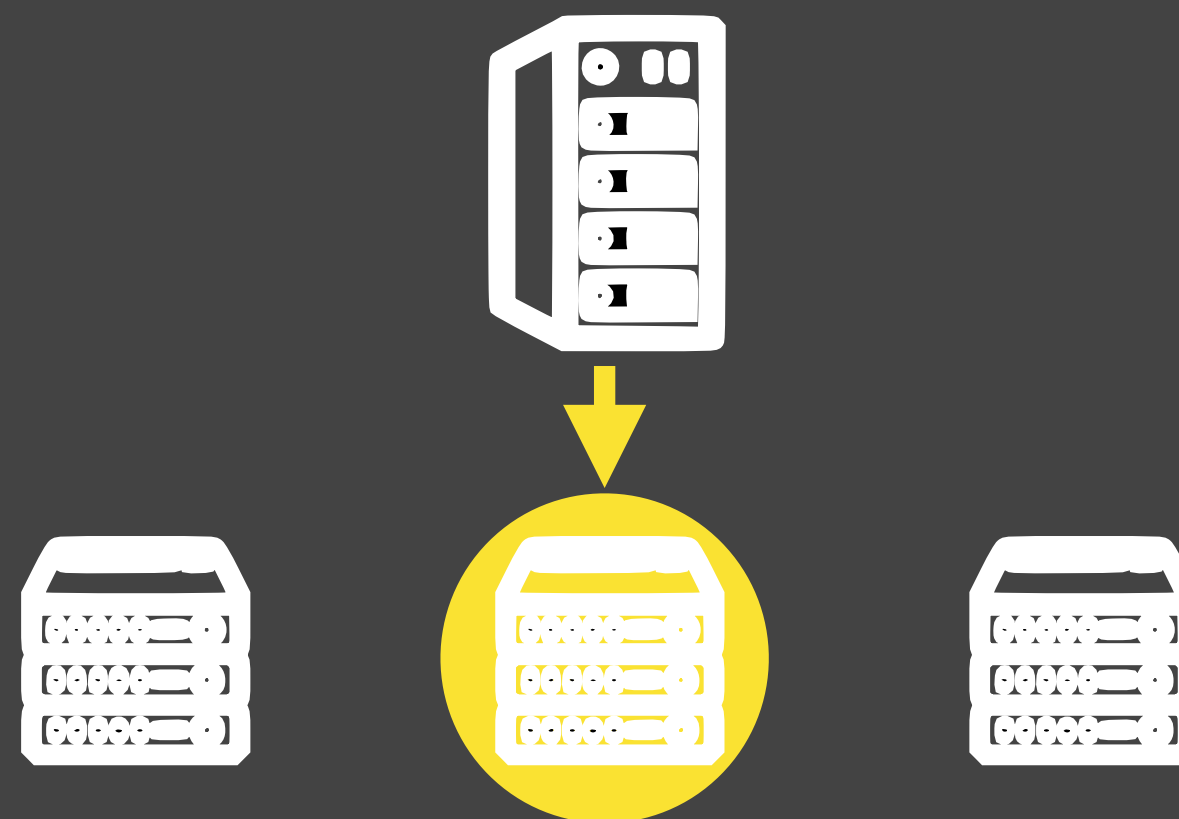
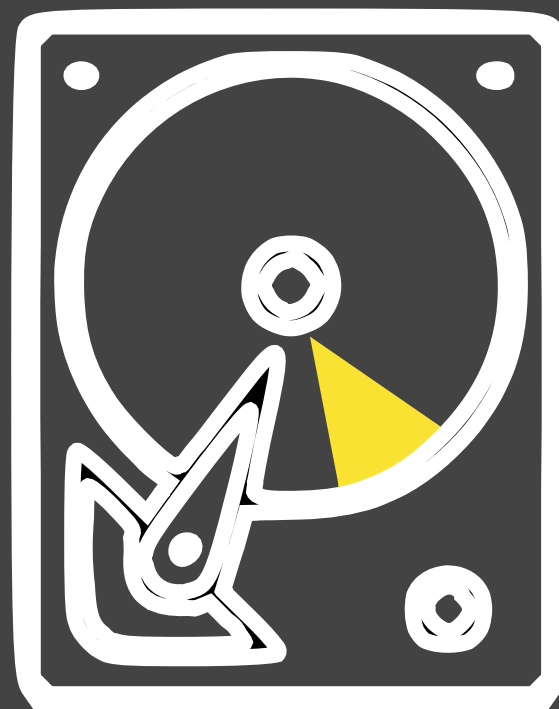


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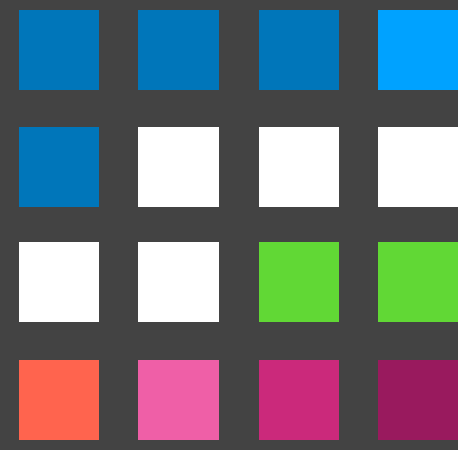
[Beckmann
Gibbons
McGuffey
SPAA 21]



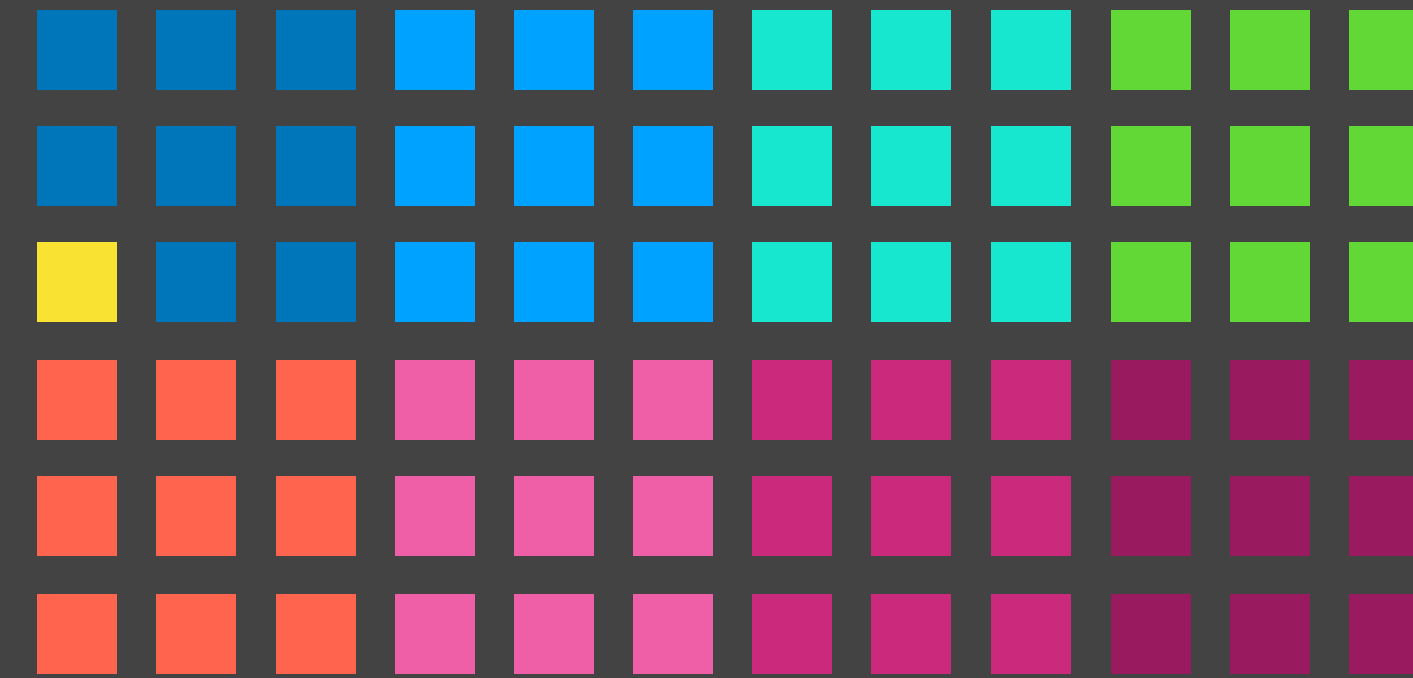
We give **near-optimal**
algorithms using [GL. 20]!

Block-Aware Caching [Coester, Naor, L., Talmon SPAA 22]

Cache of size k

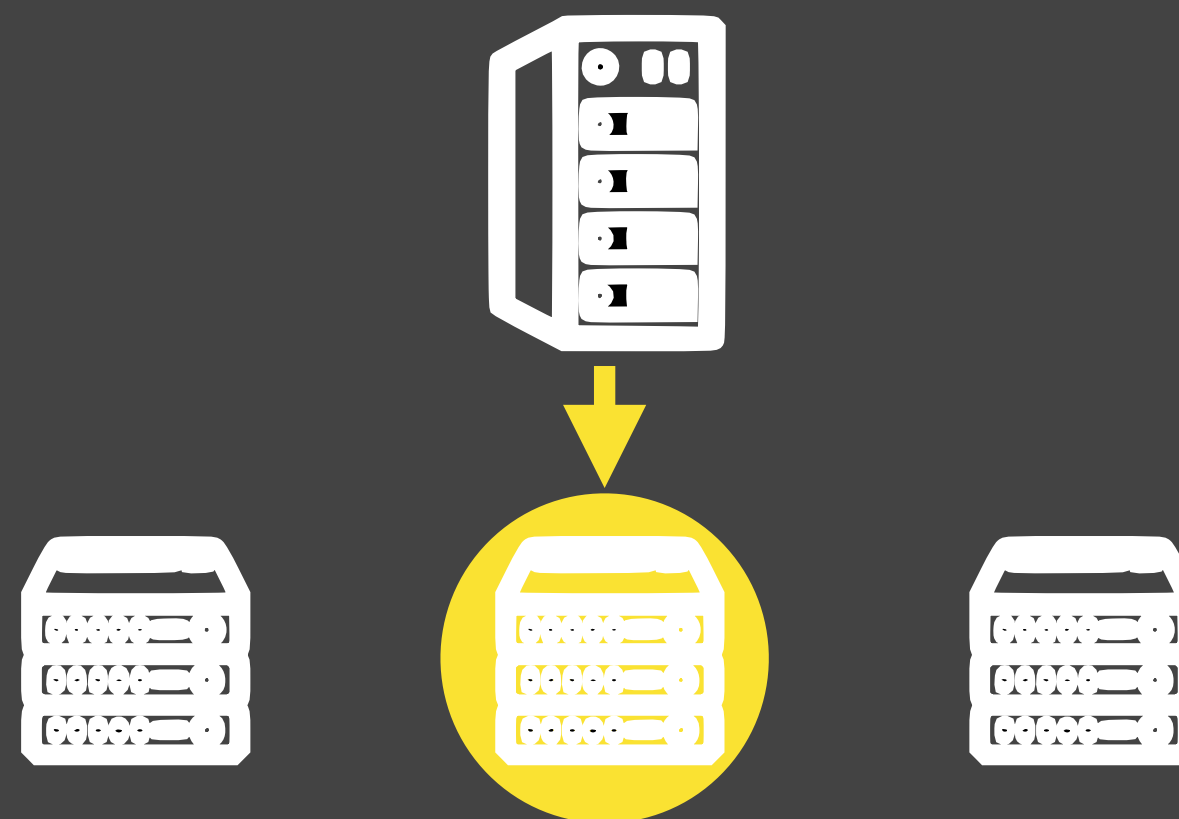
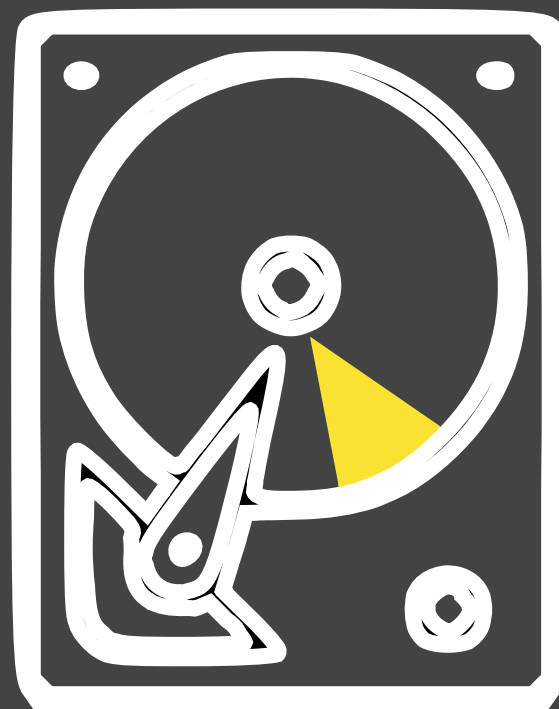


n total pages, **divided into blocks**



Goal is to minimize number of **blocks** fetched/evicted!

[Beckmann
Gibbons
McGuffey
SPAA 21]



We give **near-optimal** algos using [GL. 20]!

Reduction to **Online** submodular cover!

Take Away I

[Gupta L. SODA 20]

[Coester, Naor, L., Talmon SPAA 22]

Q: What **general**
classes of optimization
problems can we solve
online?

Take Away I

[Gupta L. SODA 20]

[Coester, Naor, L., Talmon SPAA 22]

Q: What **general** classes of optimization problems can we solve **online**?

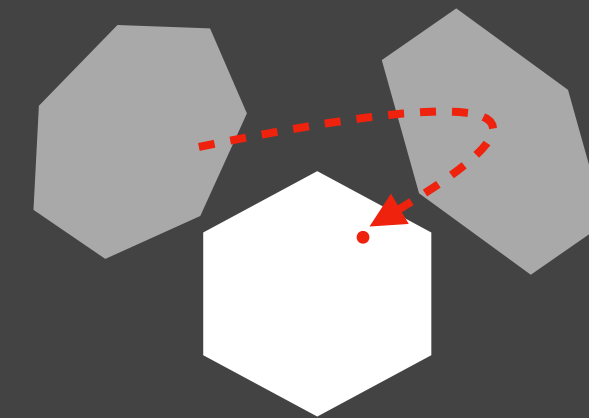
A: Any problem expressible as **Submodular Cover**!

Outline

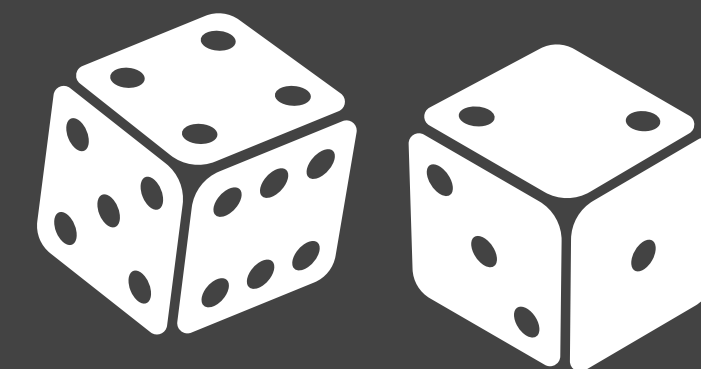
Theme I — Submodular Optimization

$$f(\text{🍕} \mid \text{🥕}) \geq f(\text{🍕} \mid \text{🥕}, \text{🍩})$$

Theme II — Stable Algorithms



Theme III — Beyond Worst-Case Analysis



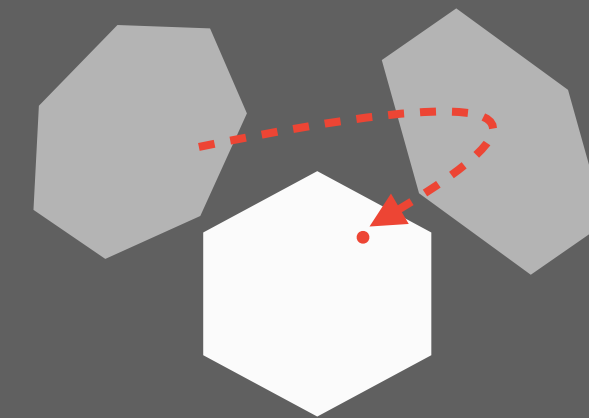
Conclusion

Outline

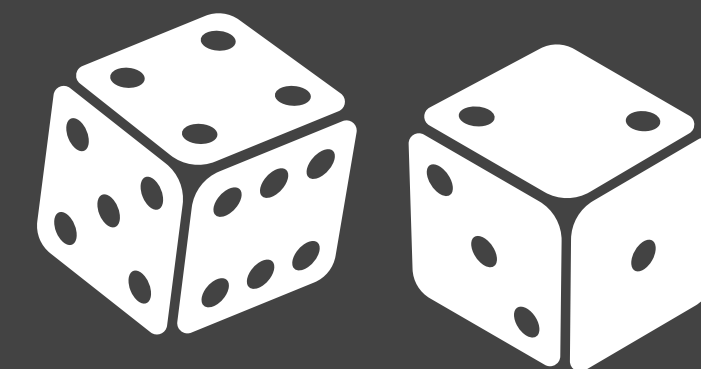
Theme I — Submodular Optimization

$$f(\text{🍕} \mid \text{🥕}) \geq f(\text{🍕} \mid \text{🥕}, \text{🍩})$$

Theme II — Stable Algorithms



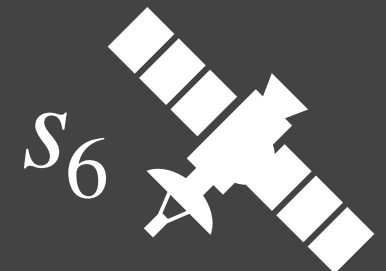
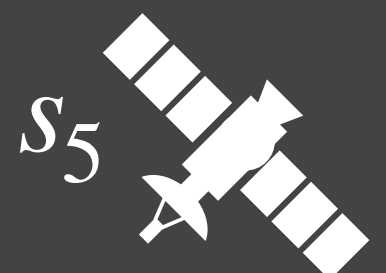
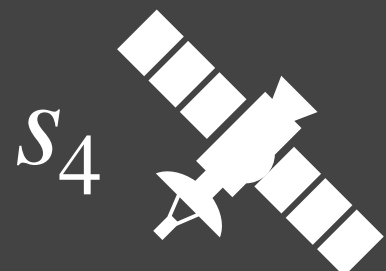
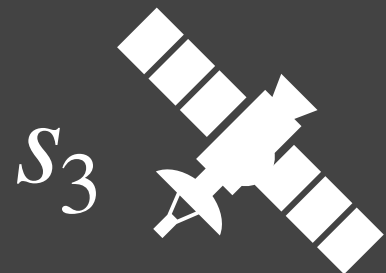
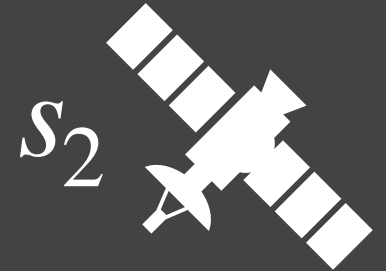
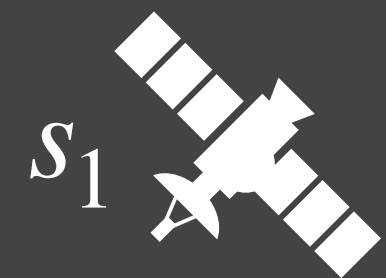
Theme III — Beyond Worst-Case Analysis



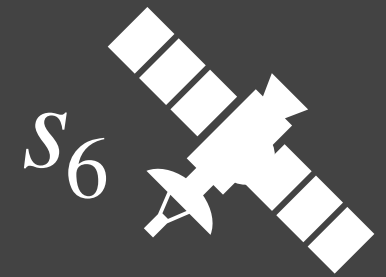
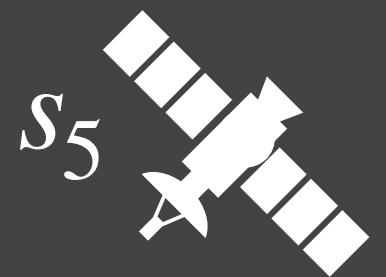
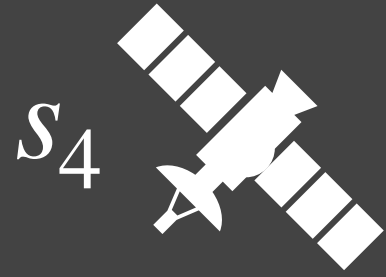
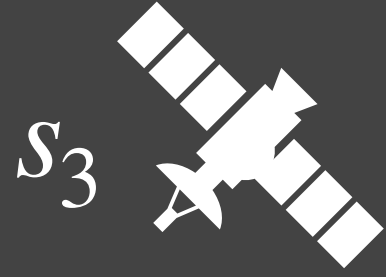
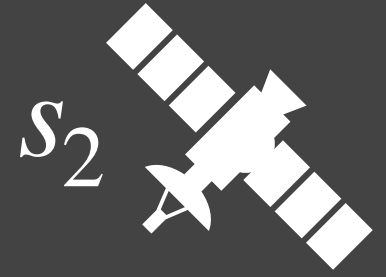
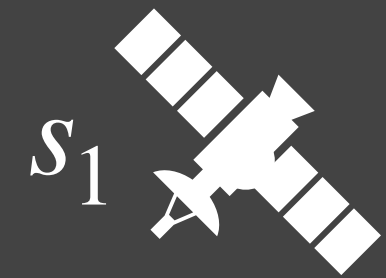
Conclusion

Theme II — Stable Algorithms

Moving to the **Dynamic** model

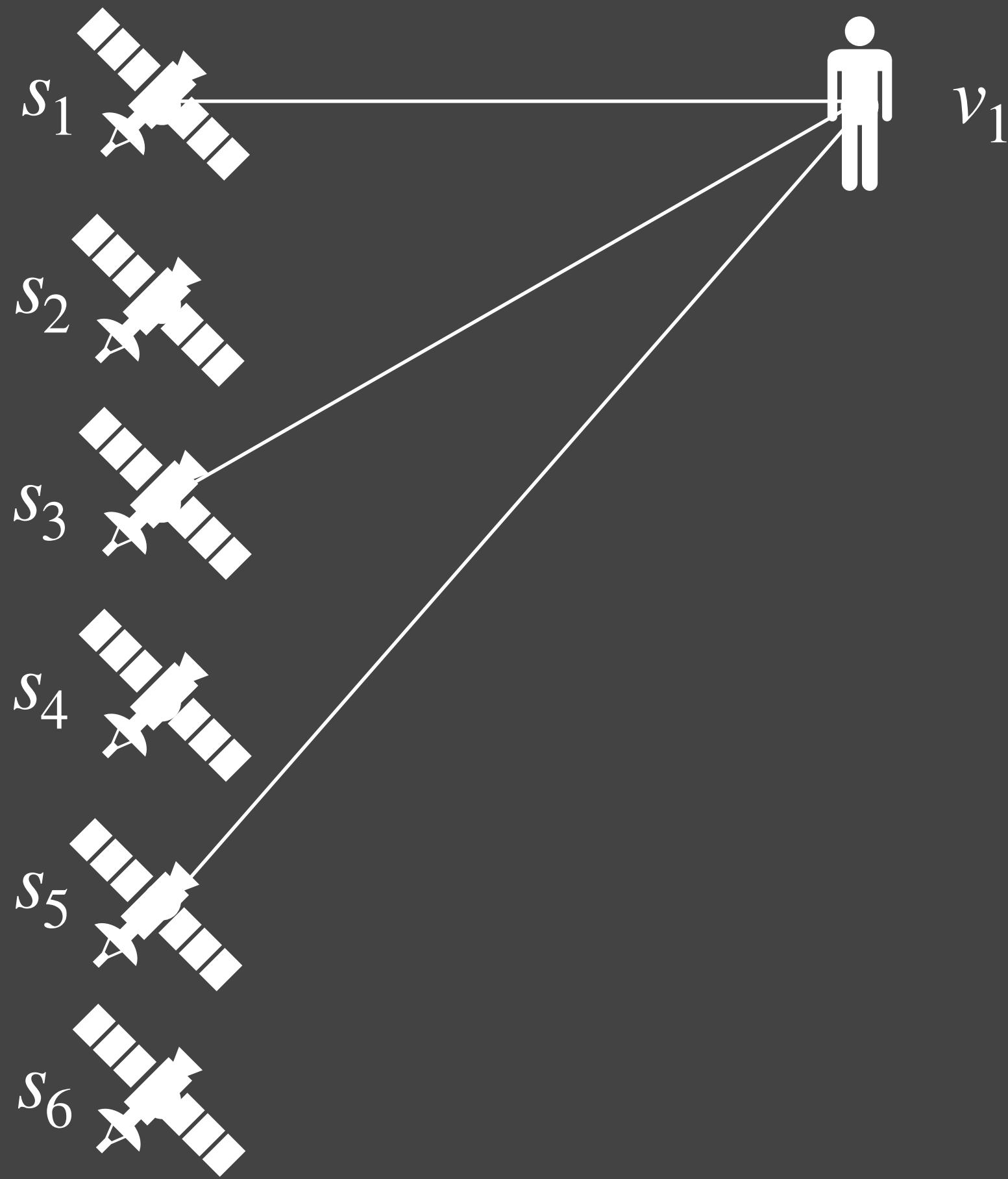


Moving to the **Dynamic** model



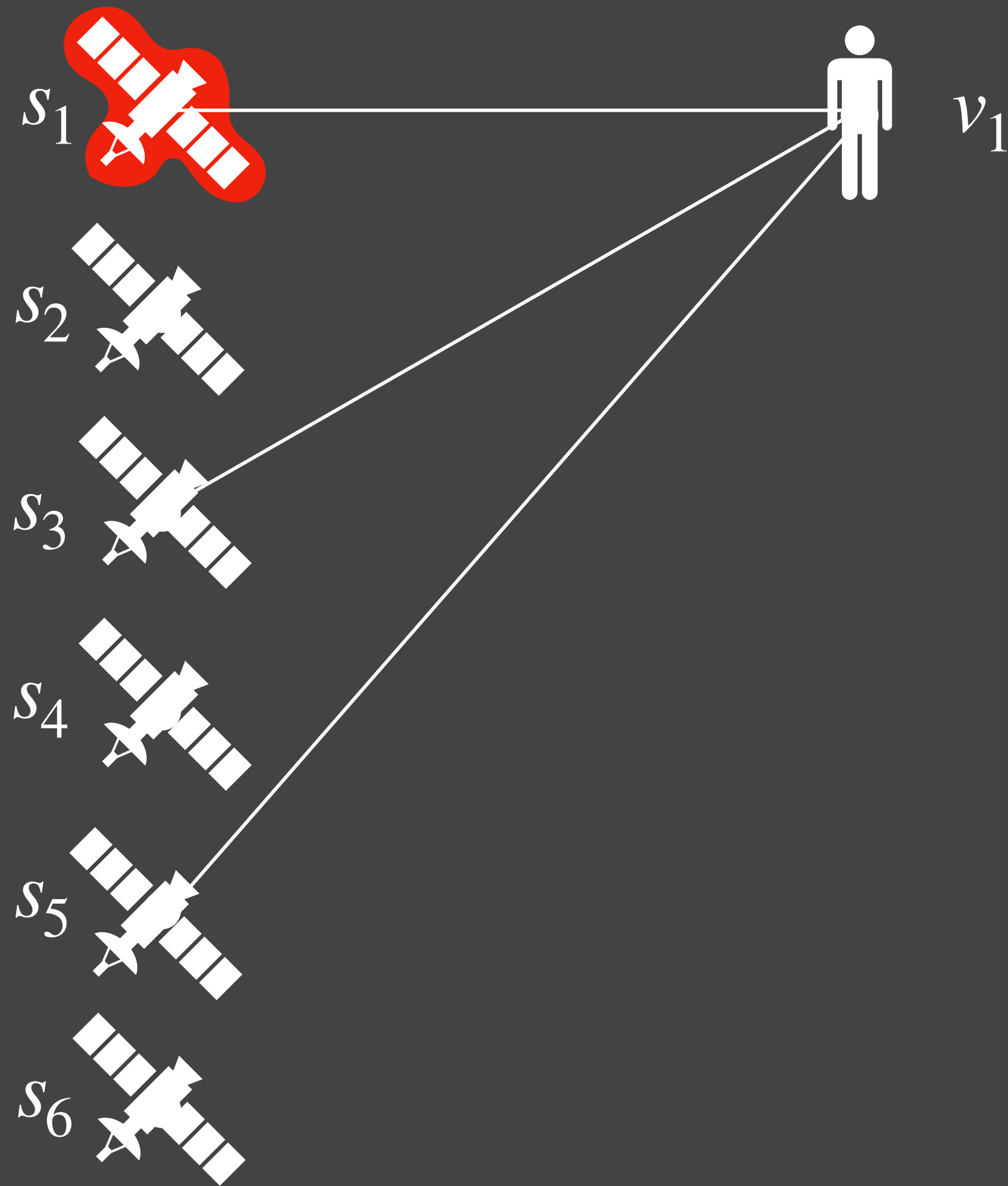
New model:
inserts AND deletes.

Moving to the **Dynamic** model



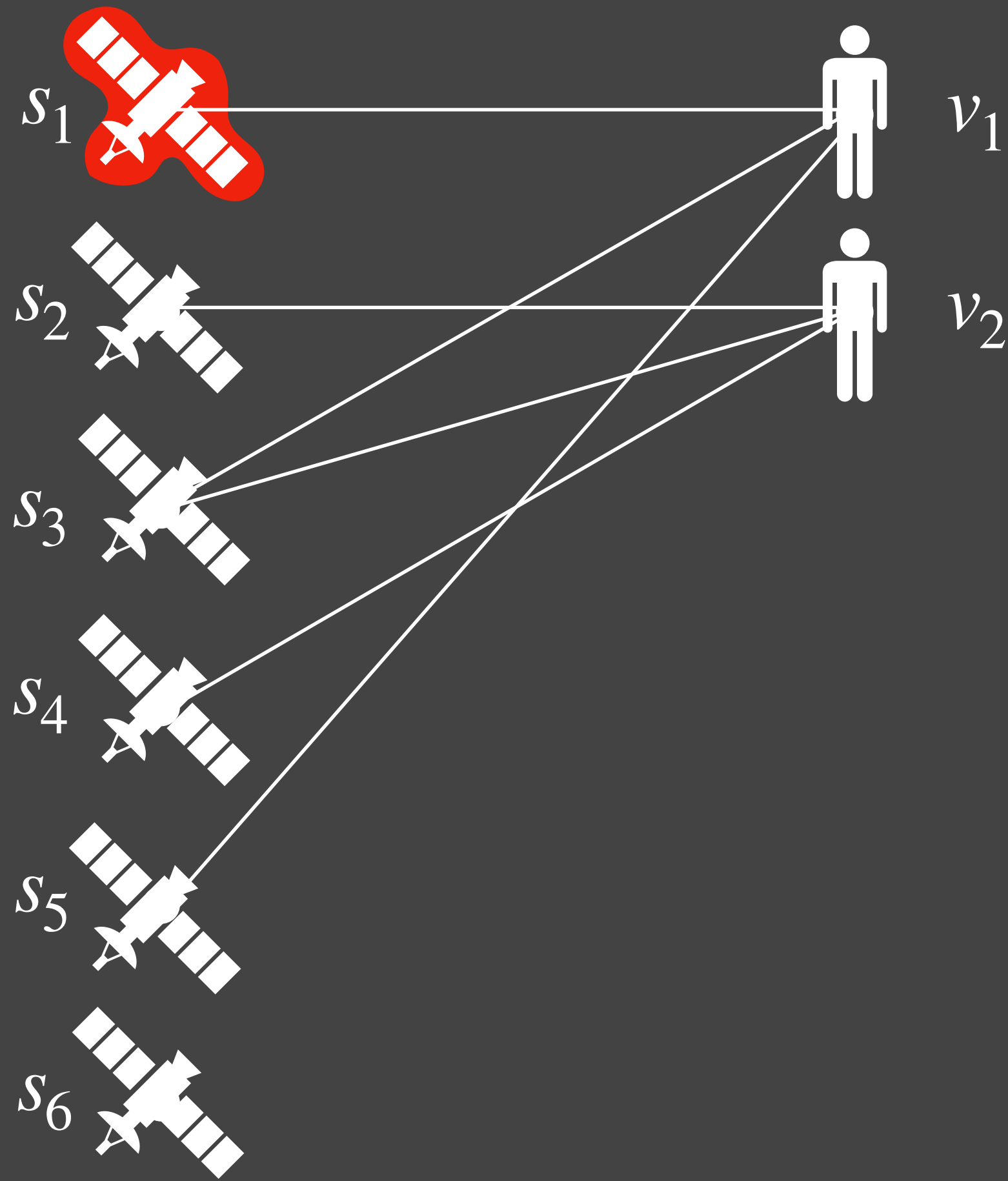
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Moving to the **Dynamic** model



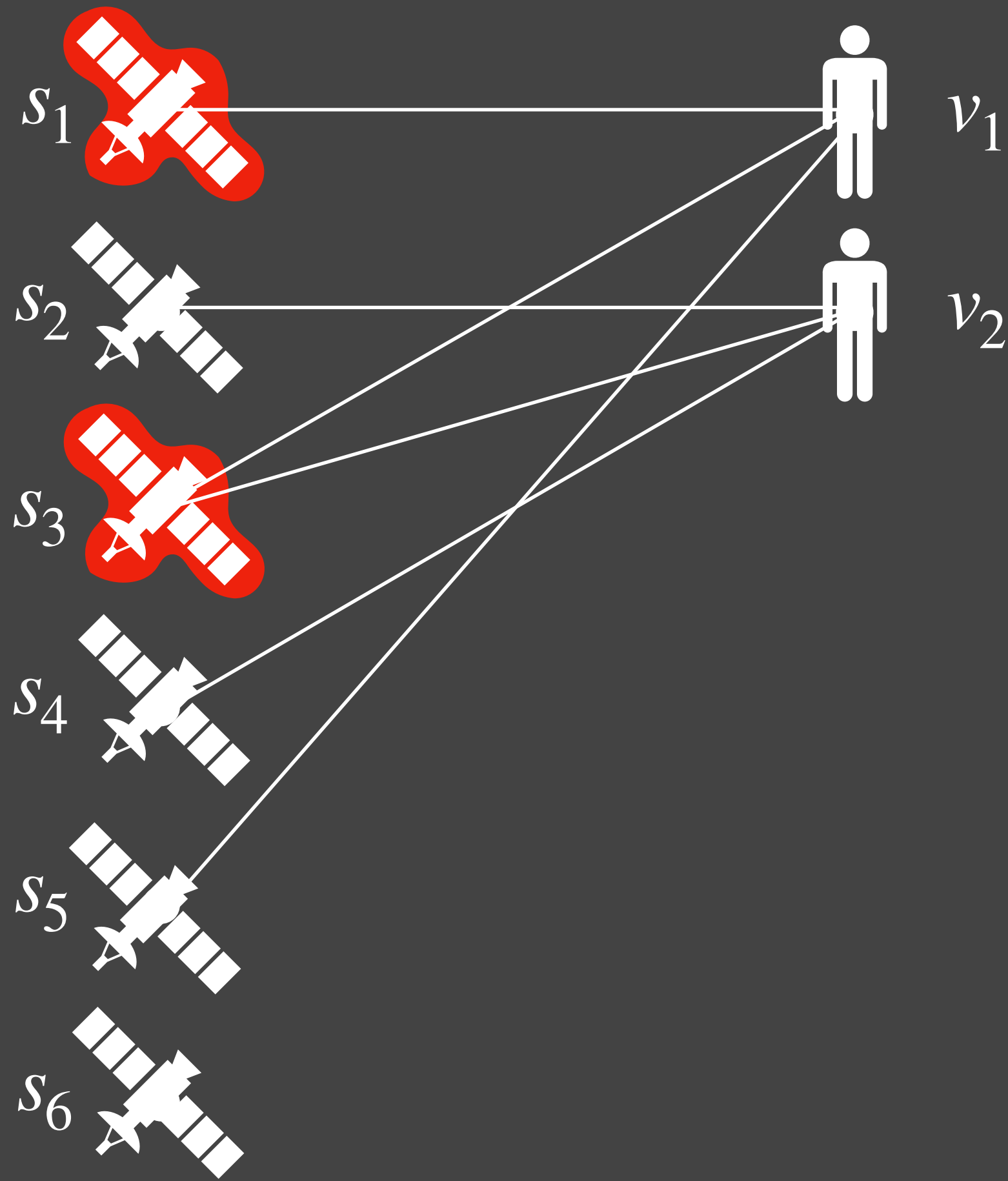
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Moving to the **Dynamic** model



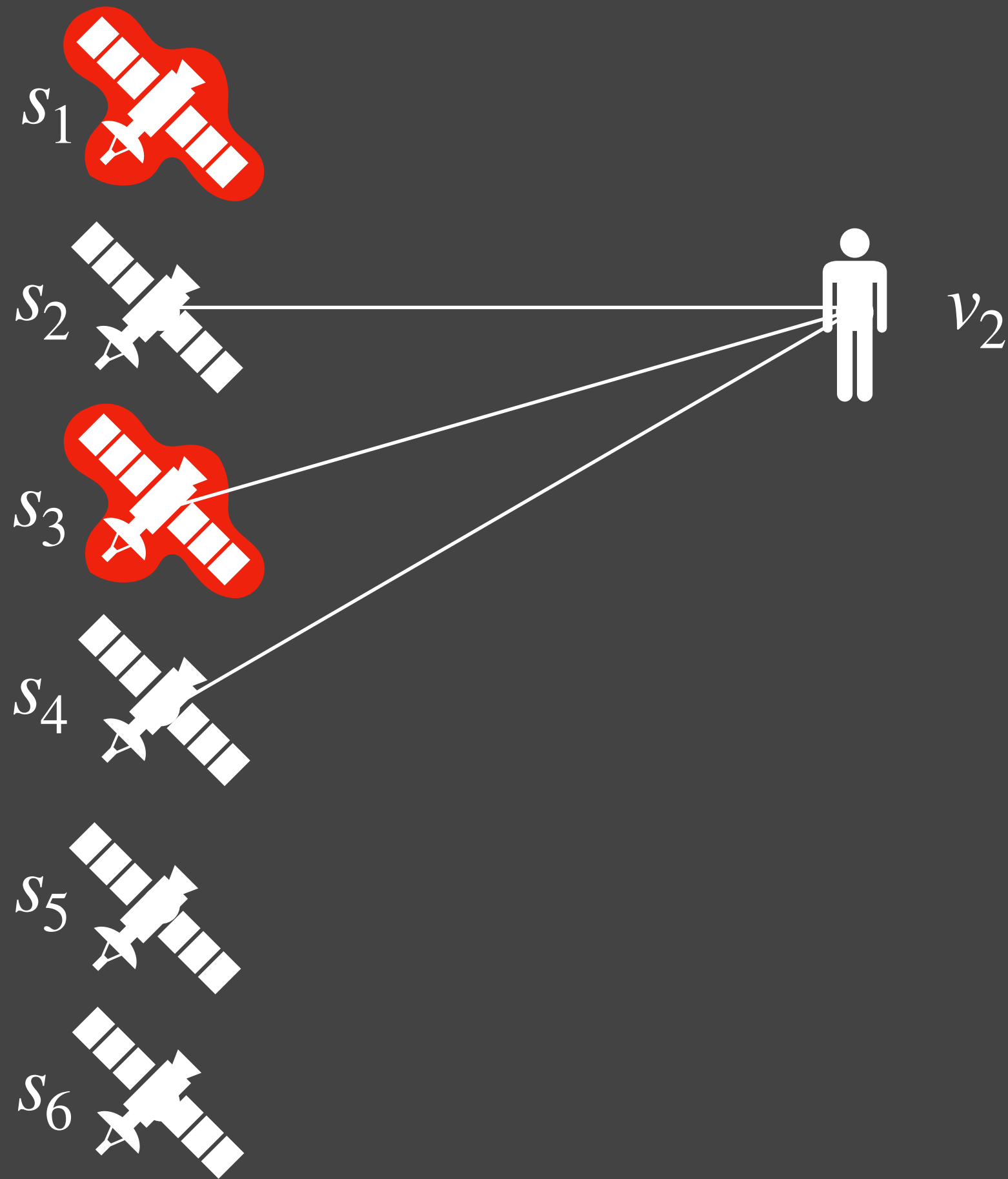
New model:
inserts AND deletes.

Moving to the **Dynamic** model



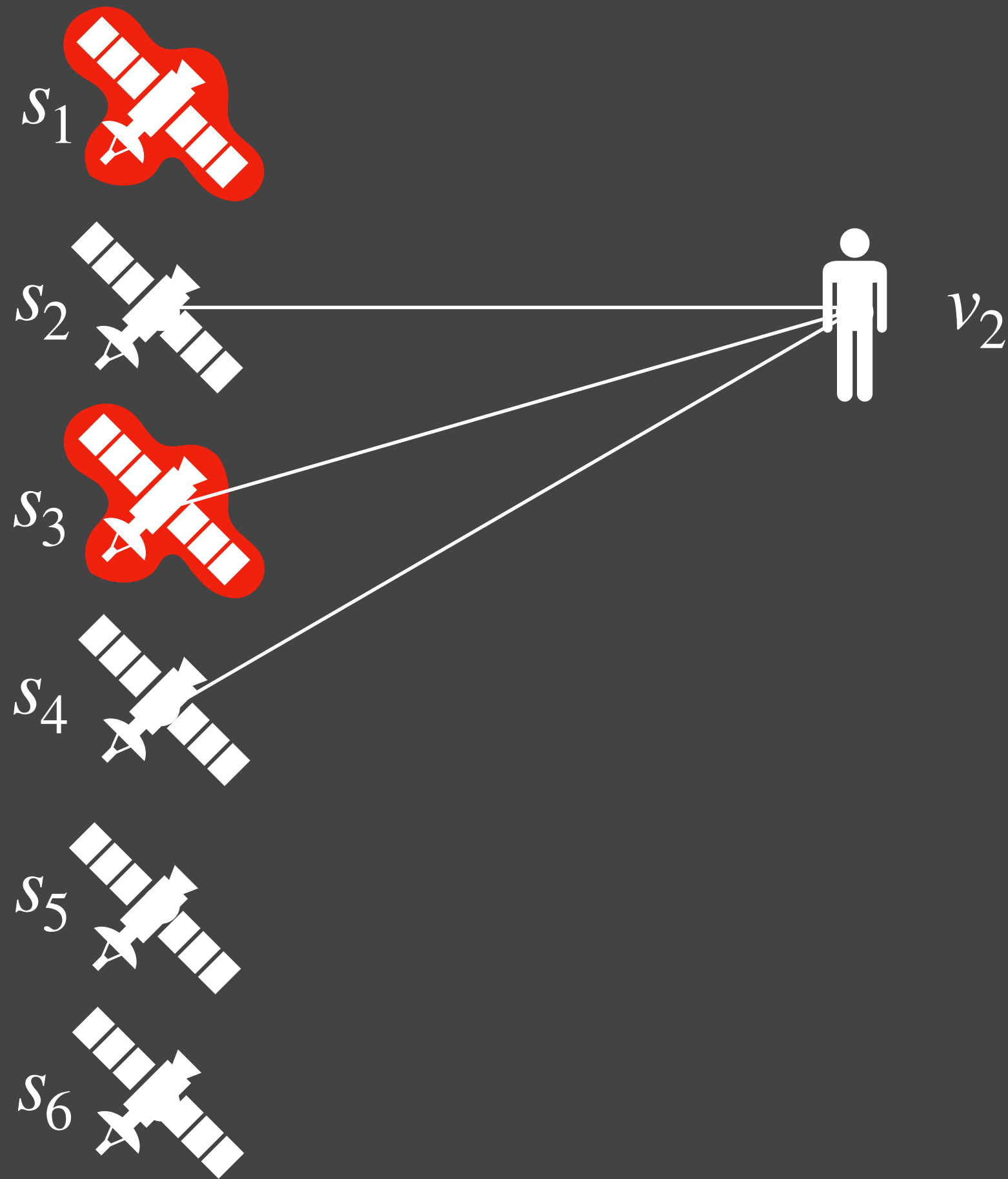
New model:
inserts AND deletes.

Moving to the **Dynamic** model



New model:
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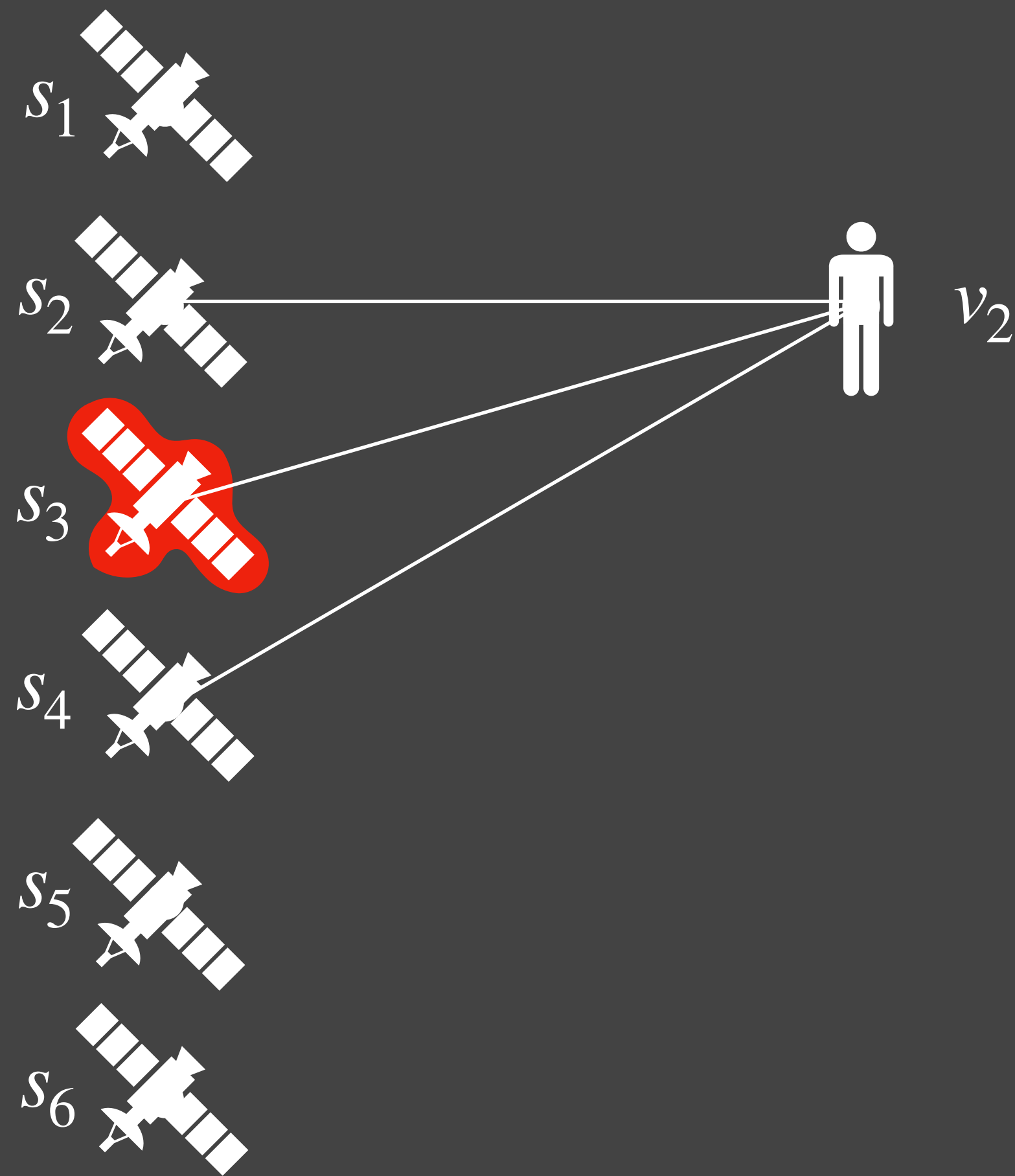
Moving to the **Dynamic** model



New model:
inserts AND deletes.

Algorithm now allowed
limited # edits, a.k.a.
recourse.

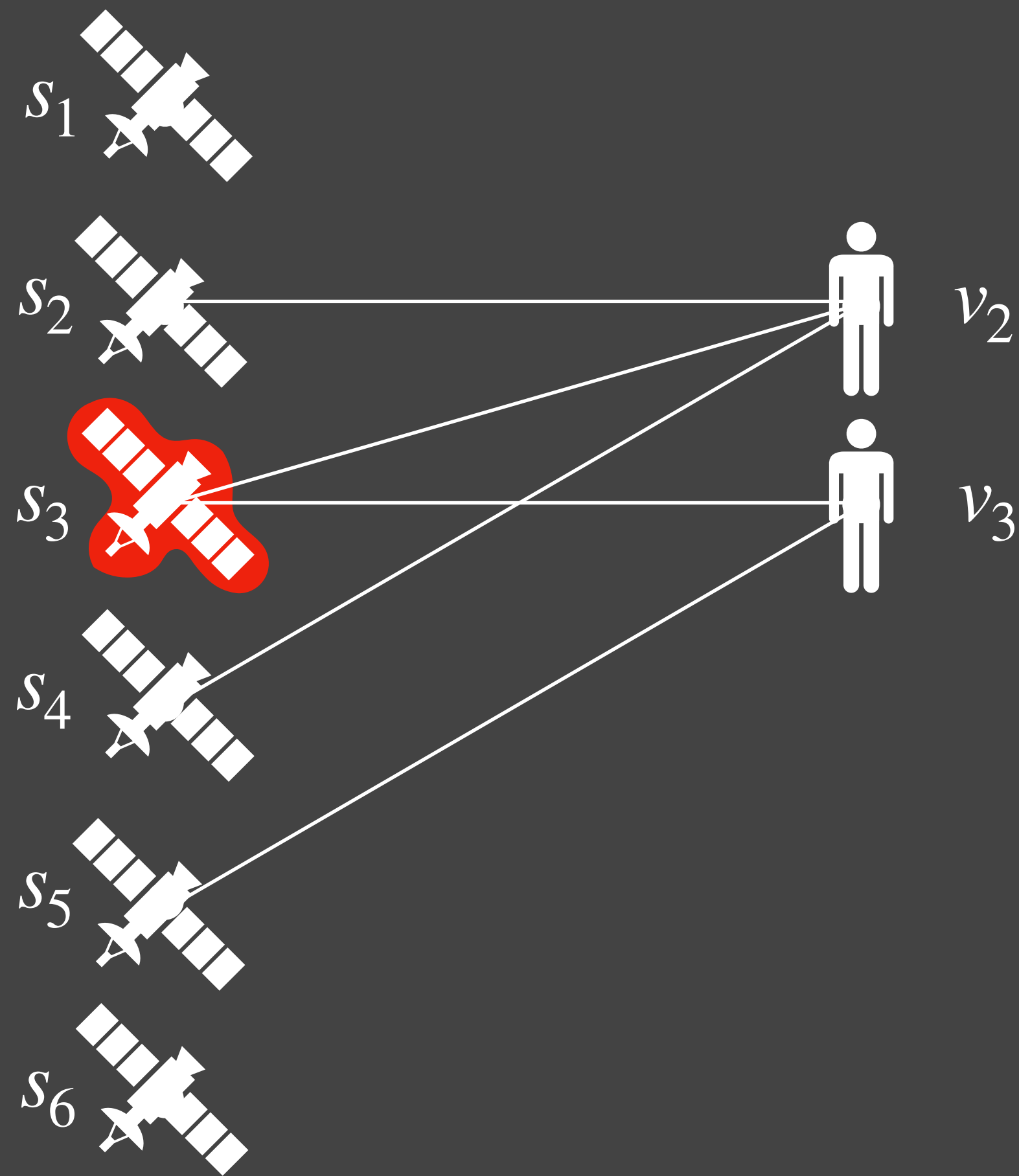
Moving to the **Dynamic** model



New model:
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Algorithm now allowed
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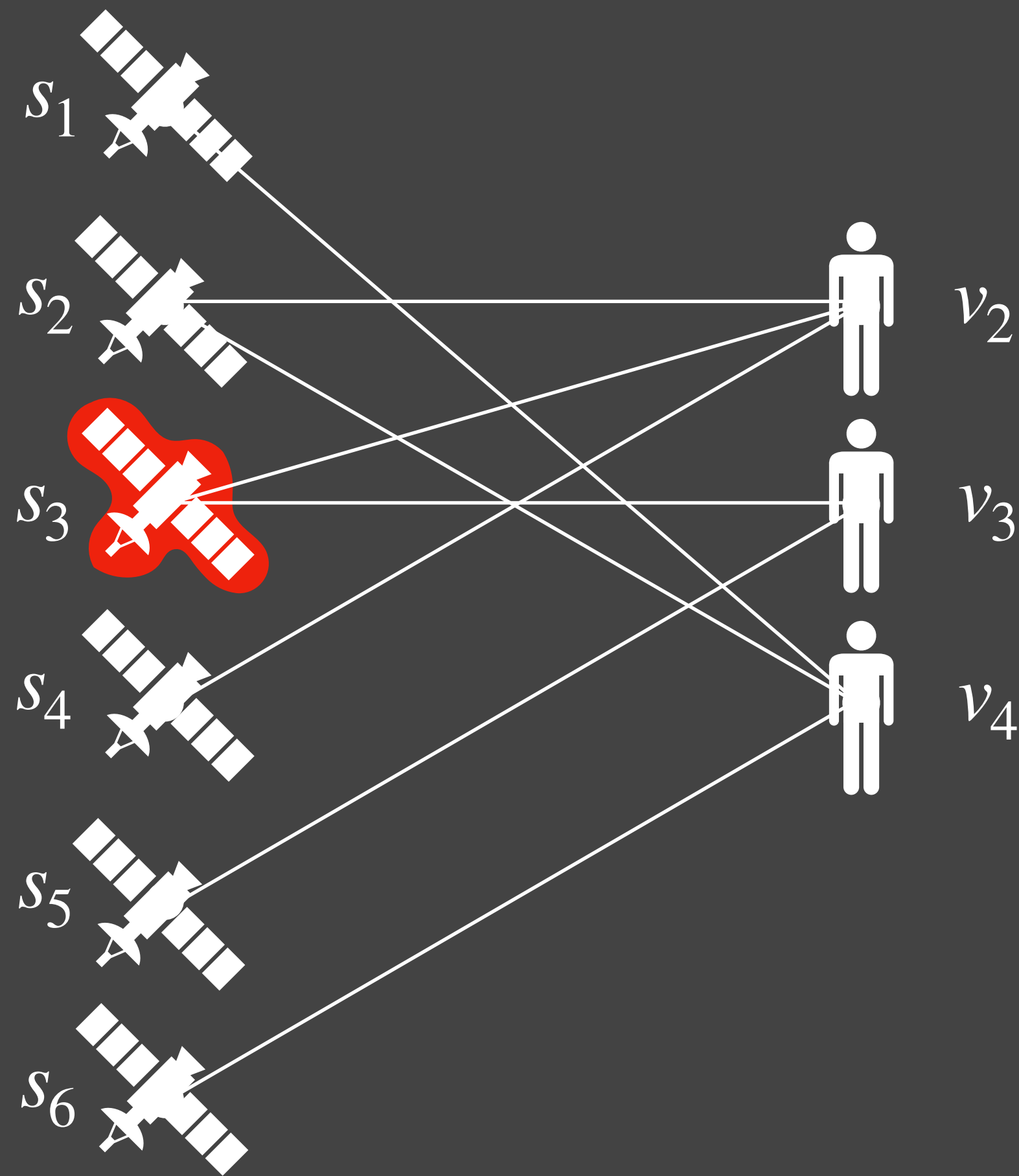
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New model:
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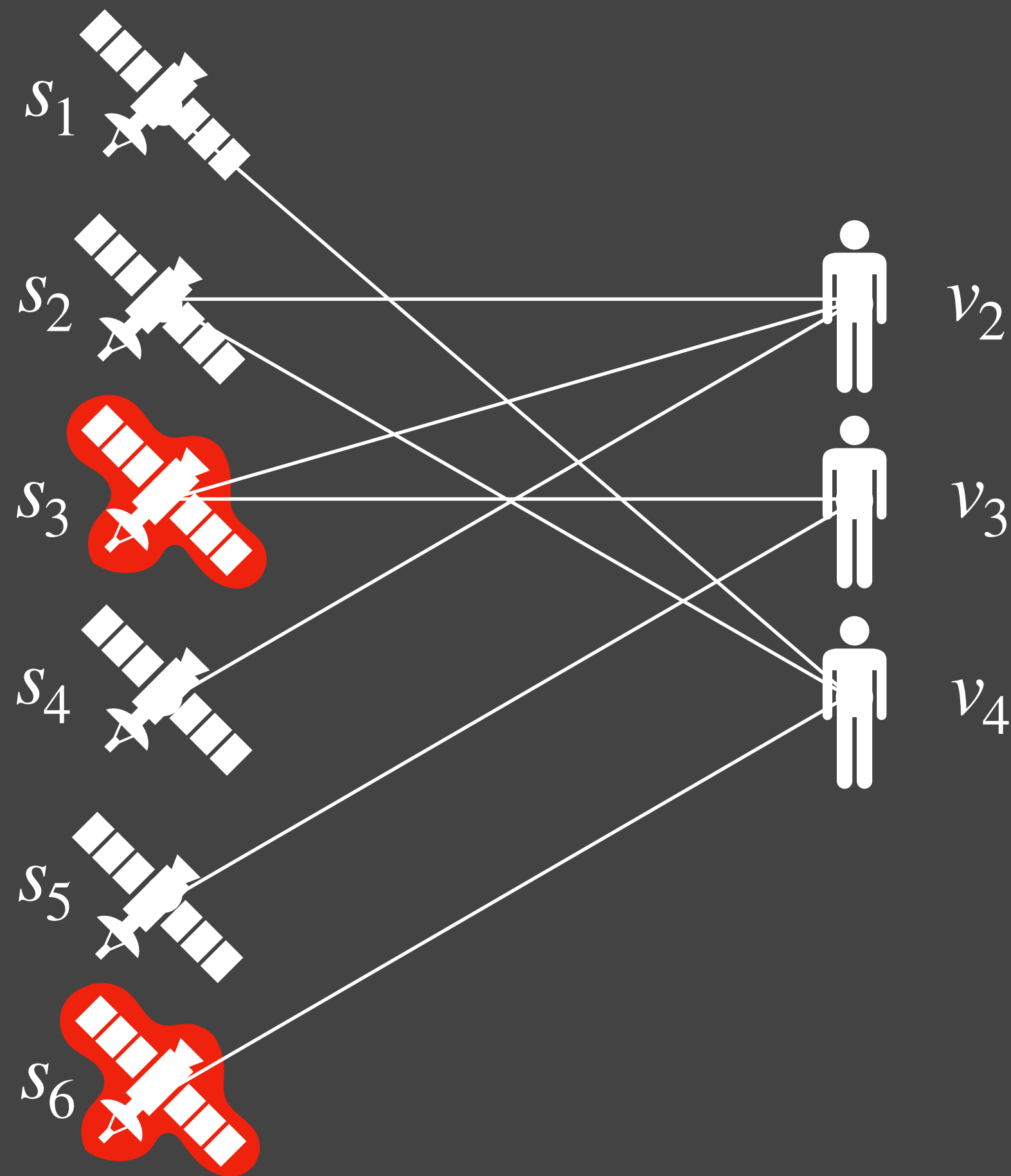
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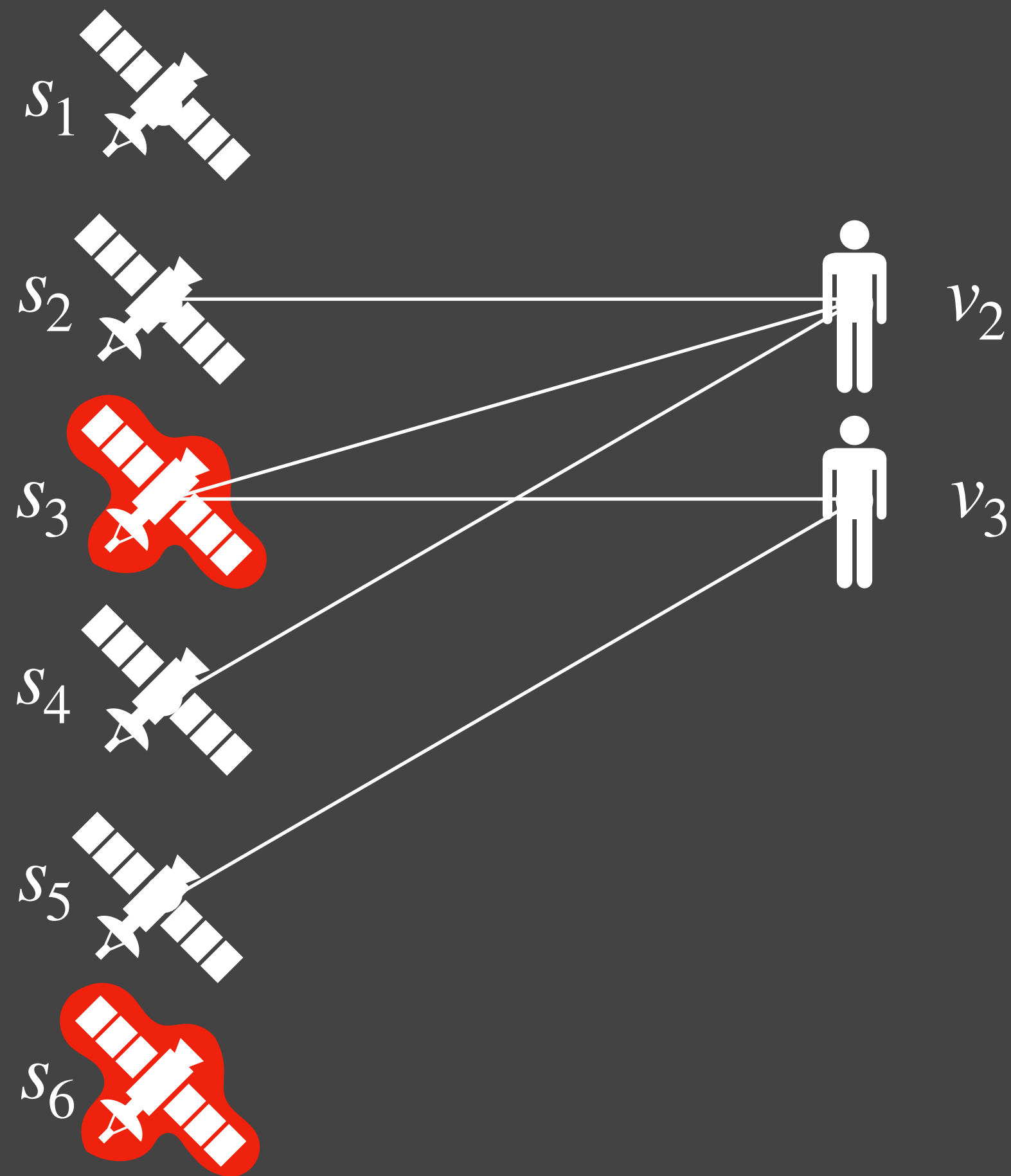
Moving to the **Dynamic** model



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Algorithm now allowed
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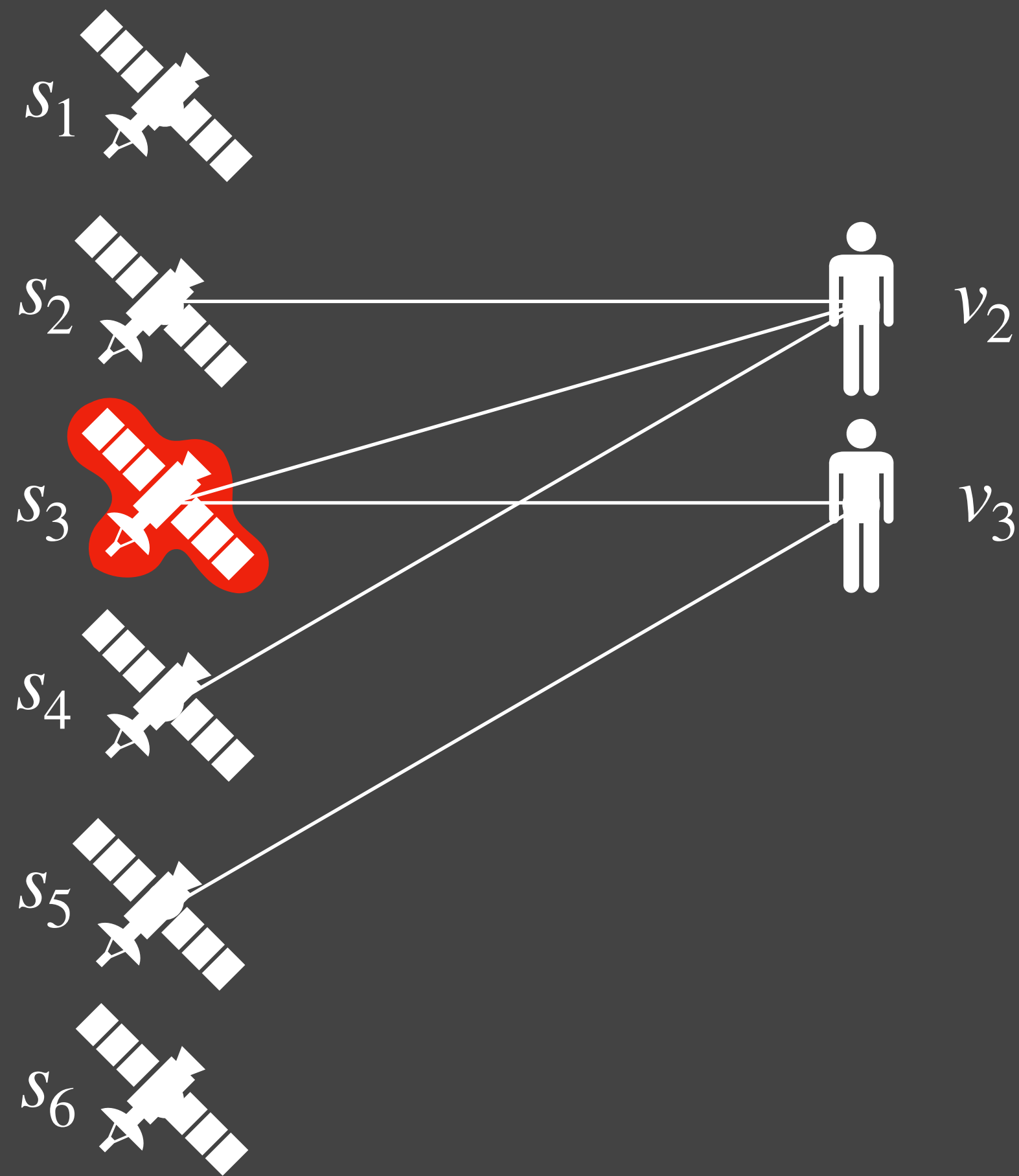
Moving to the **Dynamic** model



New model:
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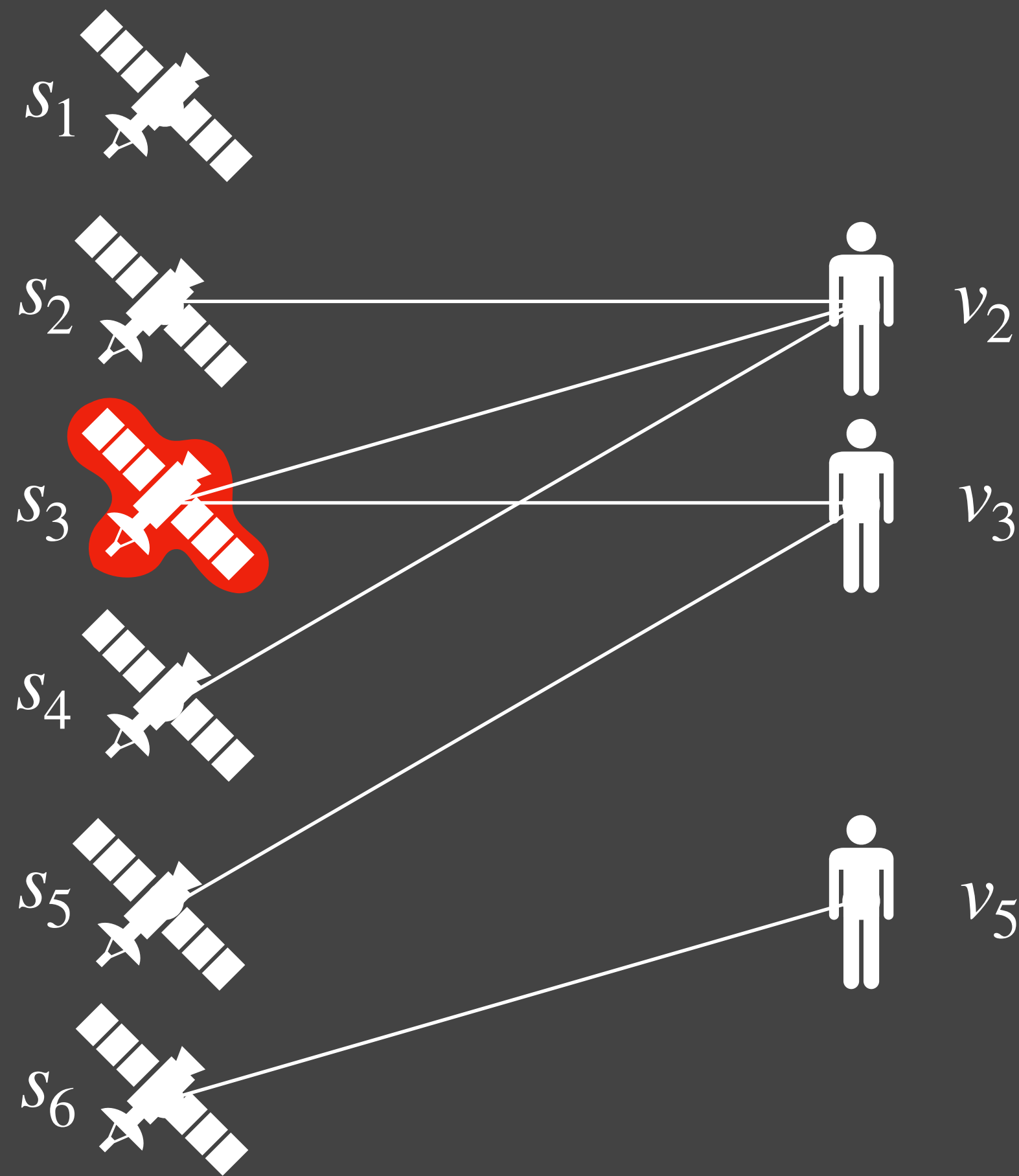
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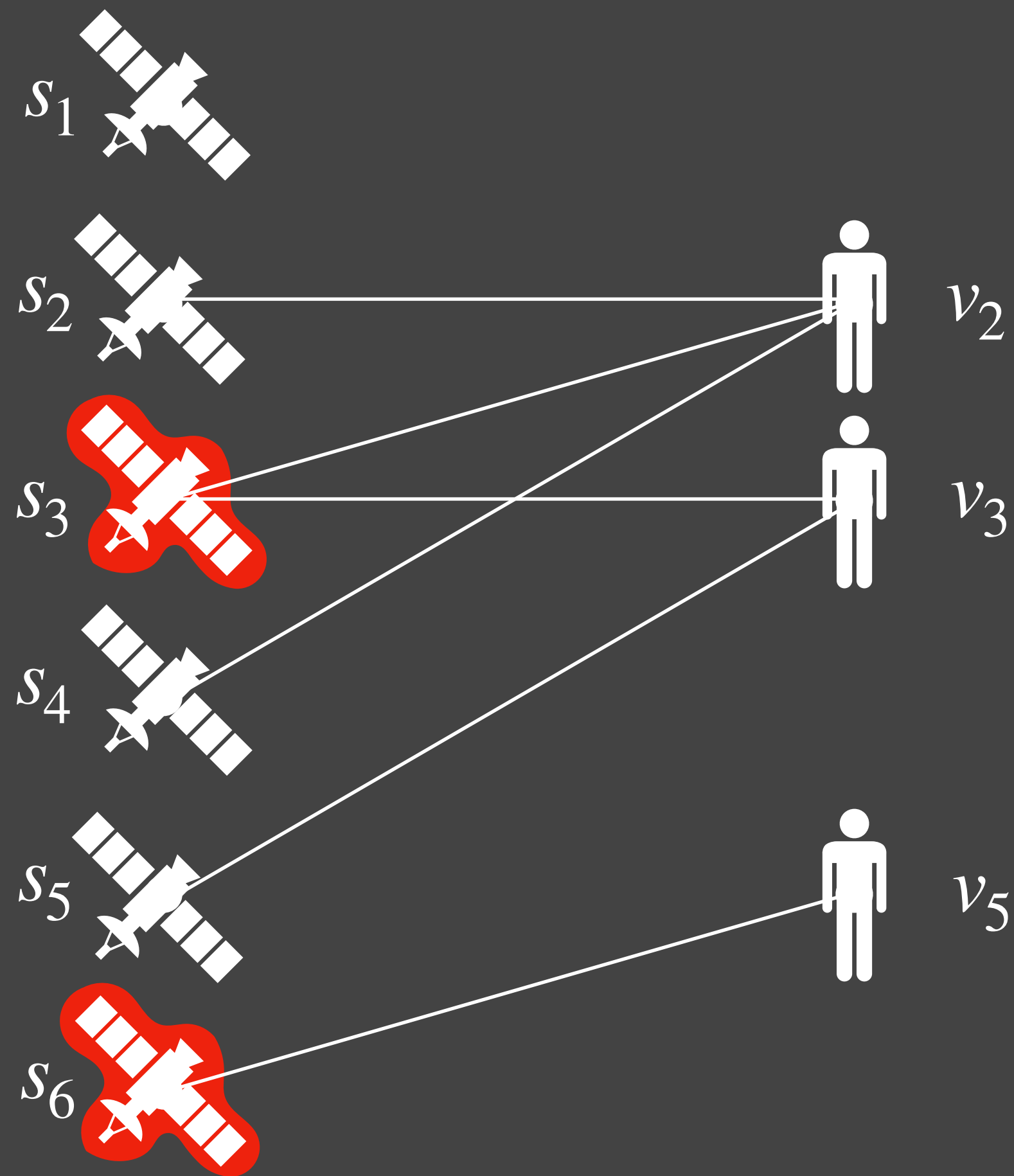
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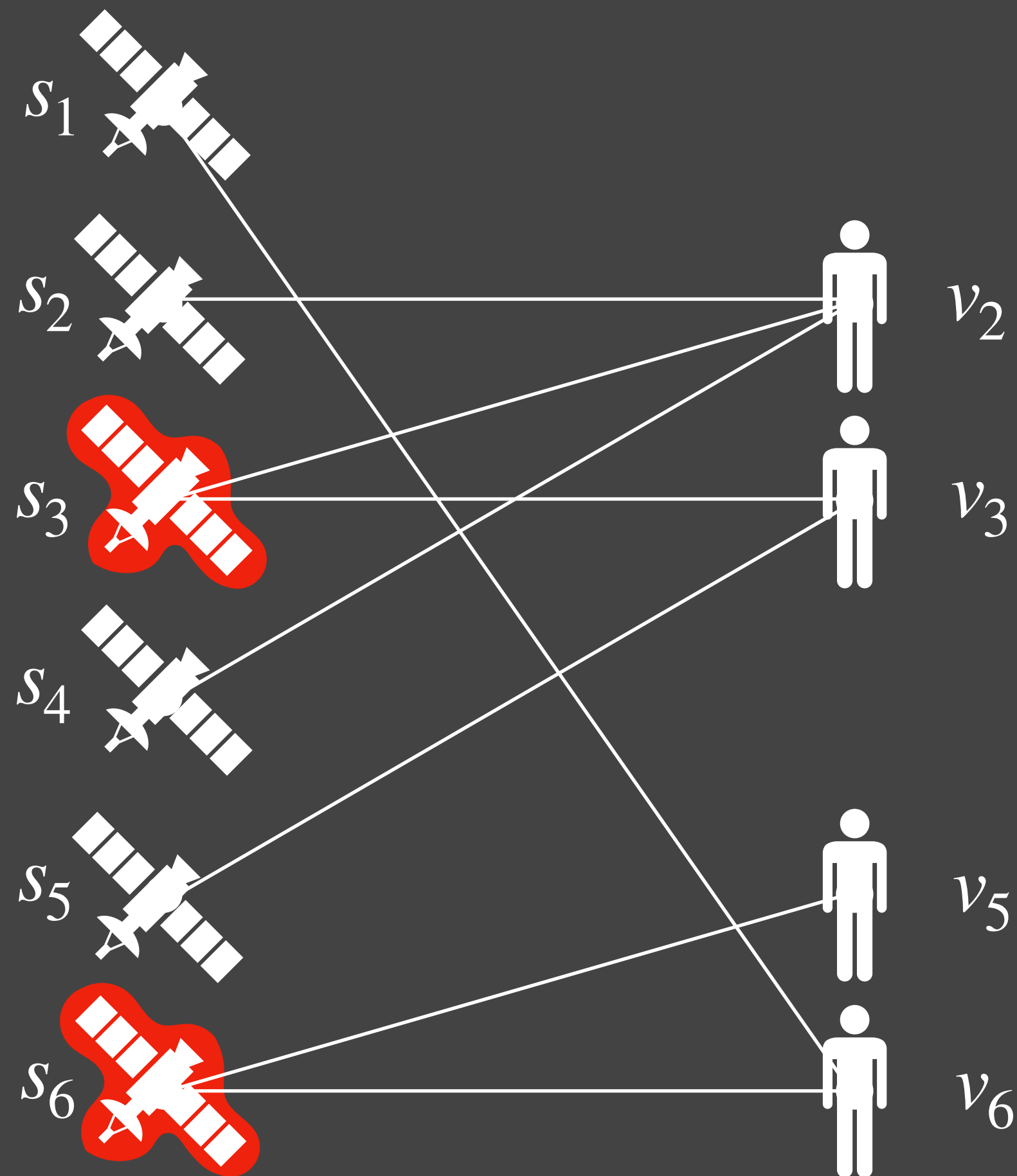
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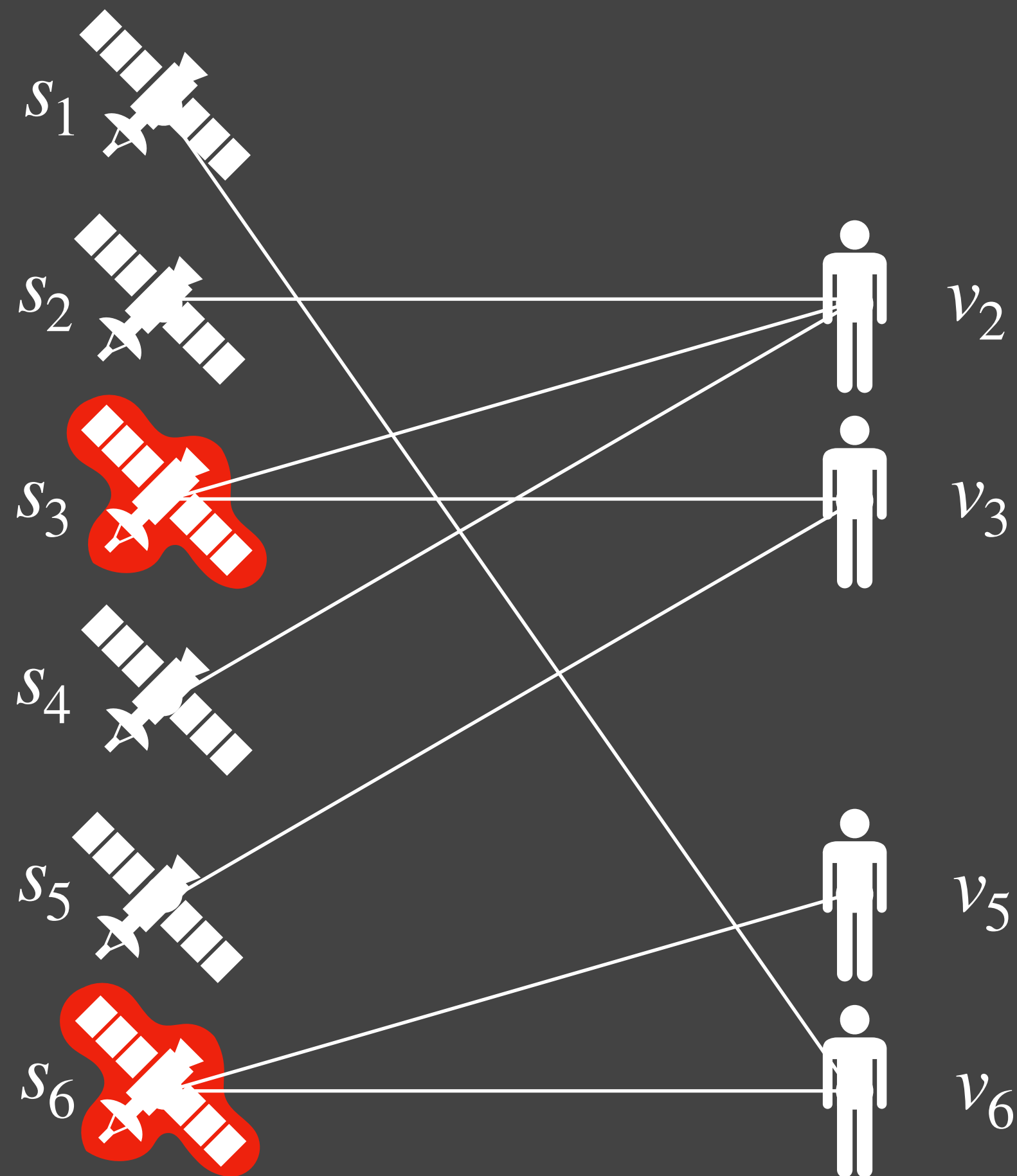
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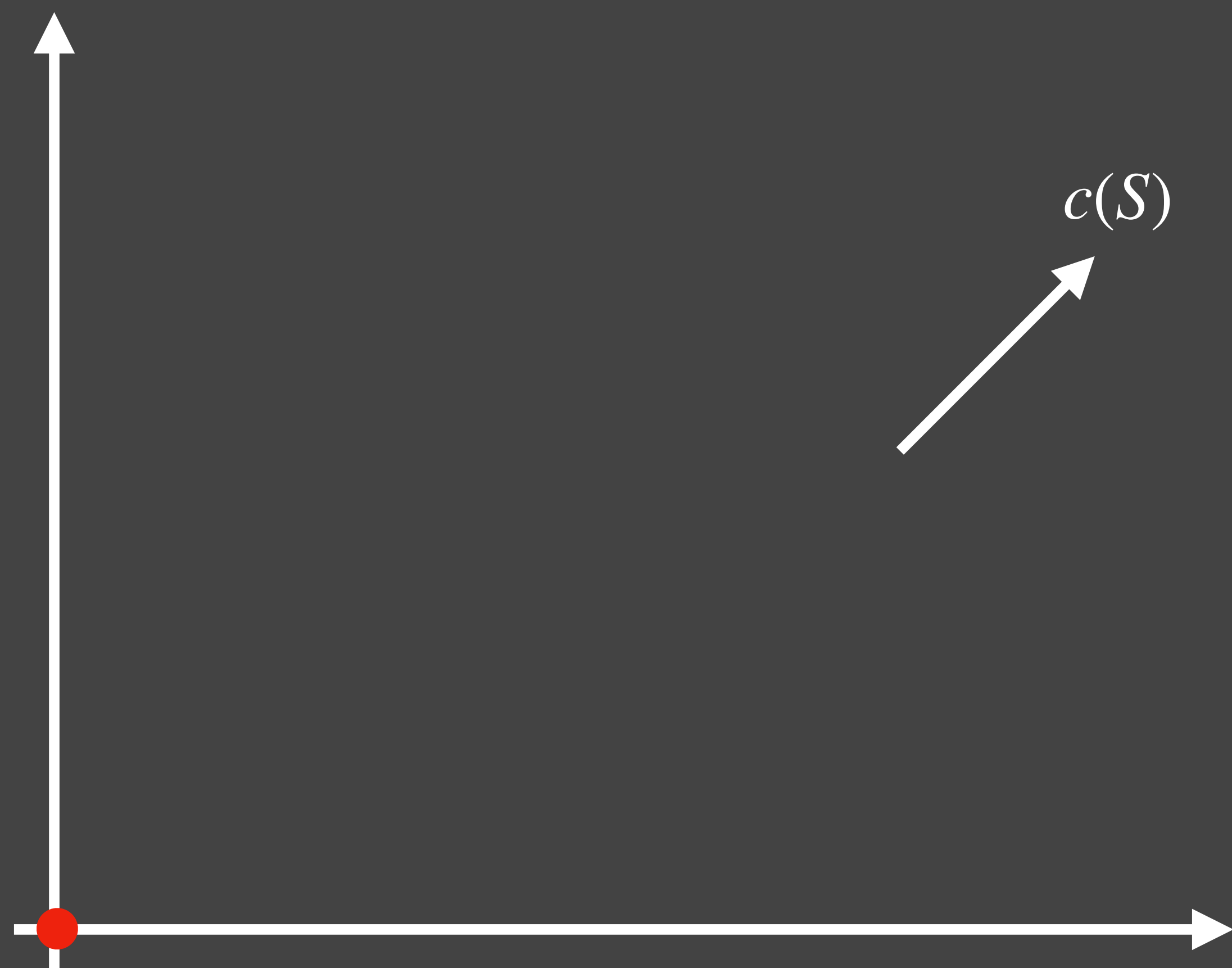
New model:
inserts AND deletes.

Algorithm now allowed
limited # edits, a.k.a.
recourse.

Q: Can we understand
recourse/approximation
tradeoffs?

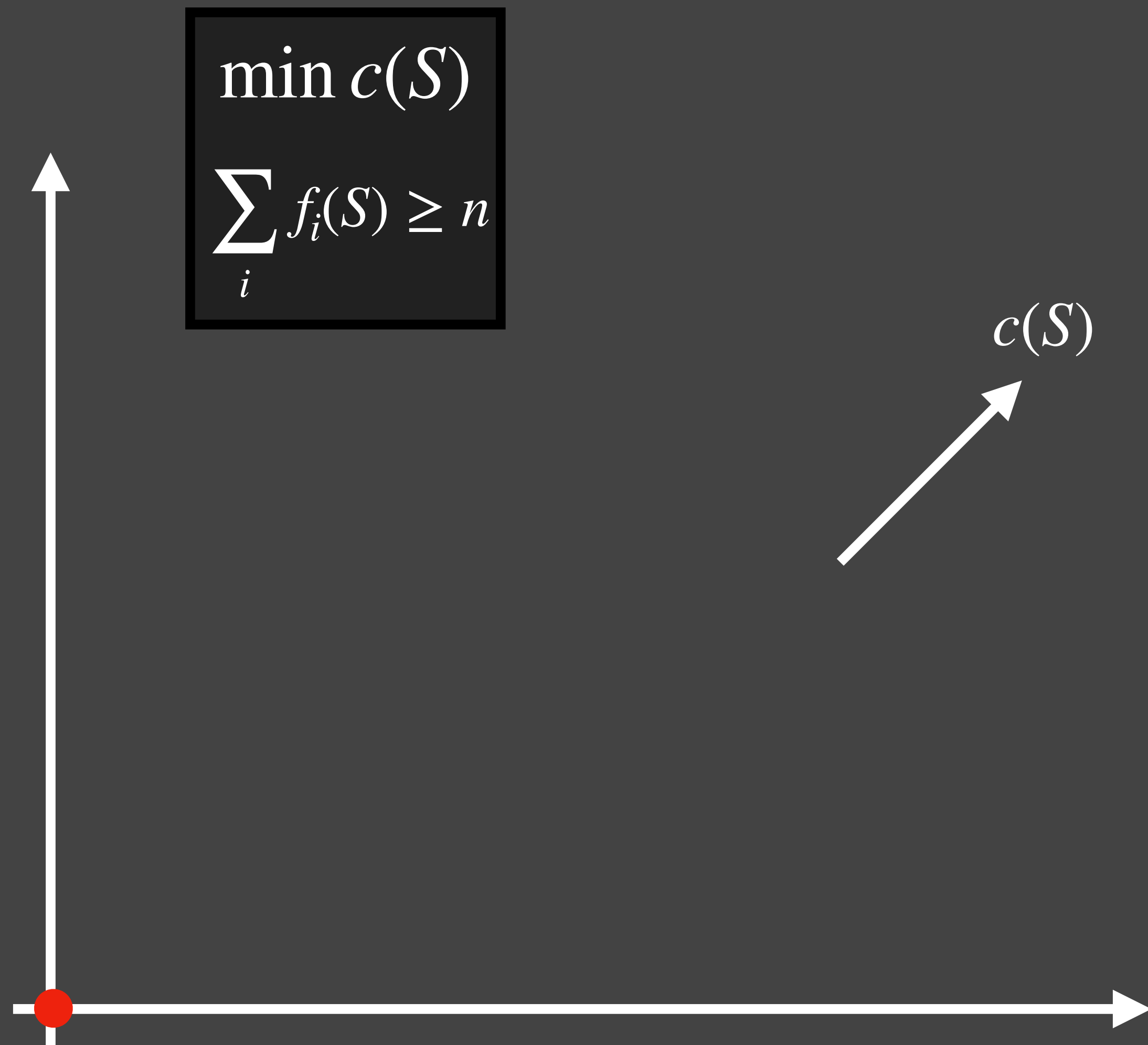
Dynamic Submodular Cover

[Gupta L. FOCS 20]



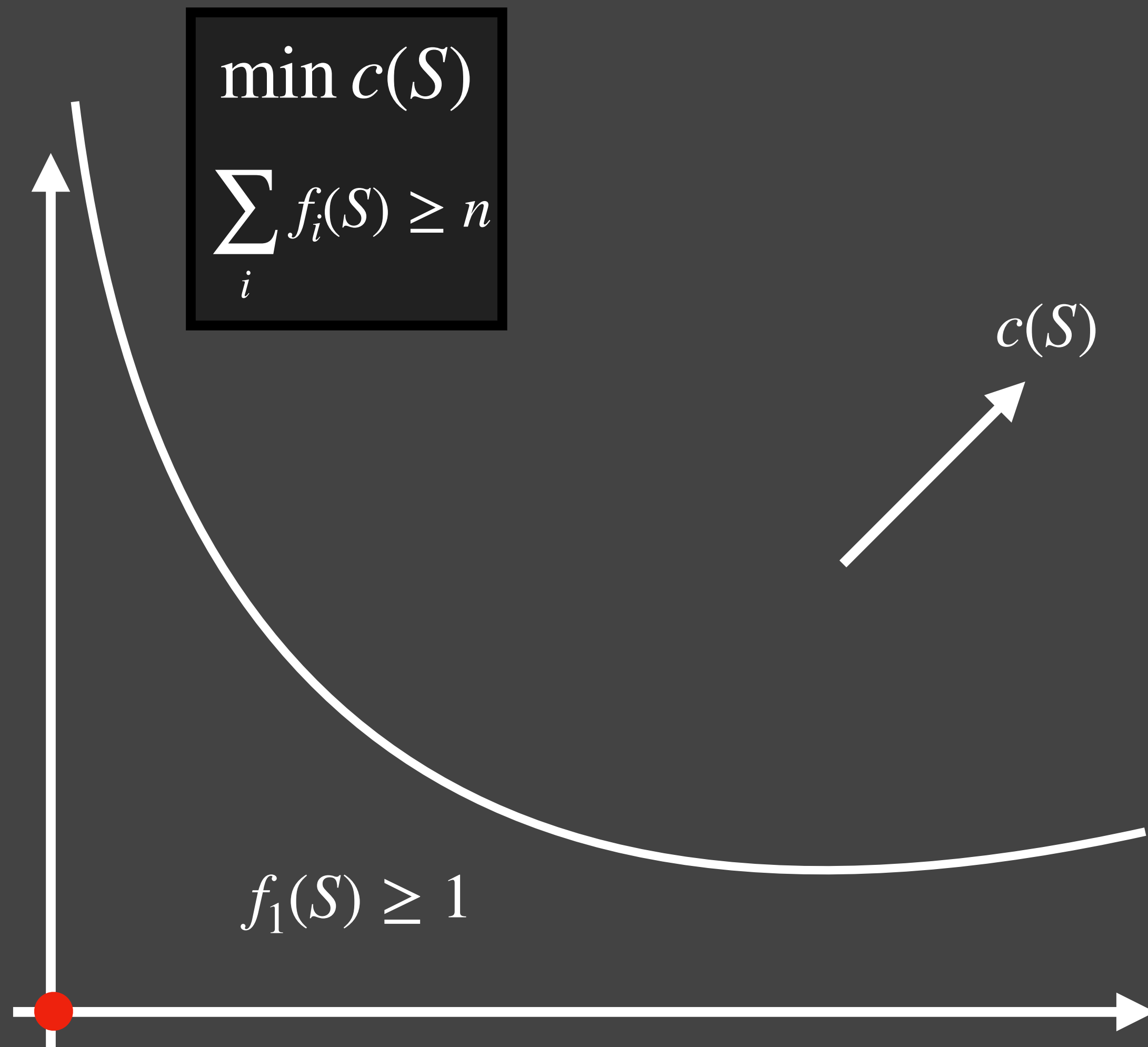
Dynamic Submodular Cover

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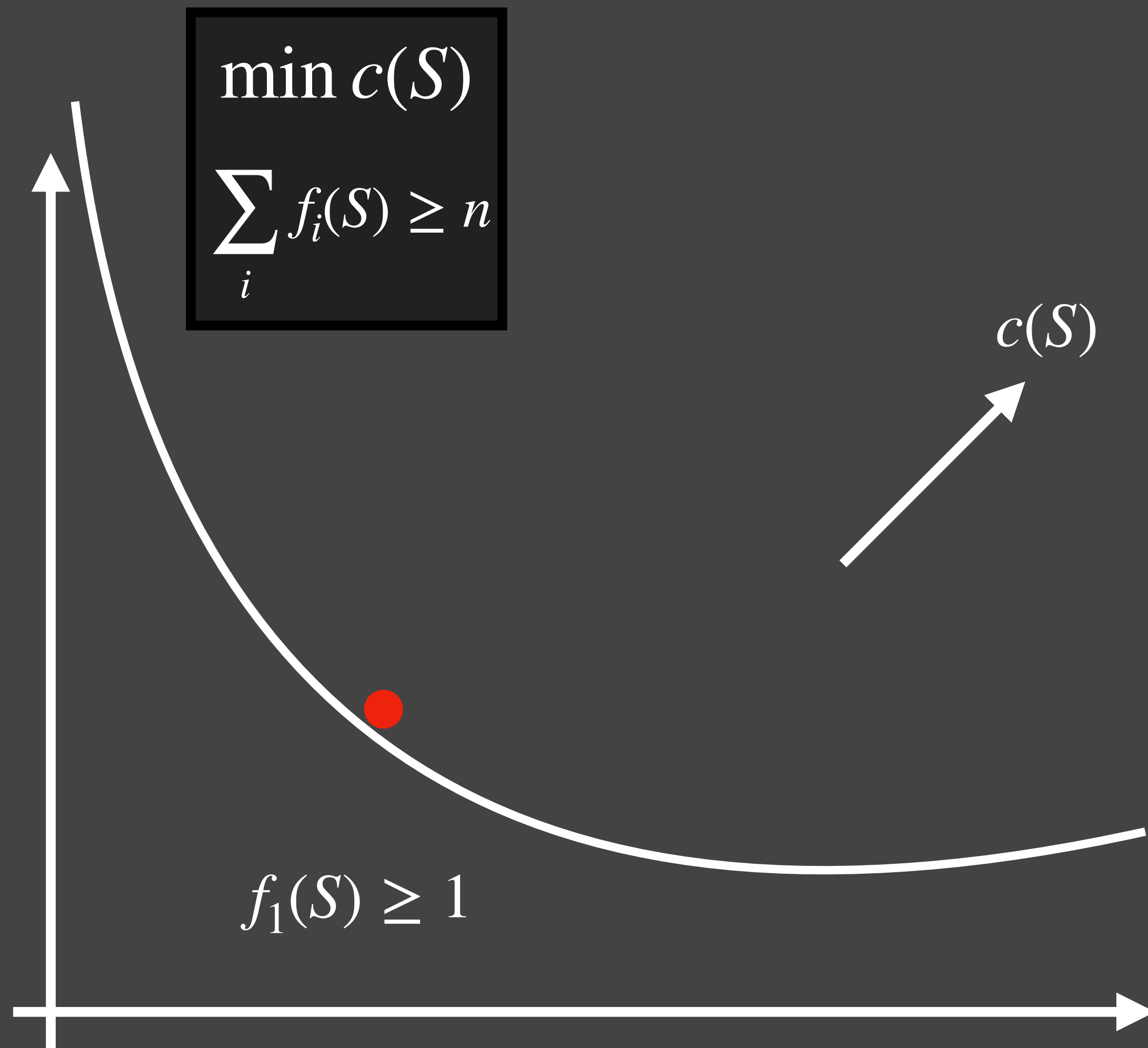
Dynamic Submodular Cover

[Gupta L. FOCS 20]



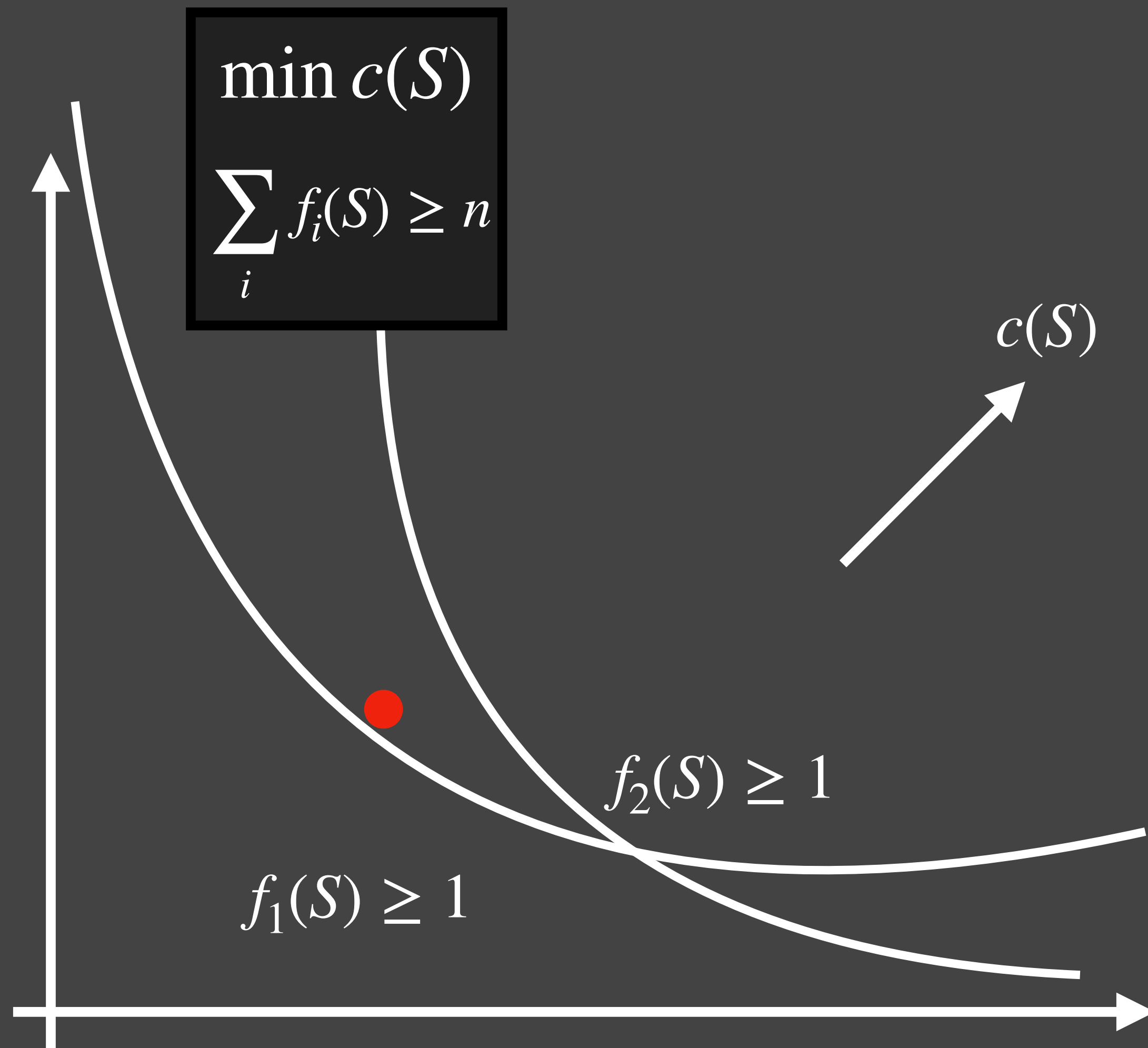
Dynamic Submodular Cover

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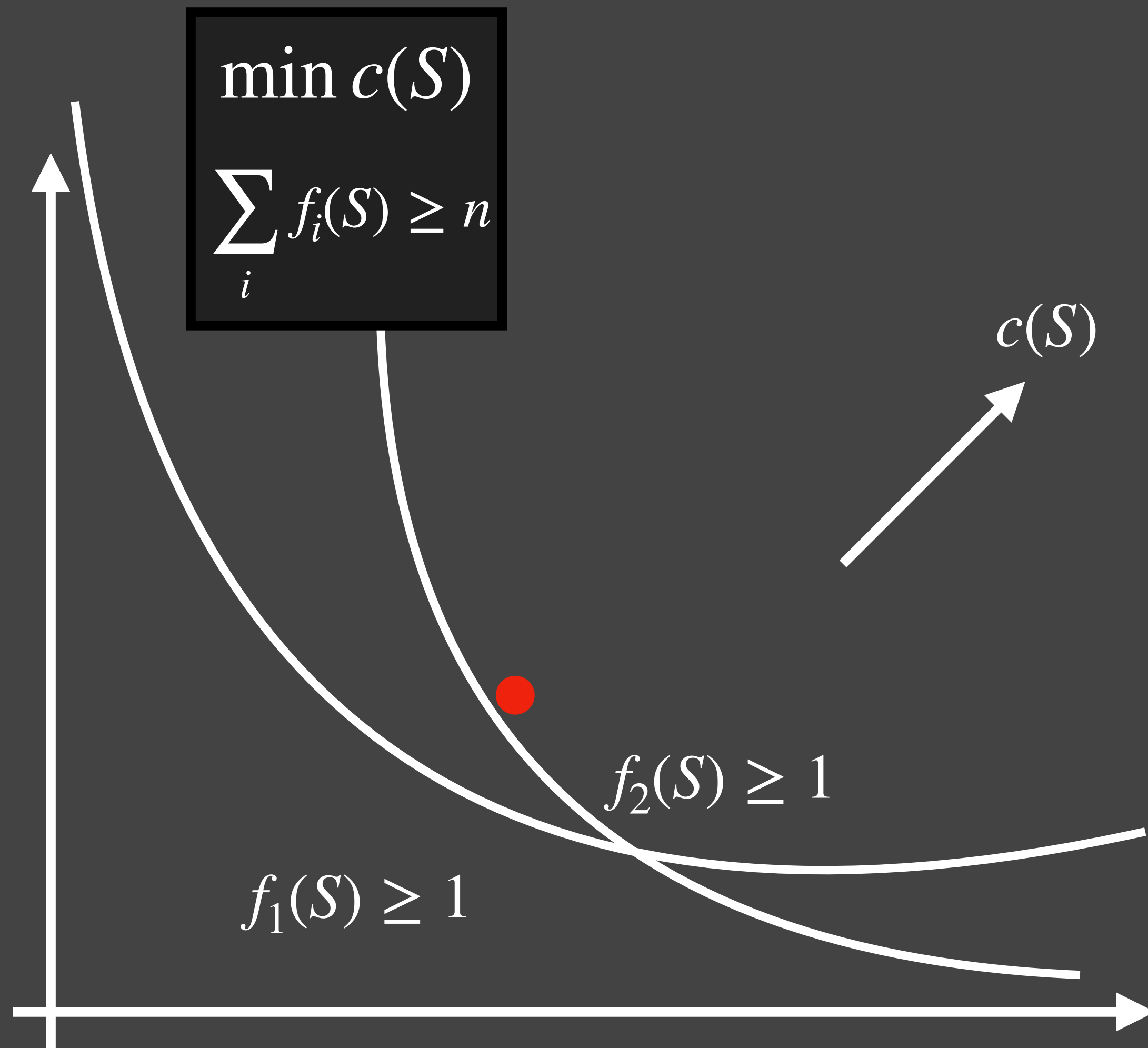
Dynamic Submodular Cover

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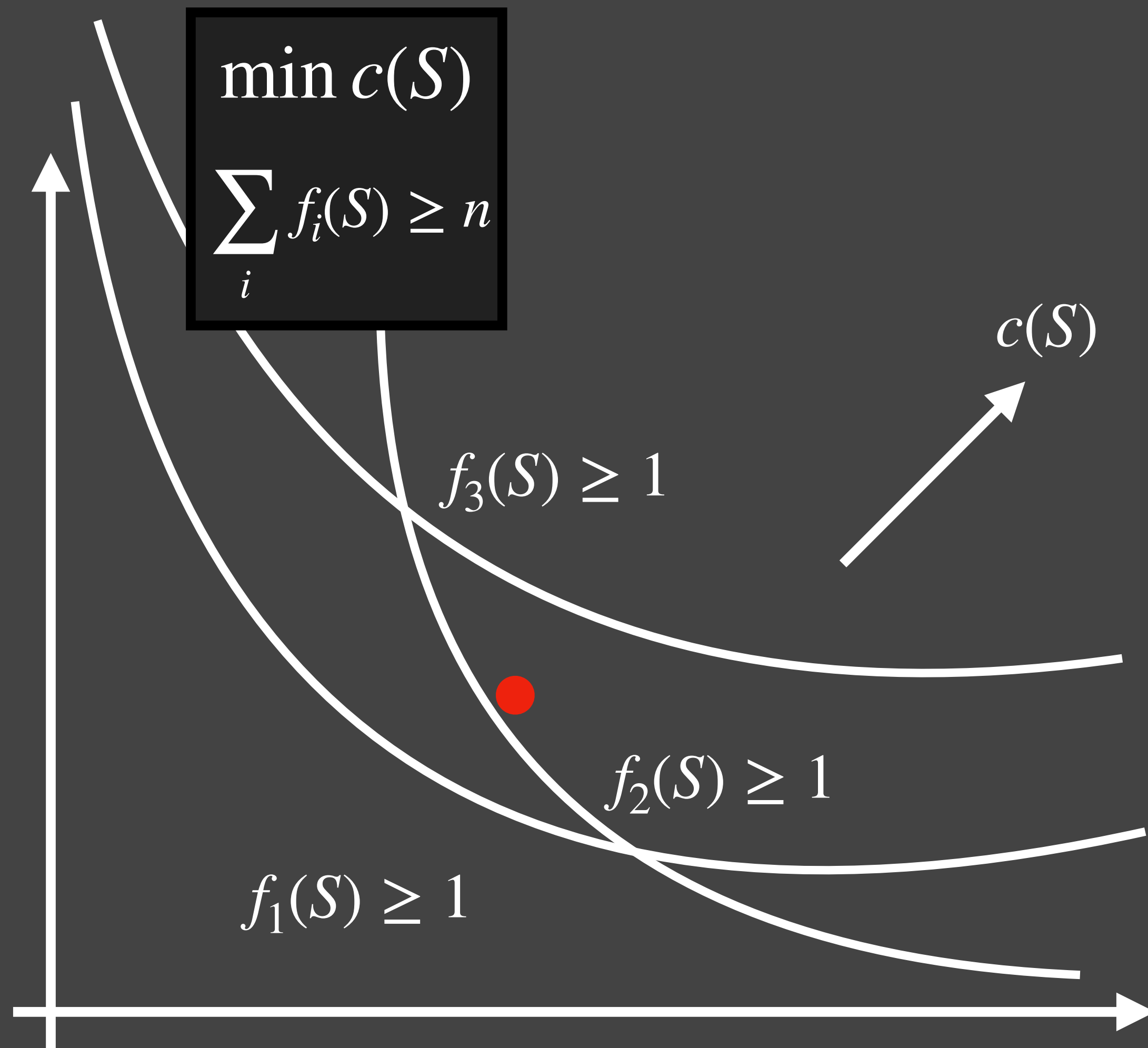
Dynamic Submodular Cover

[Gupta L. FOCS 20]



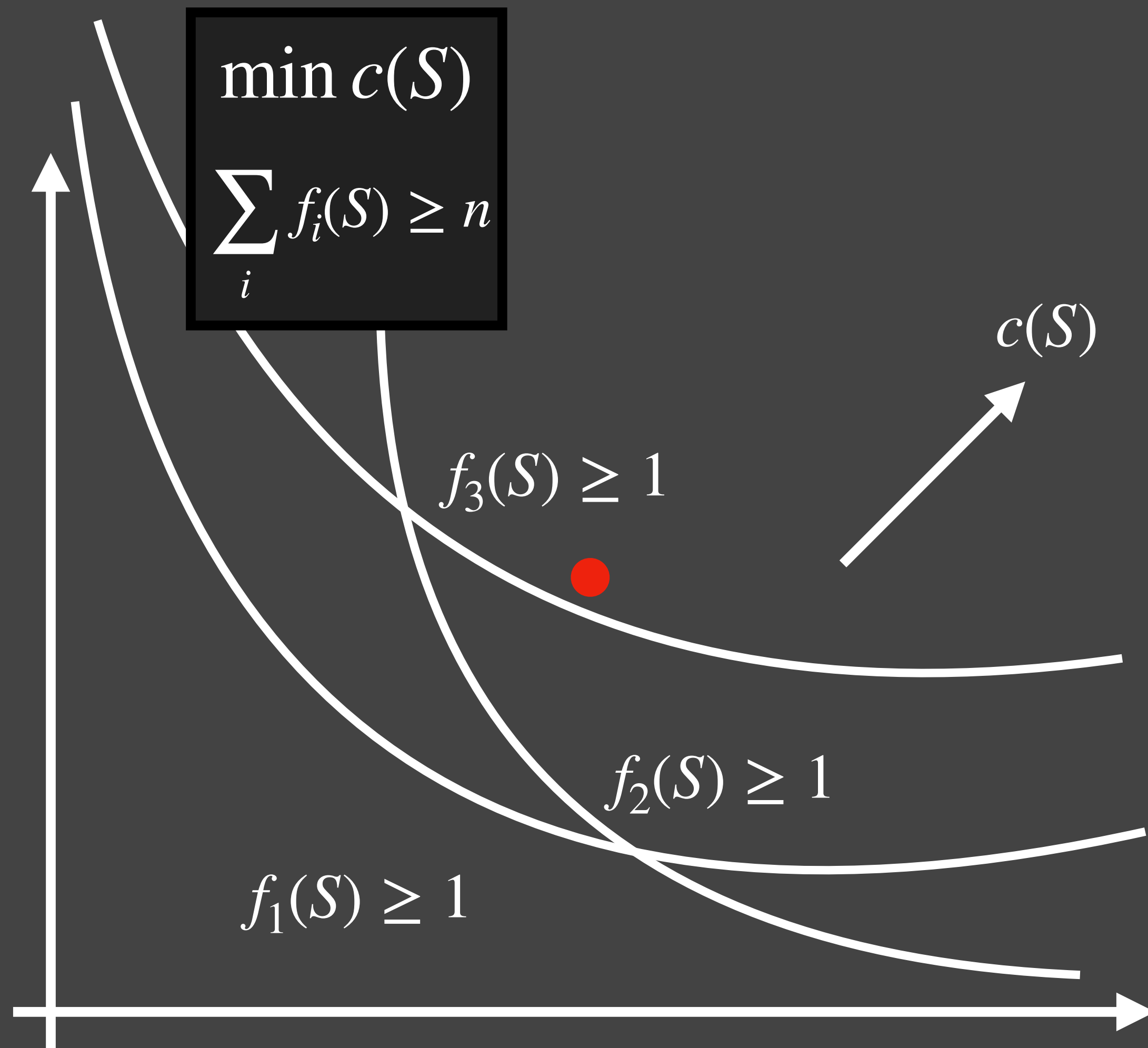
Dynamic Submodular Cover

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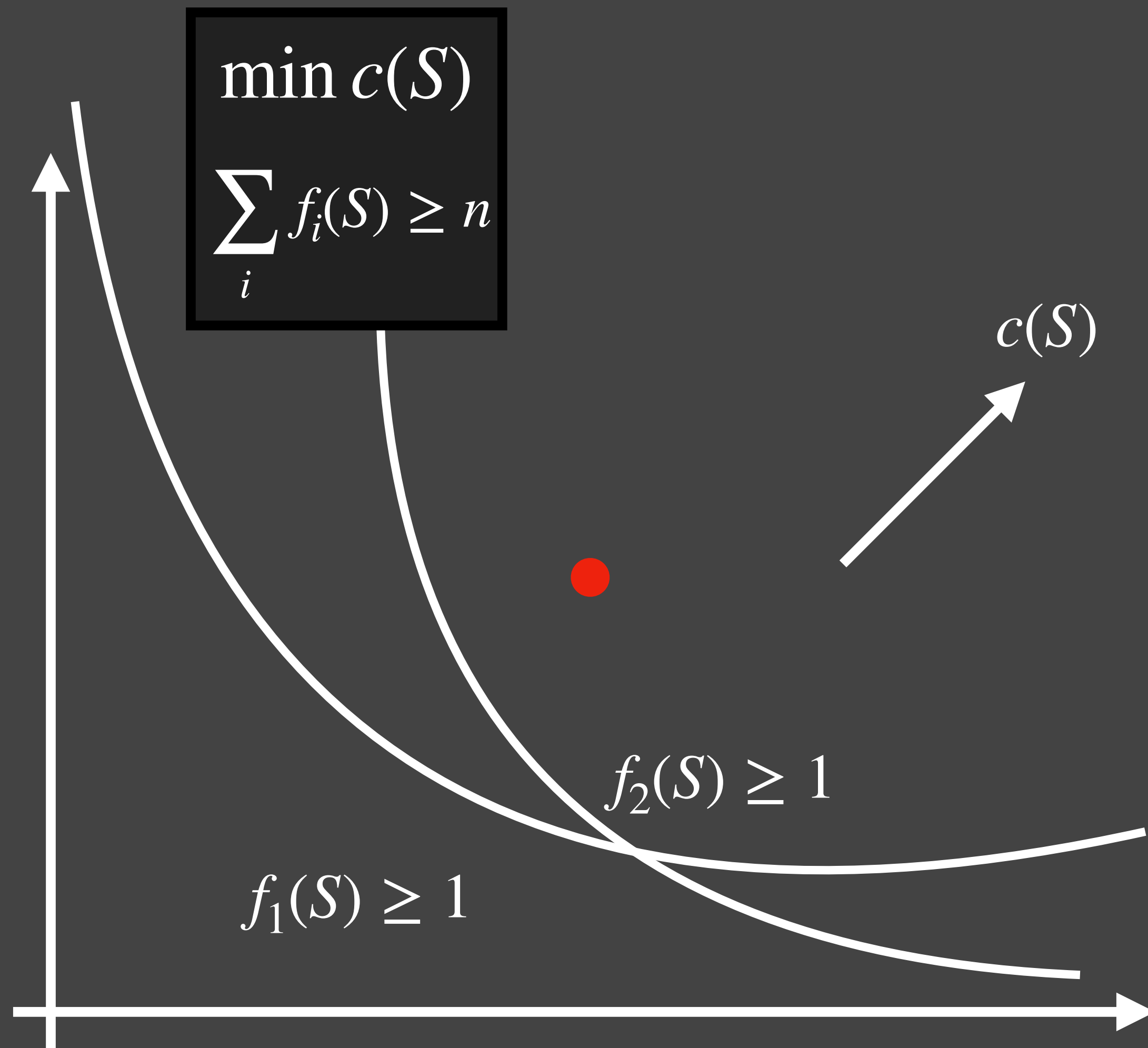
Dynamic Submodular Cover

[Gupta L. FOCS 20]



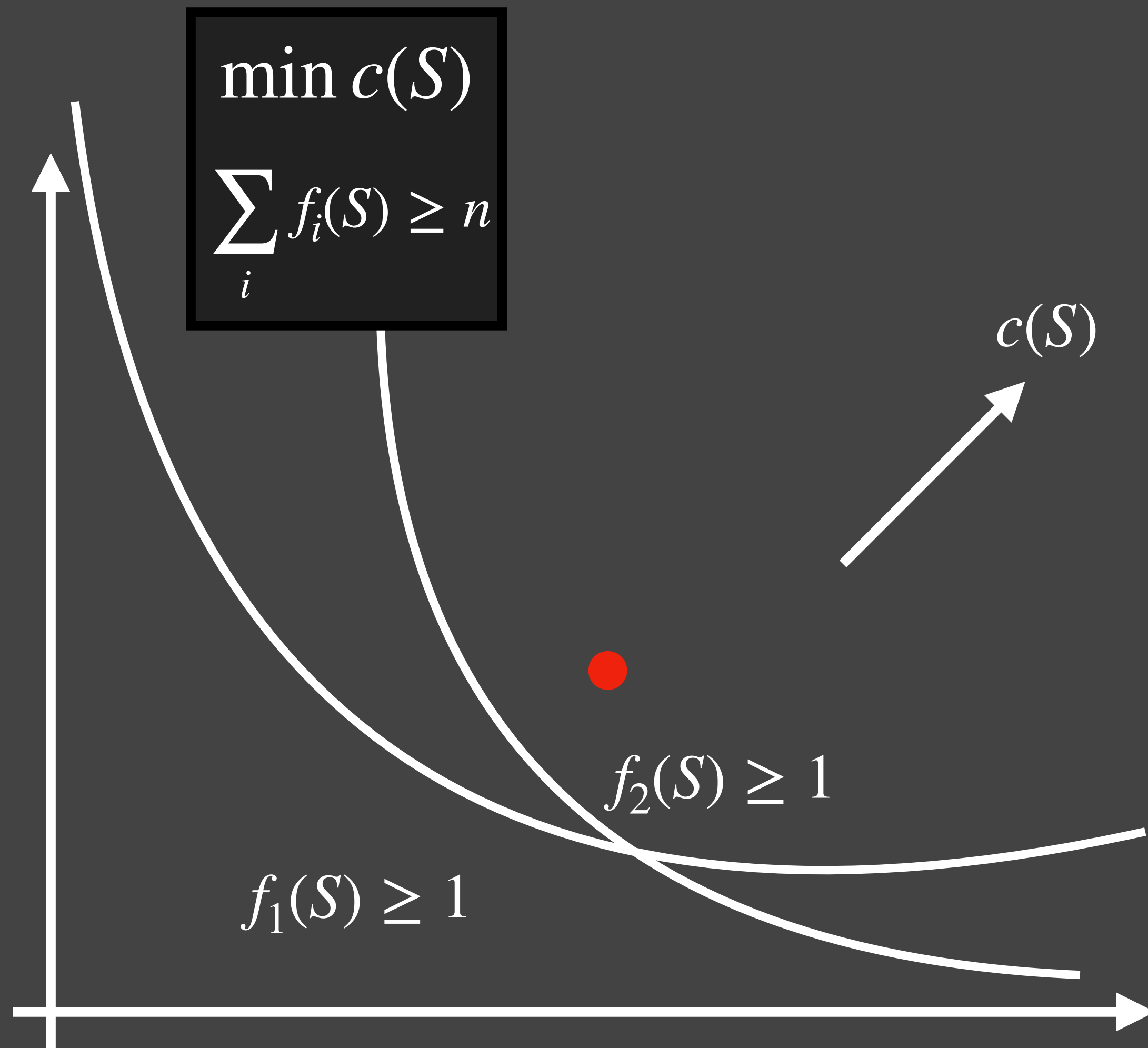
Dynamic Submodular Cover

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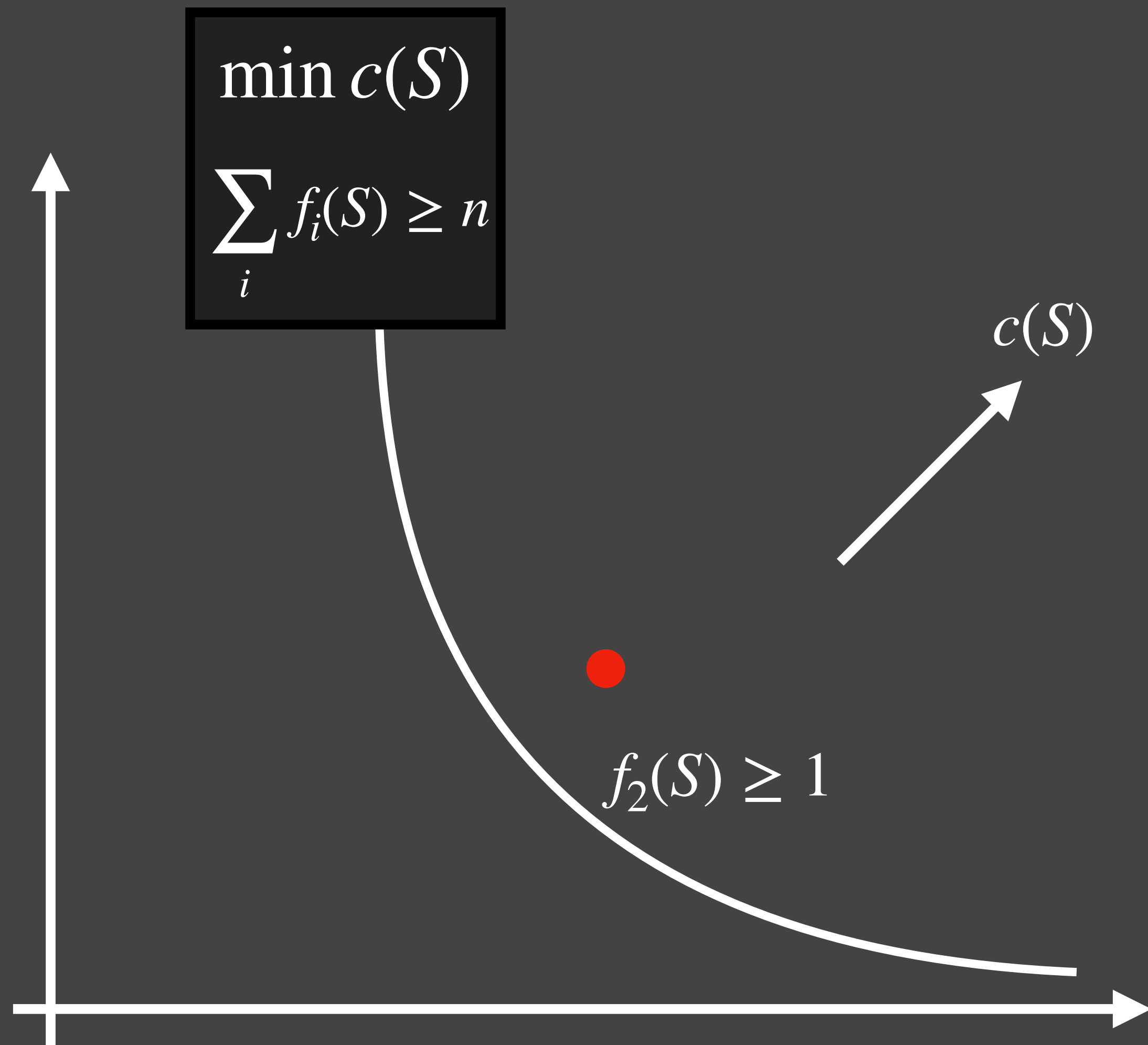
Dynamic Submodular Cover

[Gupta L. FOCS 20]



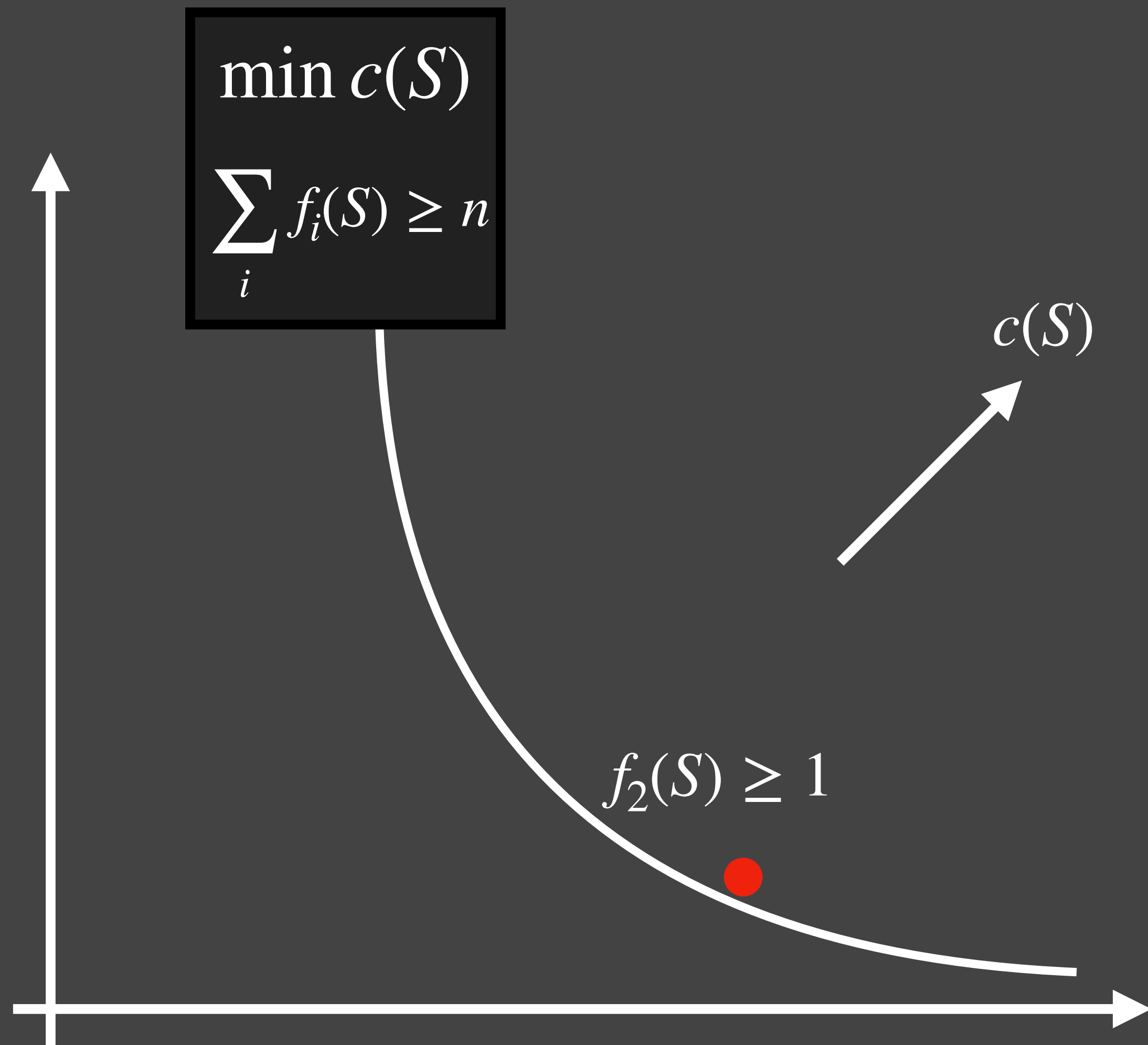
Dynamic Submodular Cover

[Gupta L. FOCS 20]



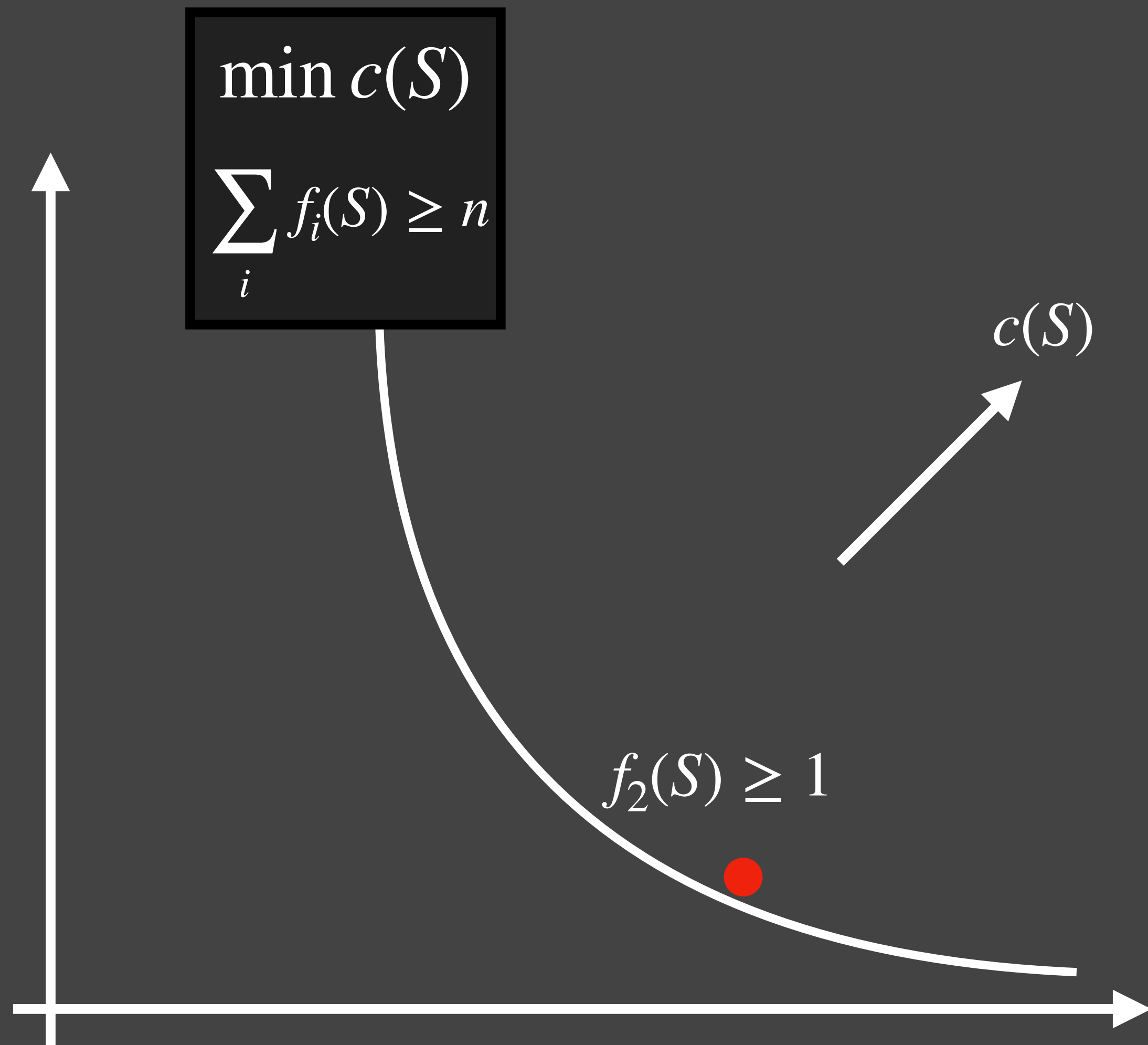
Dynamic Submodular Cover

[Gupta L. FOCS 20]



Dynamic Submodular Cover

[Gupta L. FOCS 20]



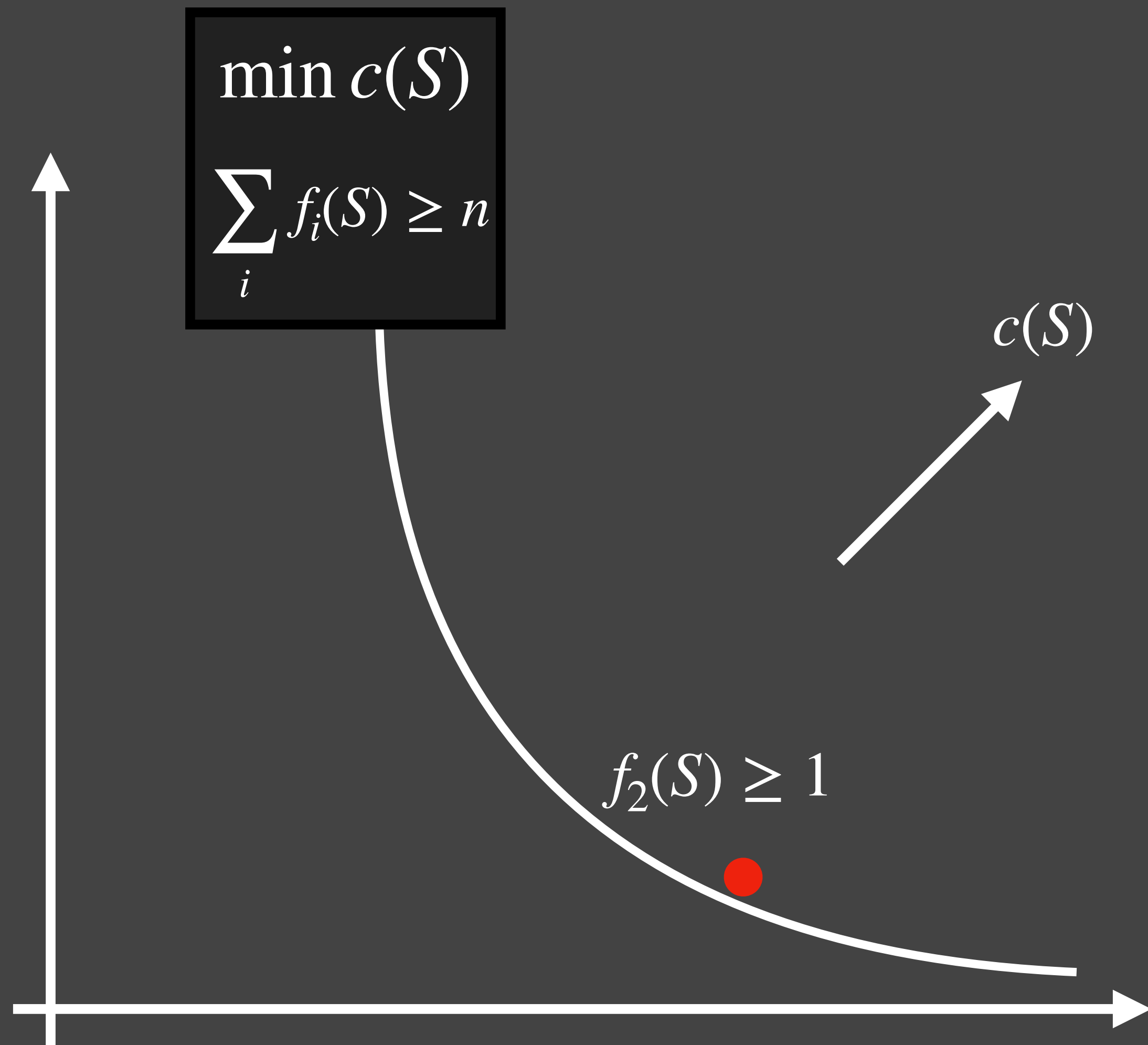
Theorem [Gupta L. FOCS 20]:

Polynomial time algo for
Dynamic Submod Cover with:

- (i) approximation $O(\log n)$.
- (ii) recourse $\tilde{O}(1)$.

Dynamic Submodular Cover

[Gupta L. FOCS 20]



Theorem [Gupta L. FOCS 20]:

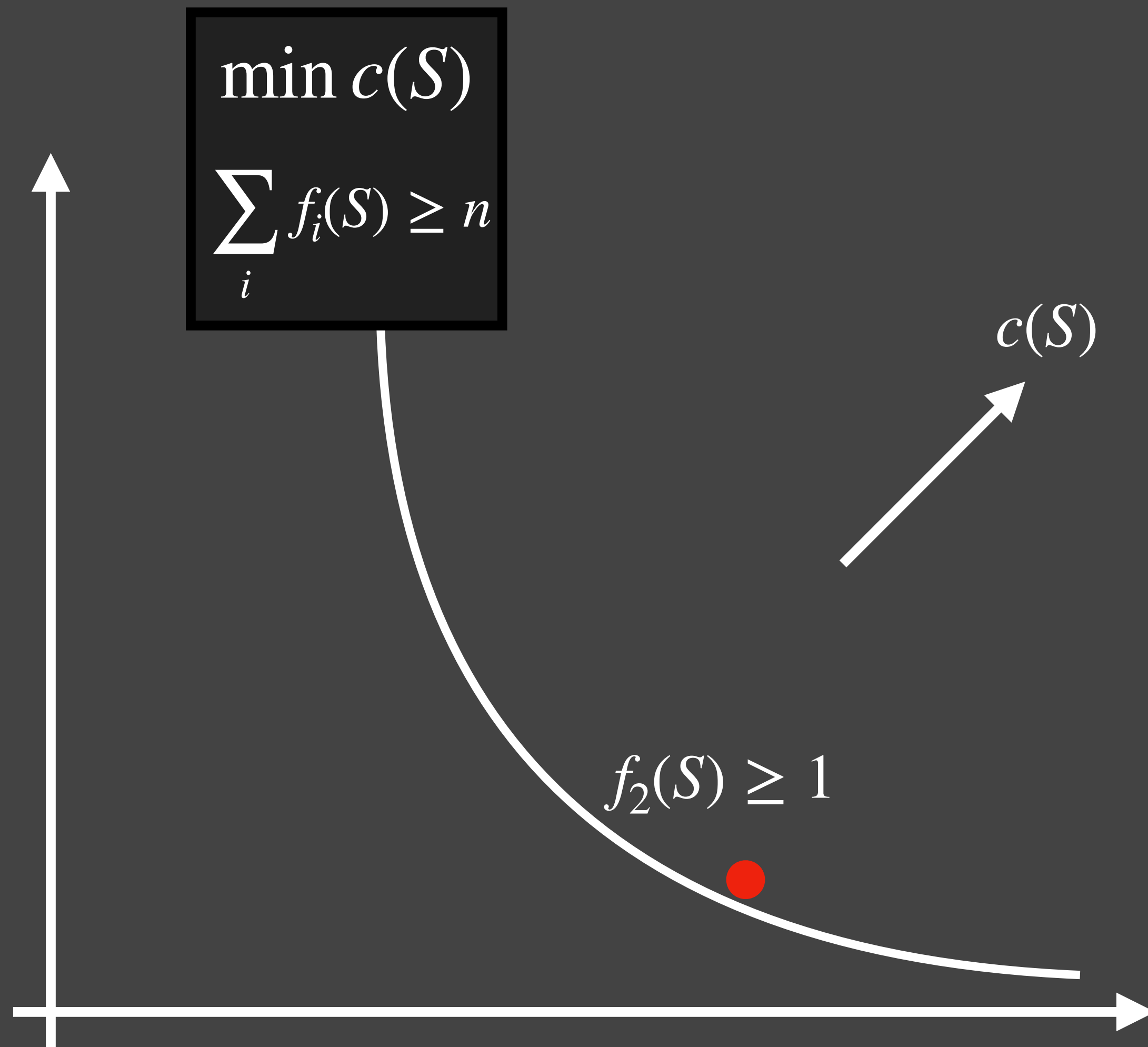
Polynomial time algo for
Dynamic Submod Cover with:

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Optimal!

Dynamic Submodular Cover

[Gupta L. FOCS 20]



Theorem [Gupta L. FOCS 20]:

Polynomial time algo for
Dynamic Submod Cover with:

- (i) approximation $O(\log n)$.
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Optimal!

Technical Ingredient:

Template for converting **greedy**
algorithms to **local search** algorithms,
+ **Tsallis Entropy** potential for
analysis!

Comparison

Online

- Inserts Only
- Decisions are irrevocable



Dynamic

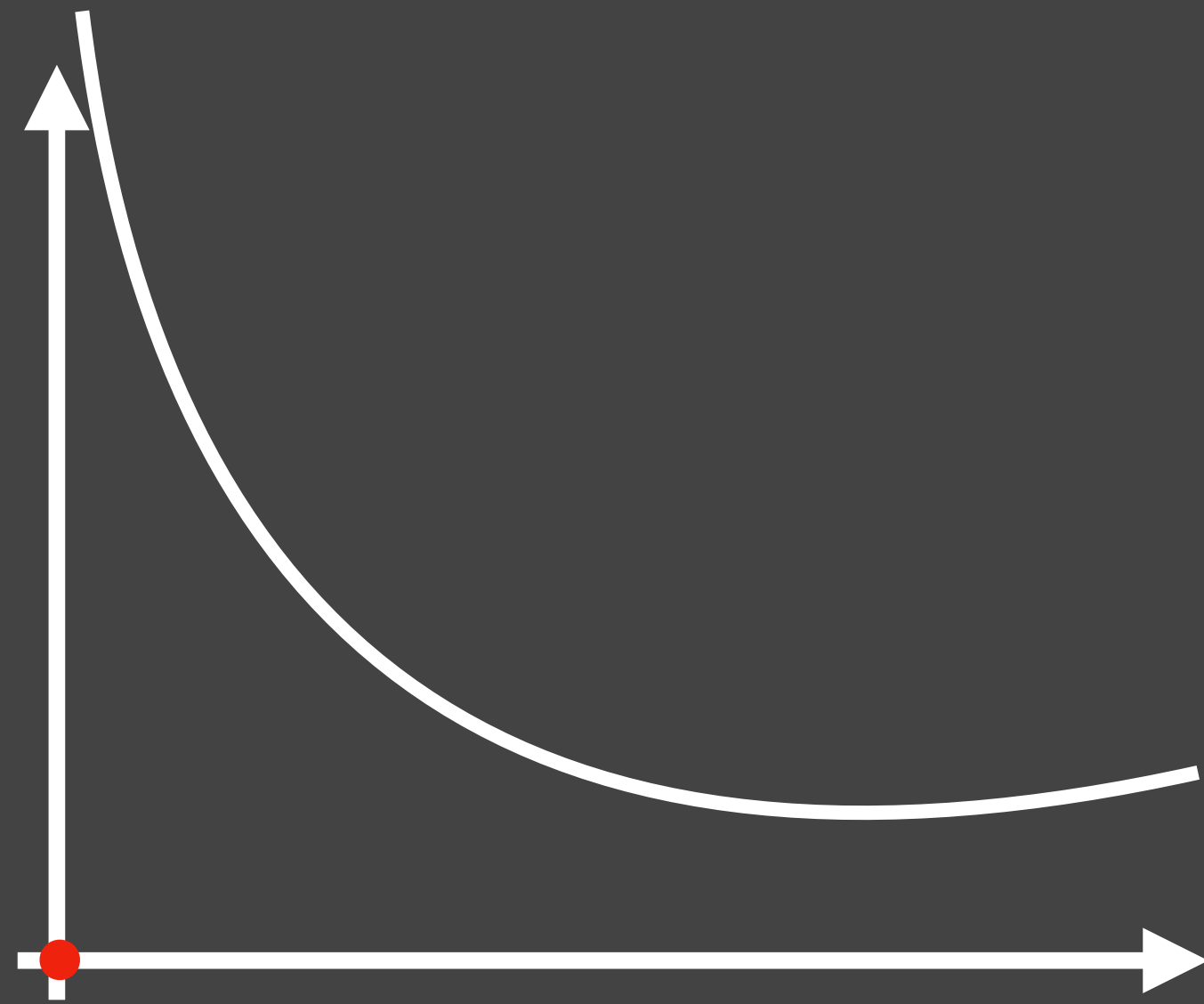
- Inserts + Deletes
- Want minimum # edits, a.k.a. recourse.



Comparison

Online

- Inserts Only
- Decisions are irrevocable



Dynamic

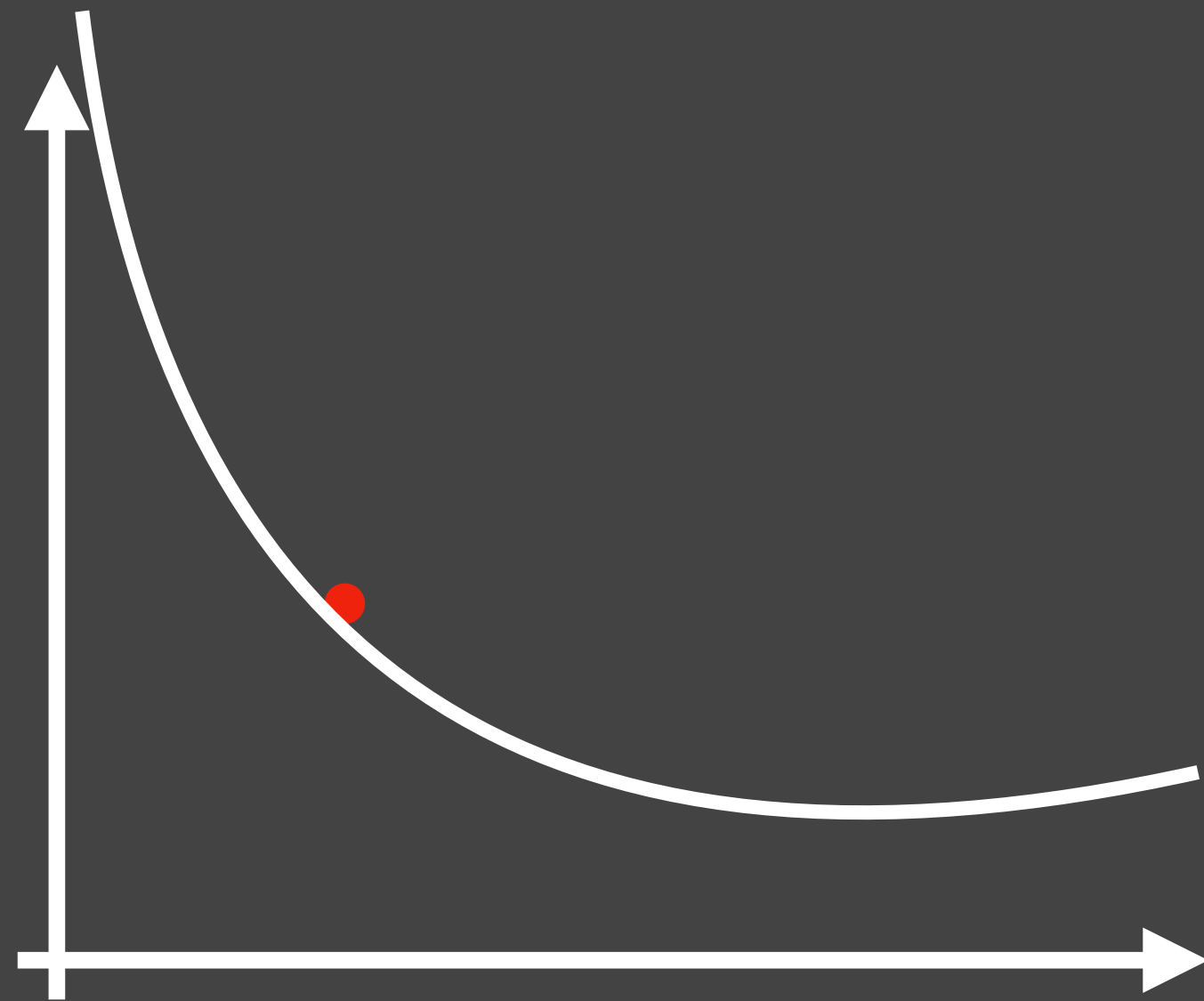
- Inserts + Deletes
- Want minimum # edits, a.k.a. recourse.



Comparison

Online

- Inserts Only
- Decisions are irrevocable



Dynamic

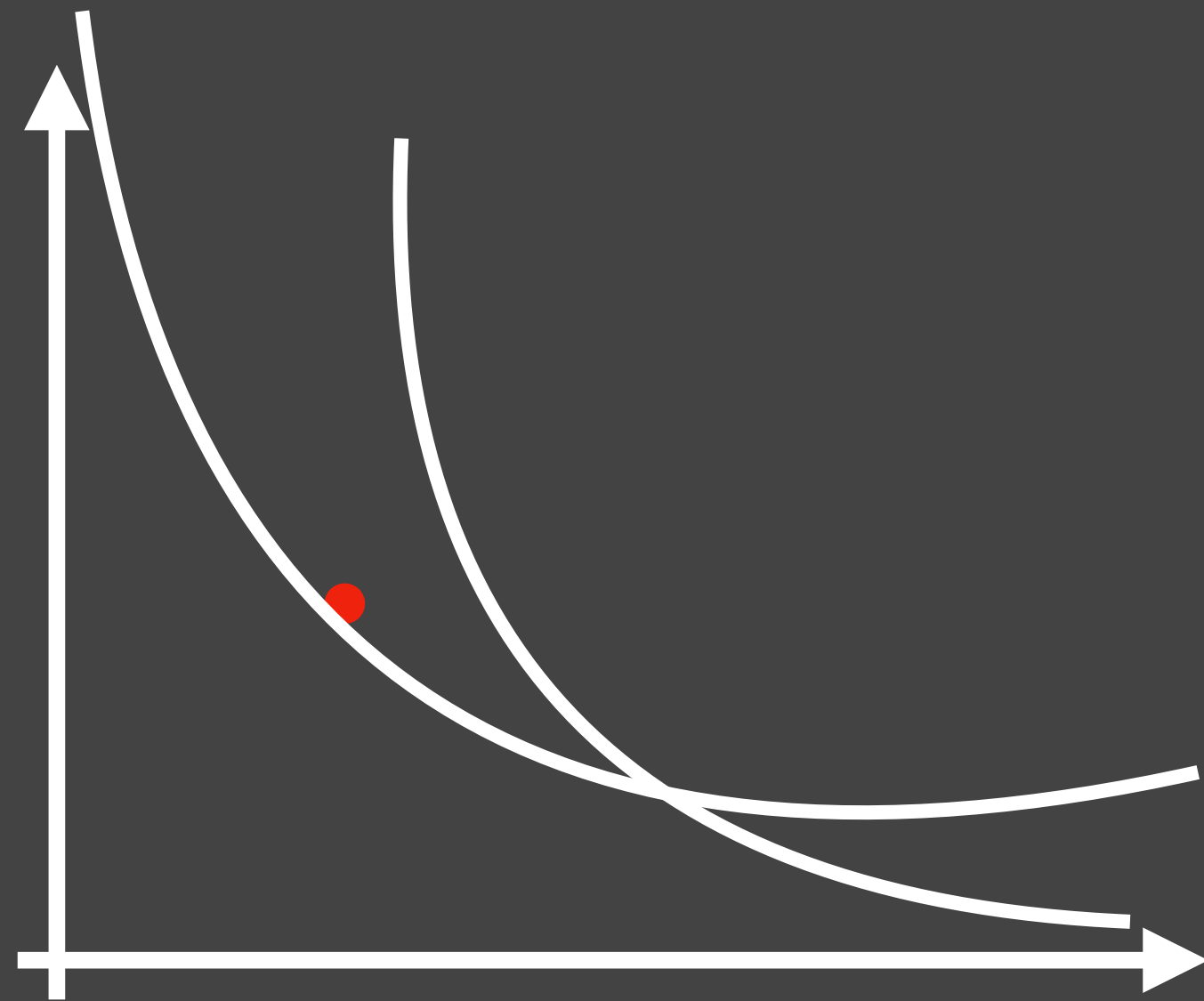
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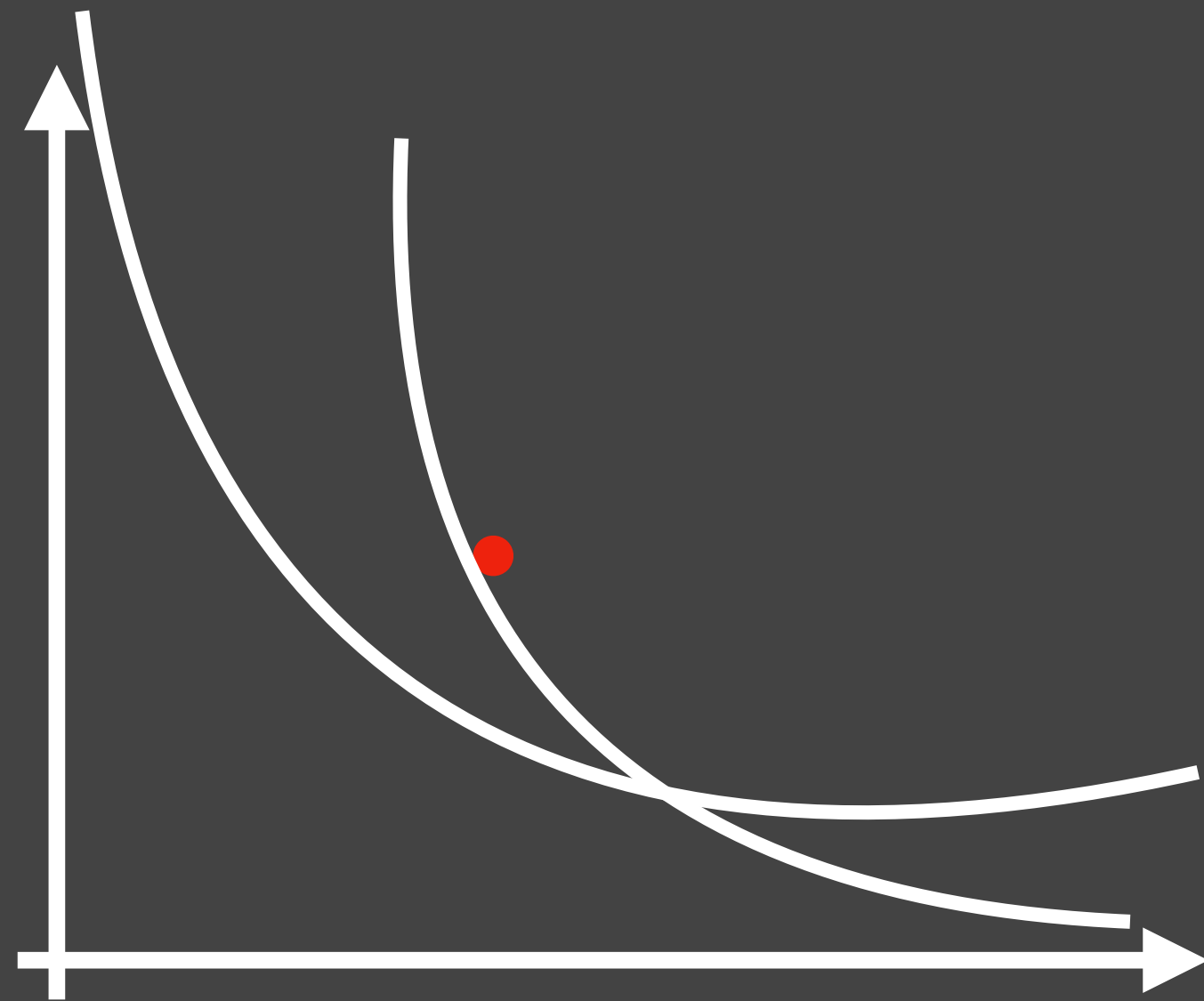
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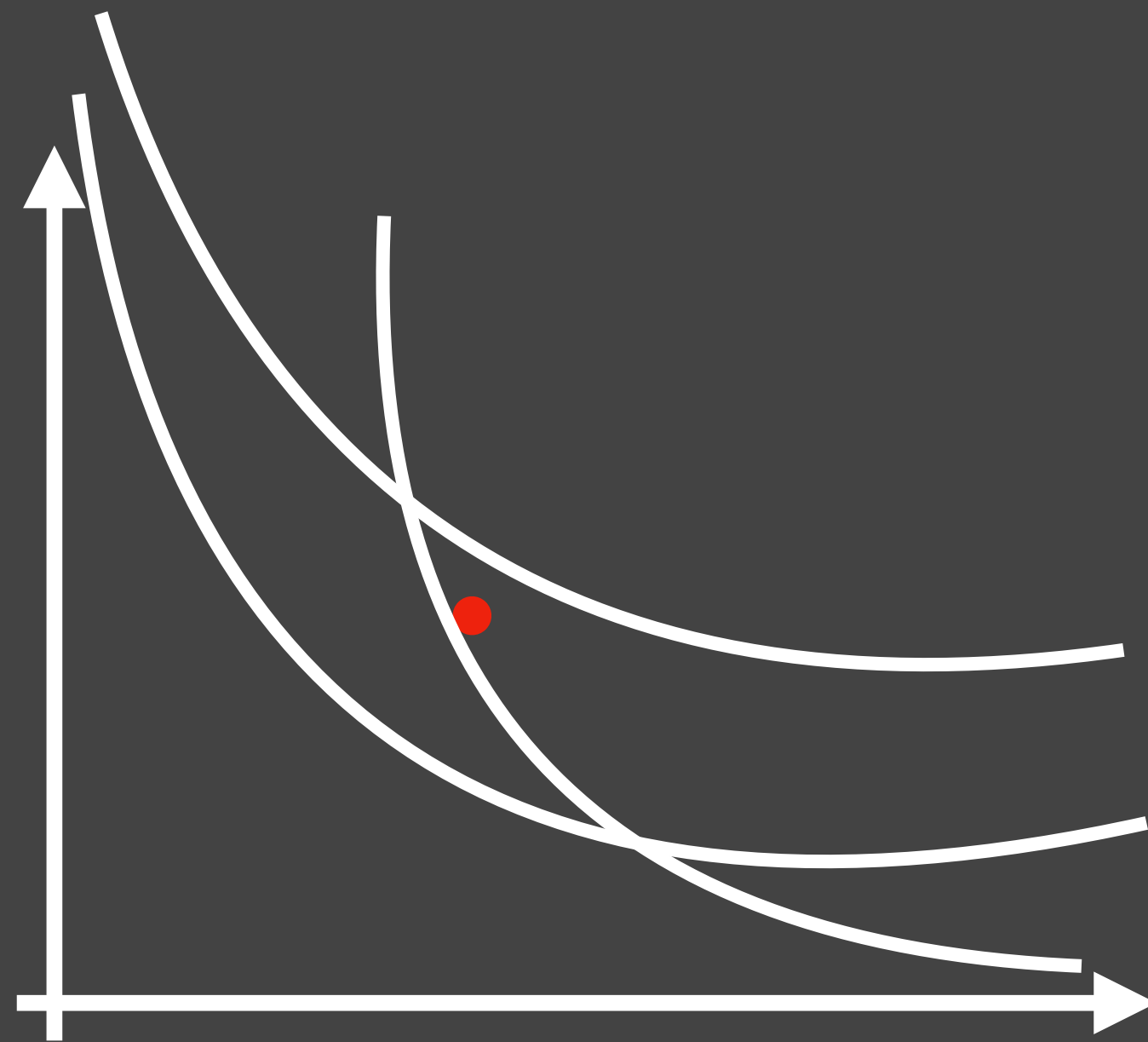
- Inserts + Deletes
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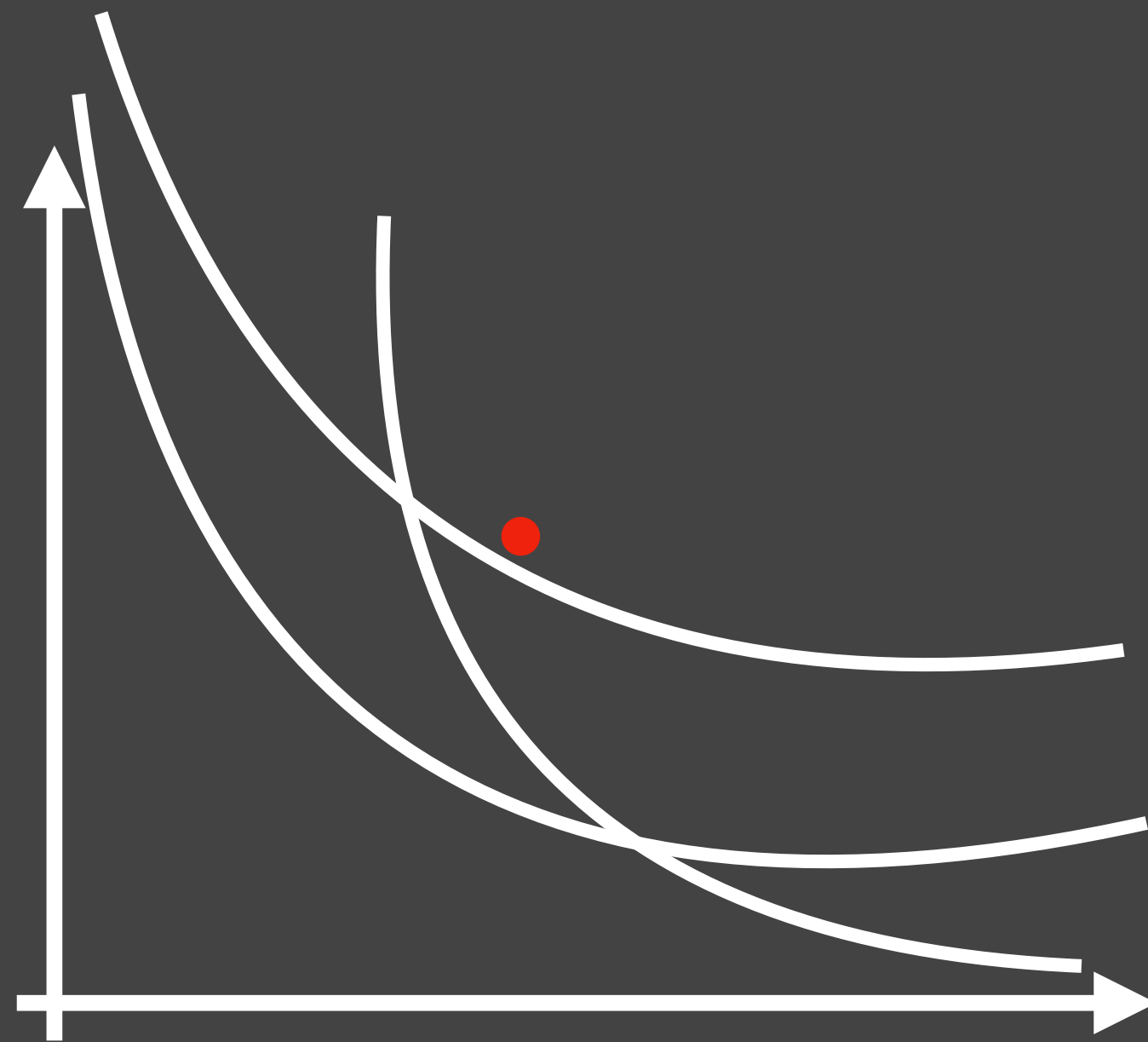
- Inserts + Deletes
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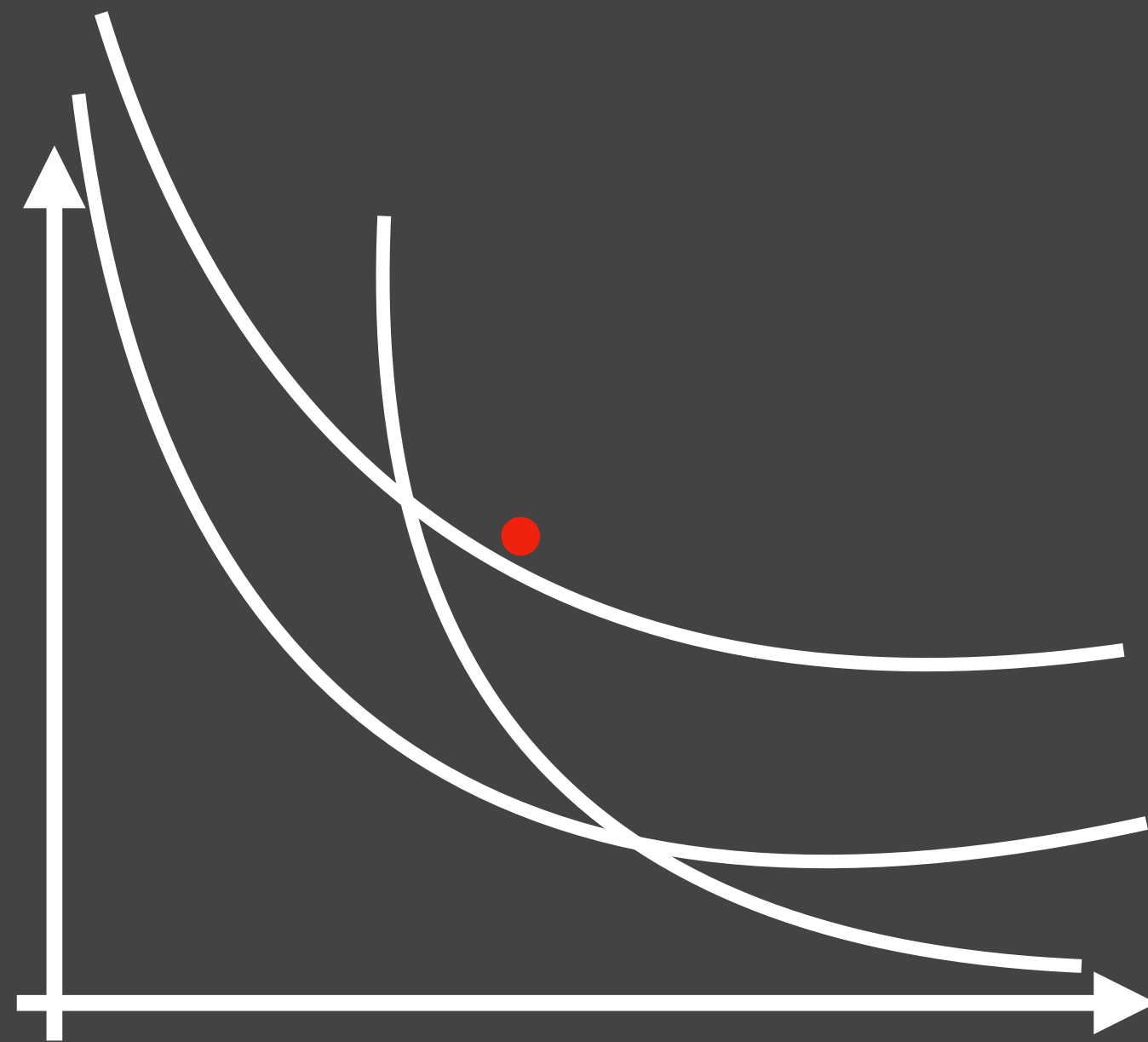
- Inserts + Deletes
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Comparison

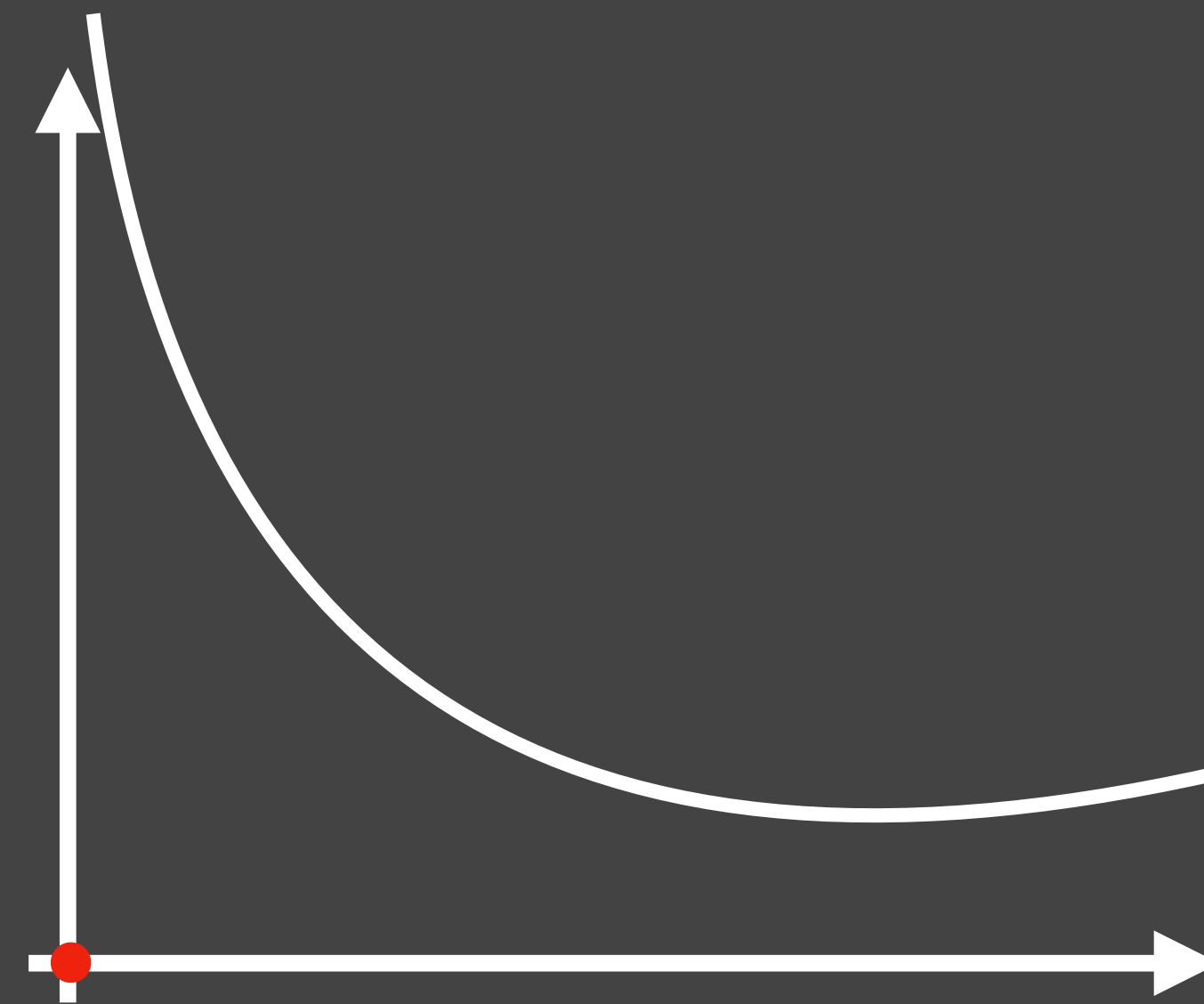
Online

- Inserts Only
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Dynamic

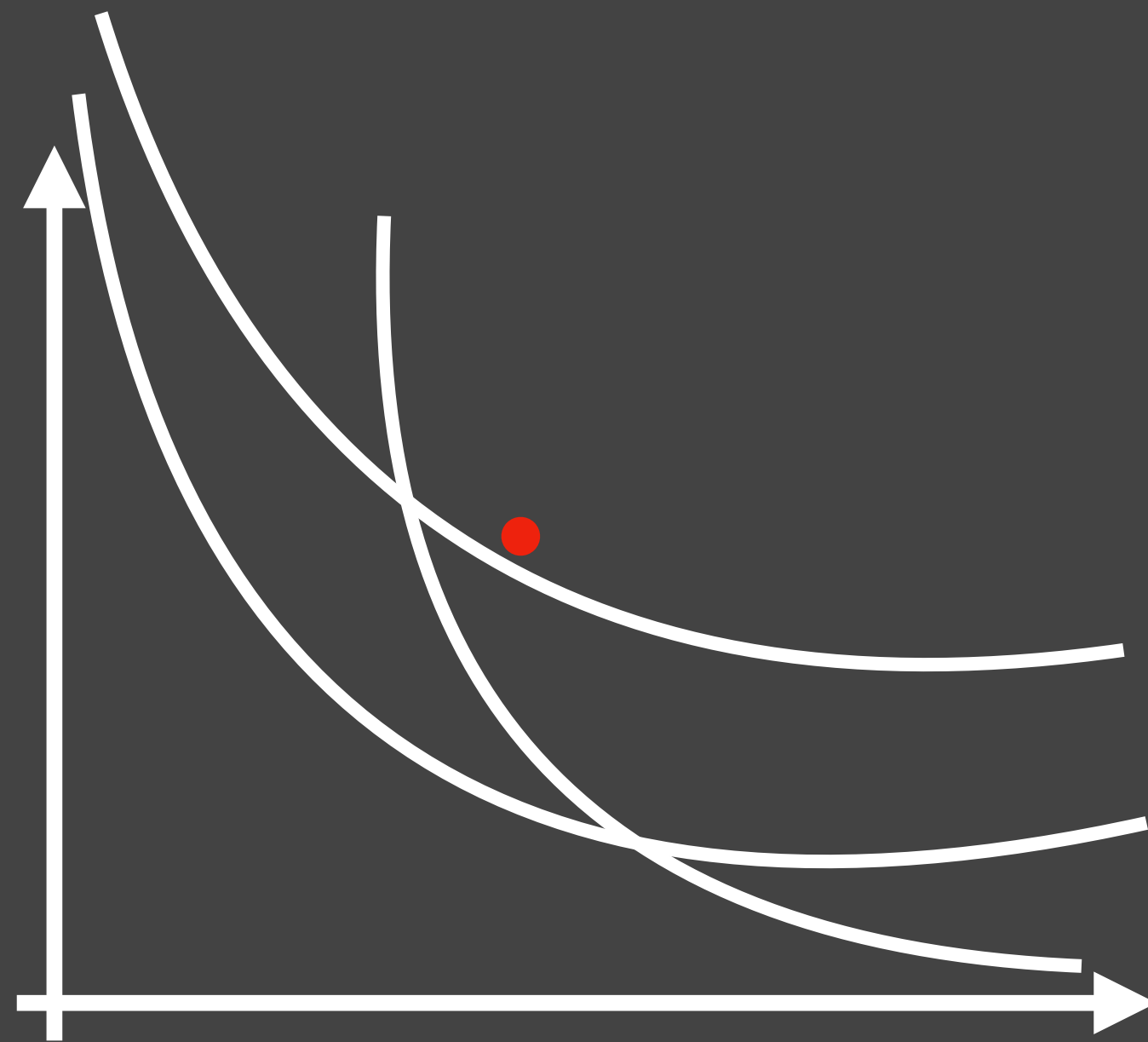
- Inserts + Deletes
- Want minimum # edits, a.k.a. recourse.



Comparison

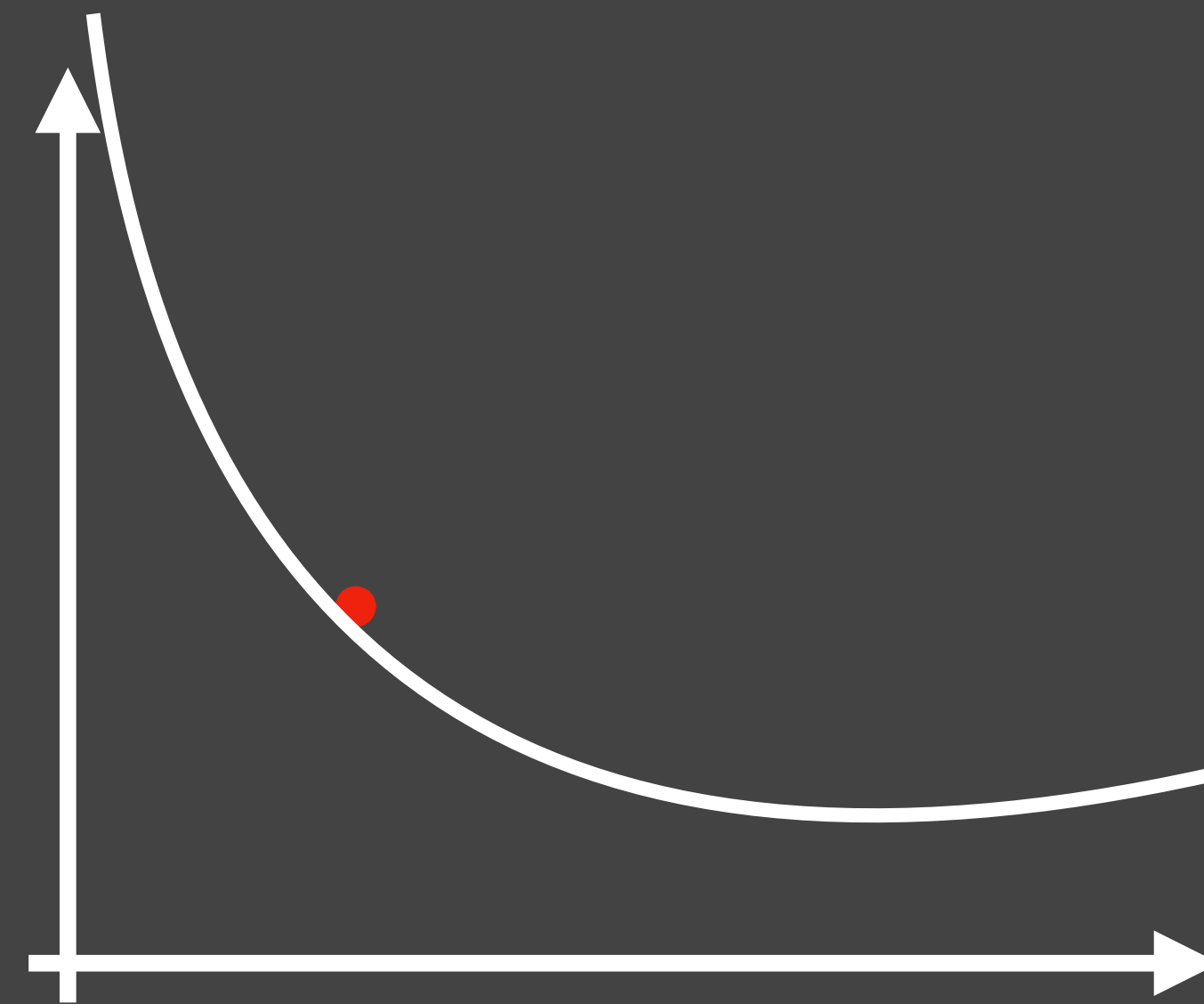
Online

- Inserts Only
- Decisions are irrevocable



Dynamic

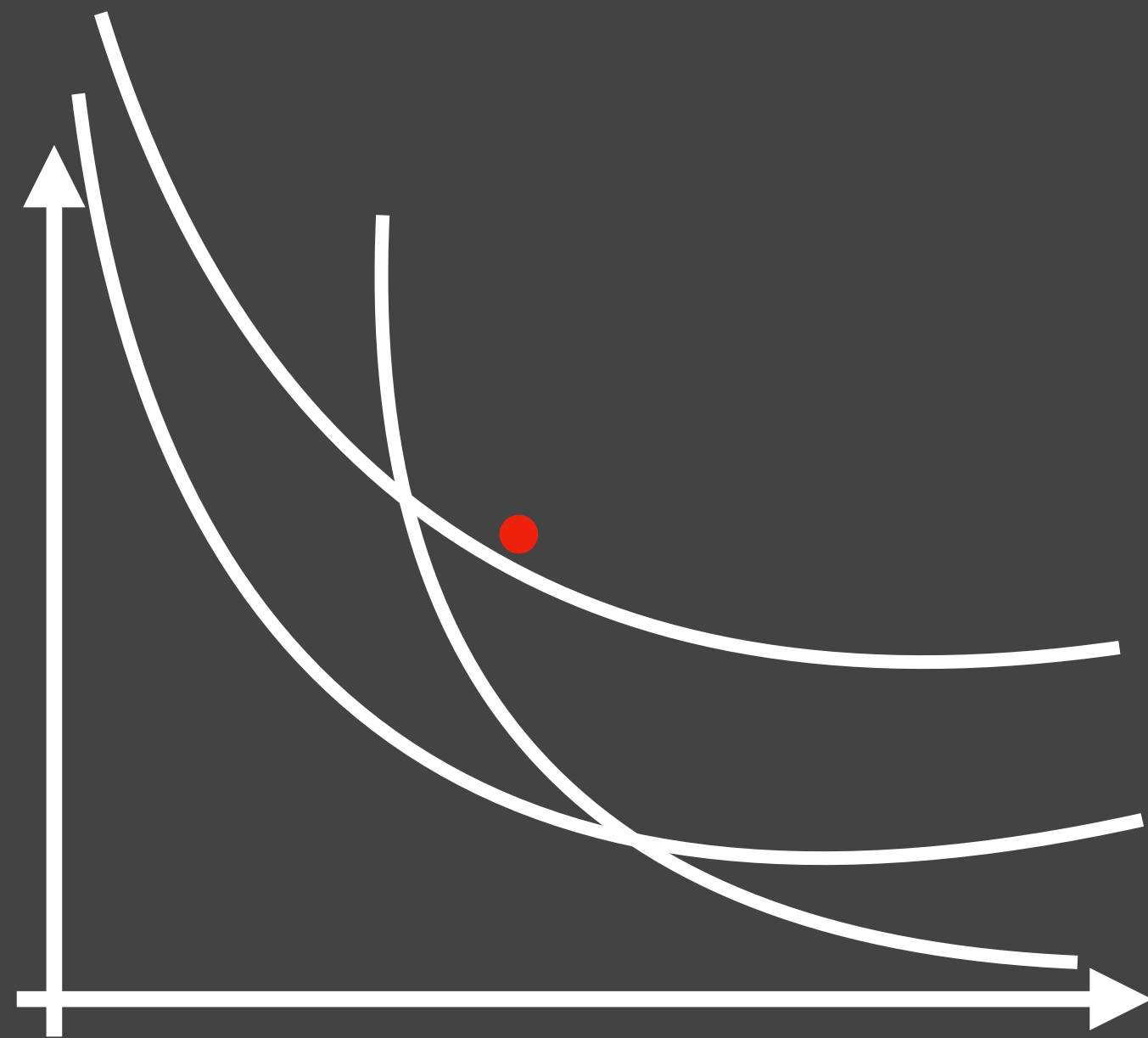
- Inserts + Deletes
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Comparison

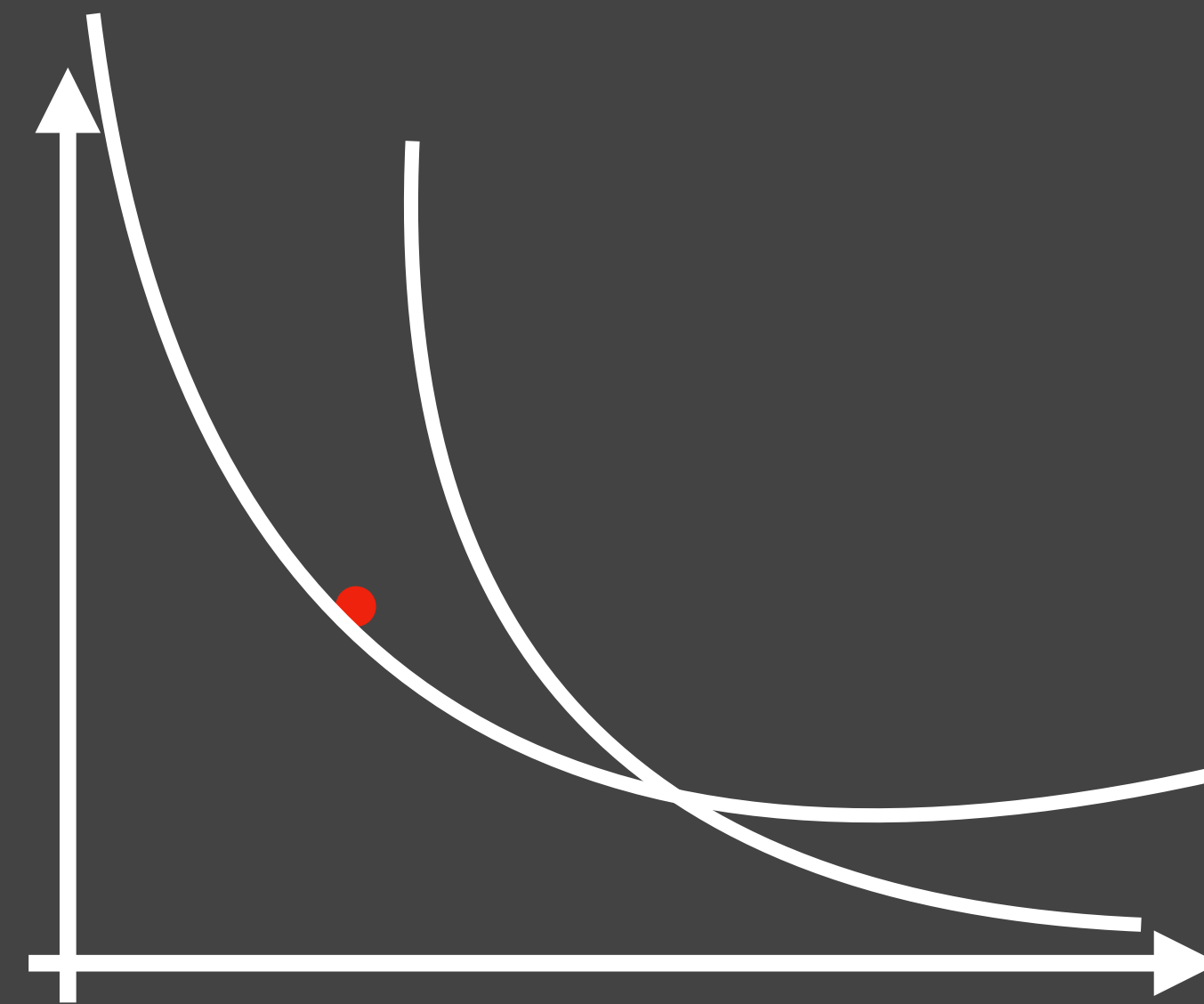
Online

- Inserts Only
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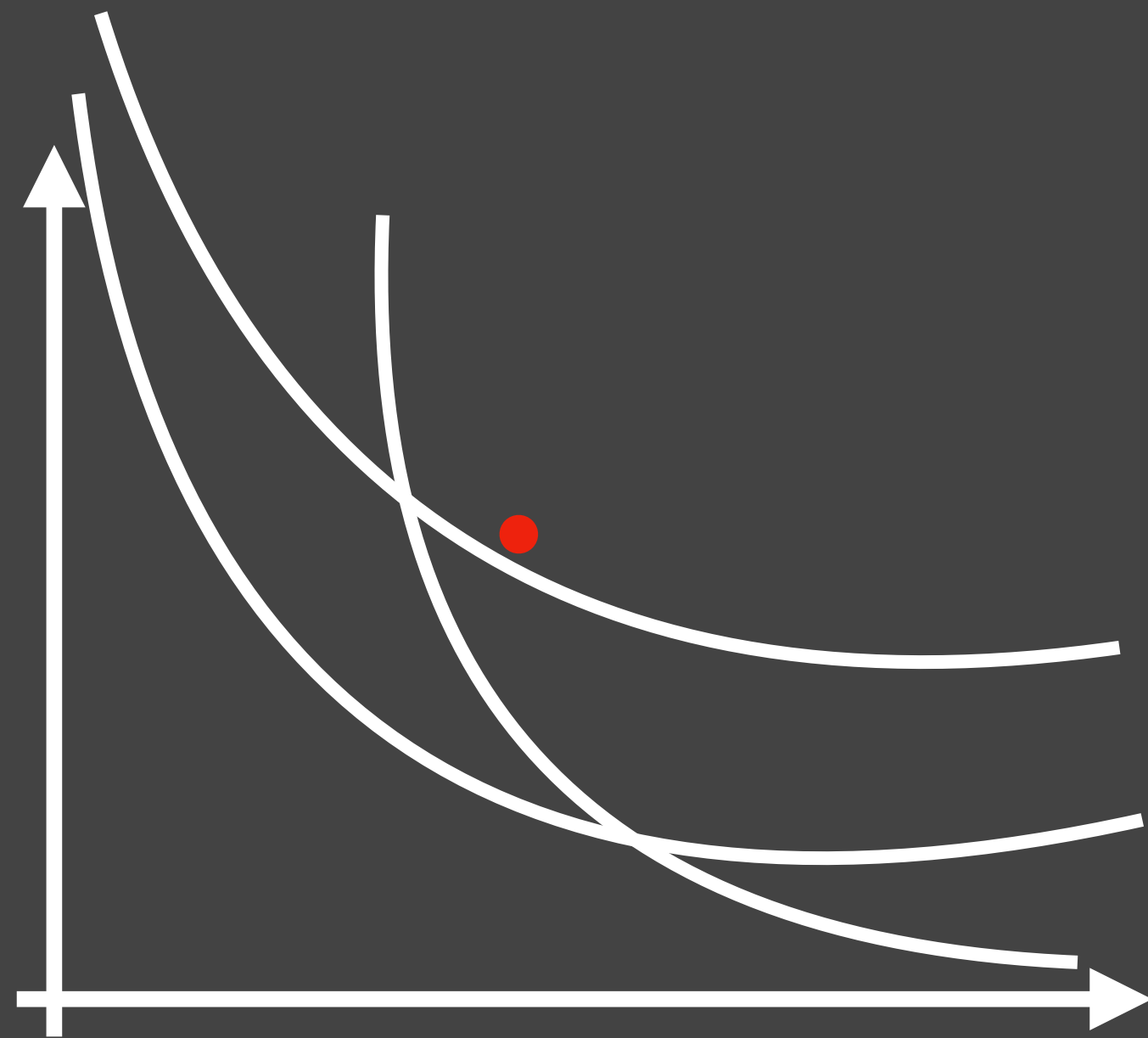
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Comparison

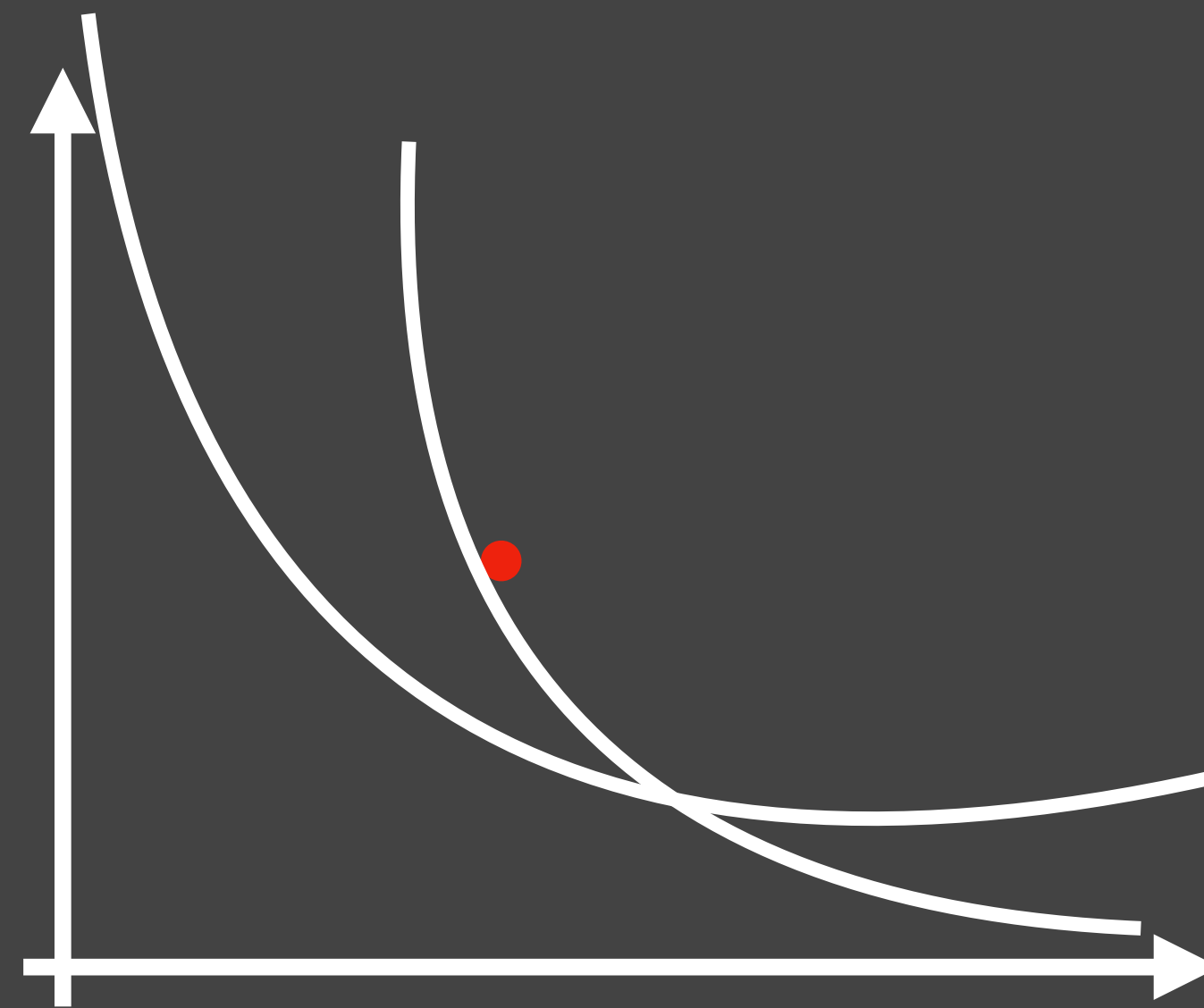
Online

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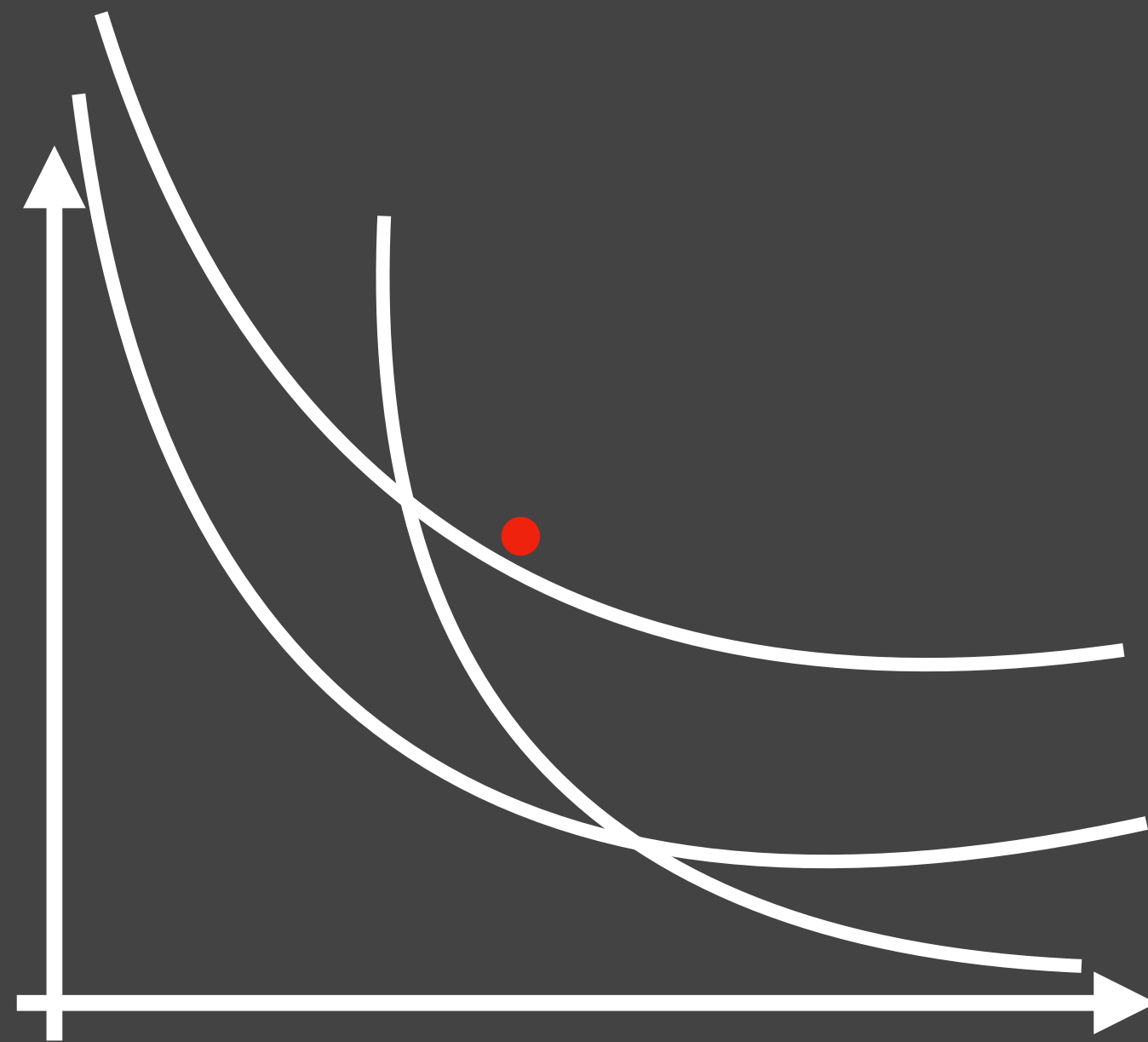
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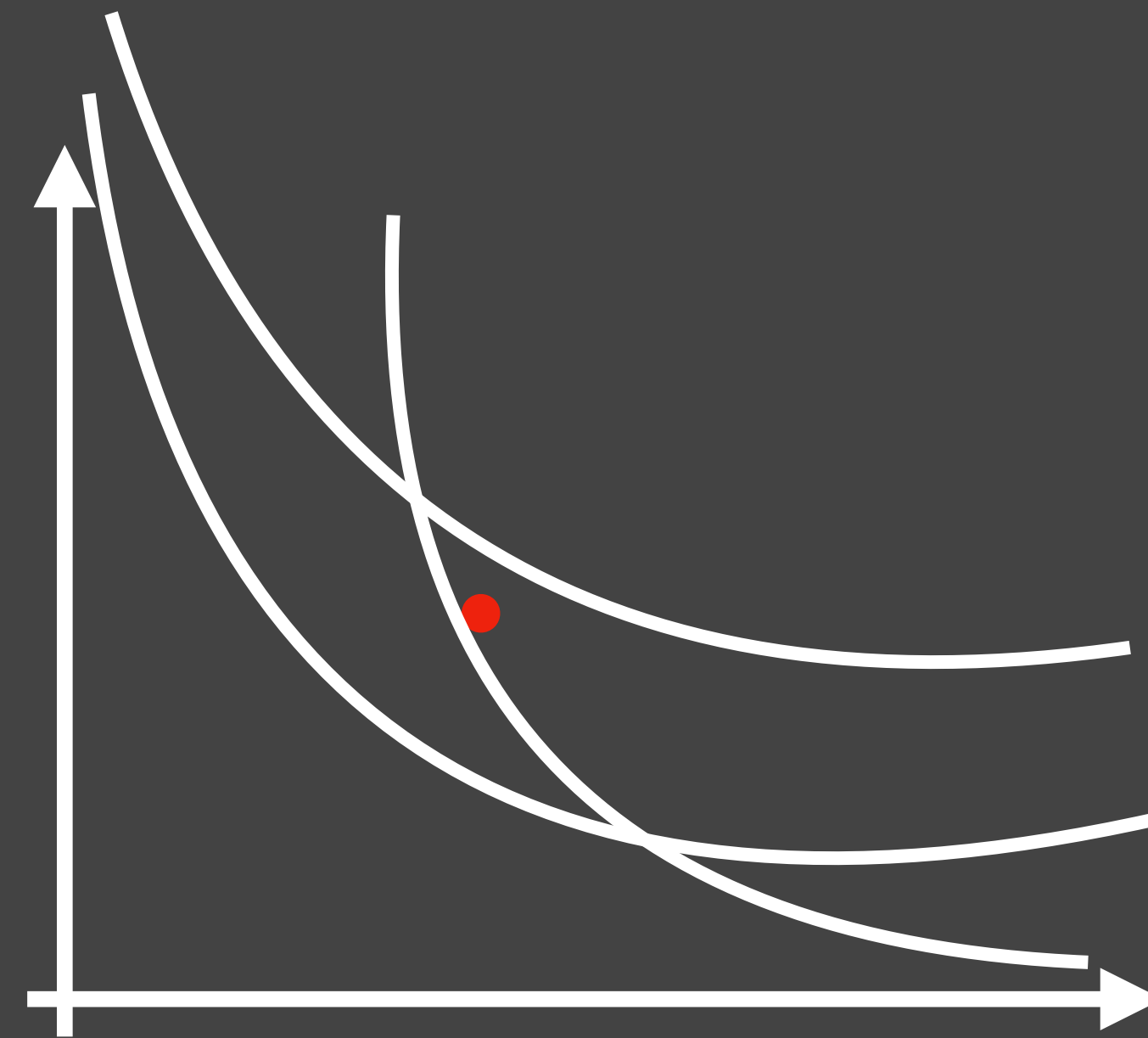
Online

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Dynamic

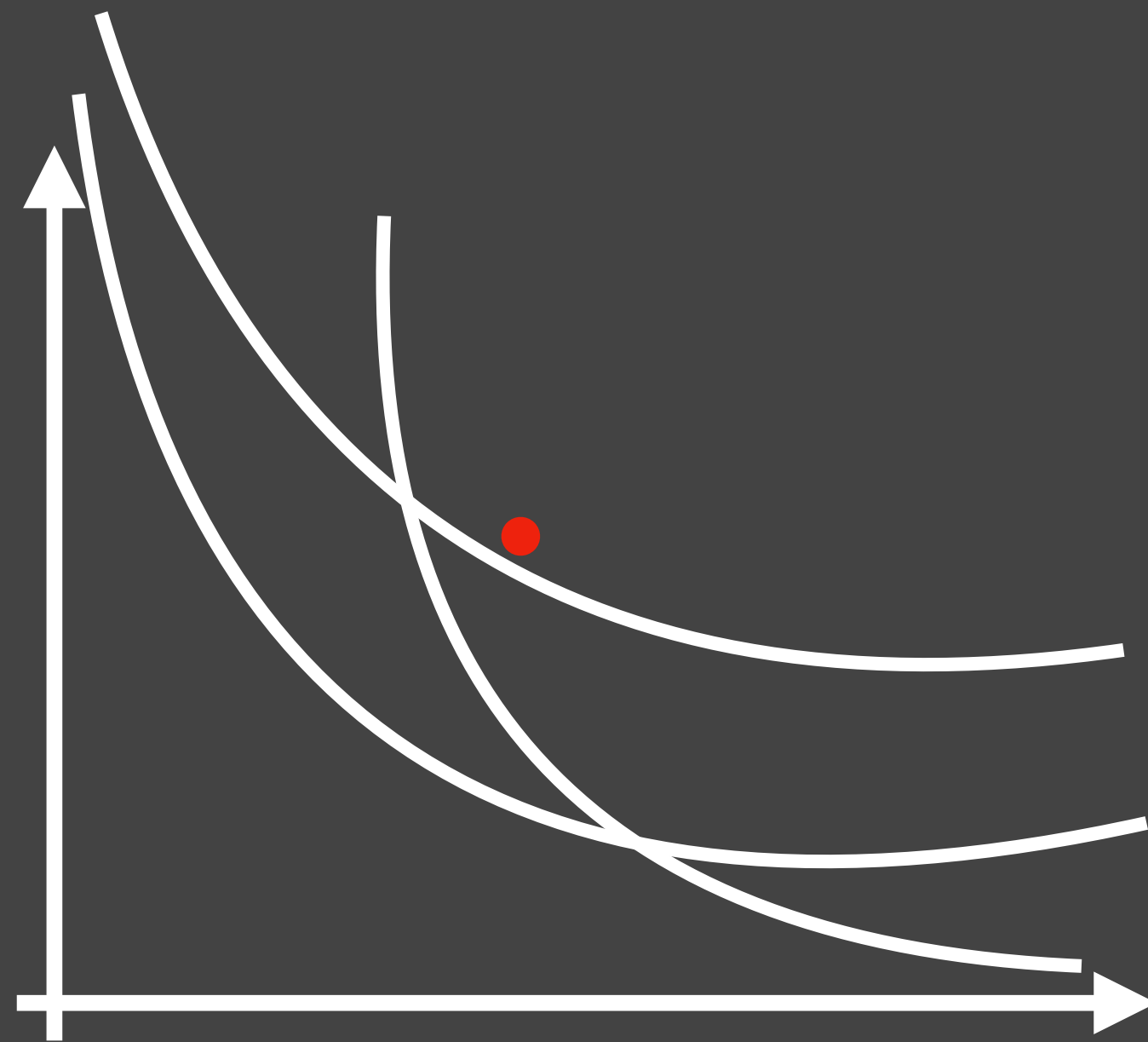
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Comparison

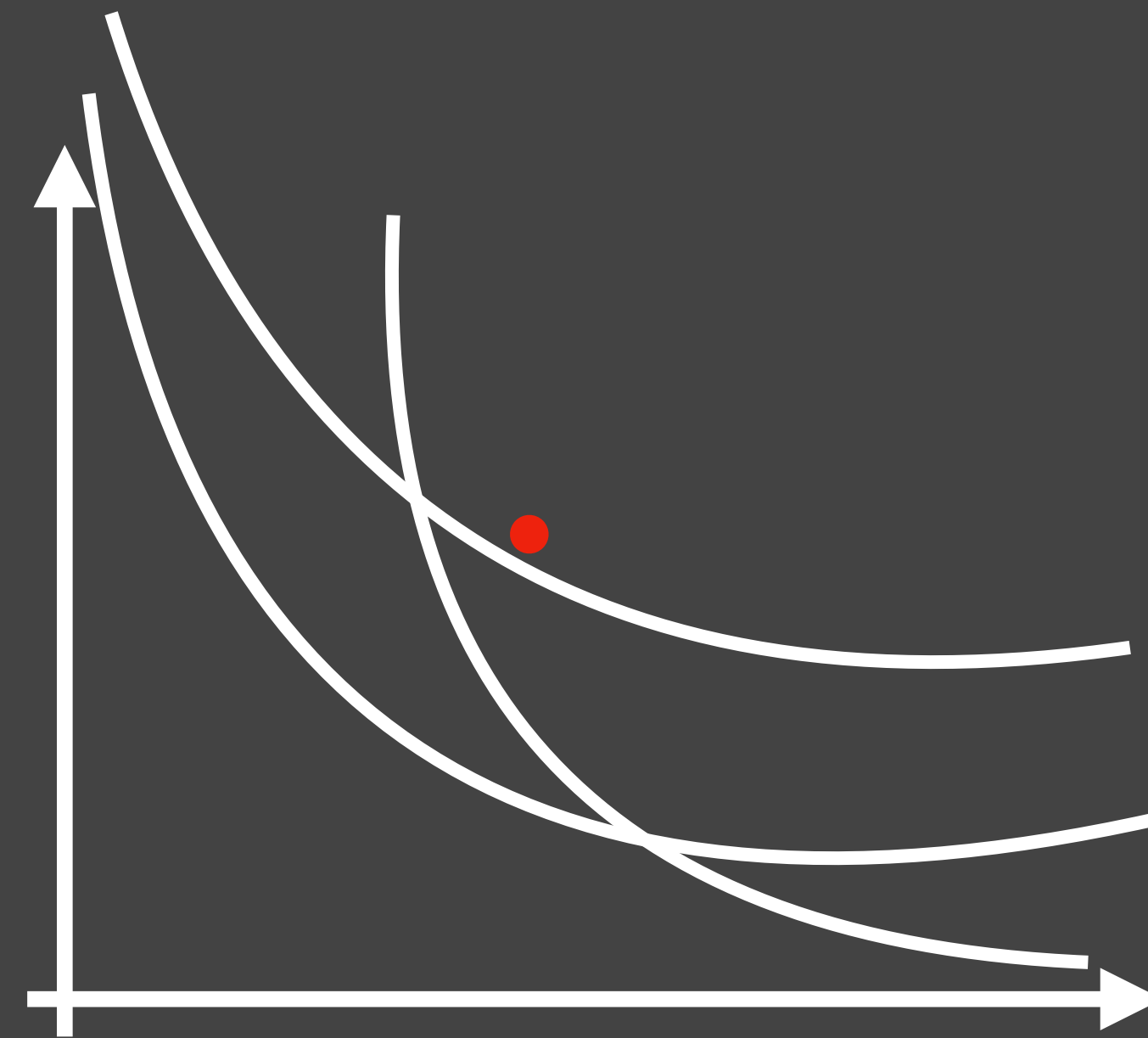
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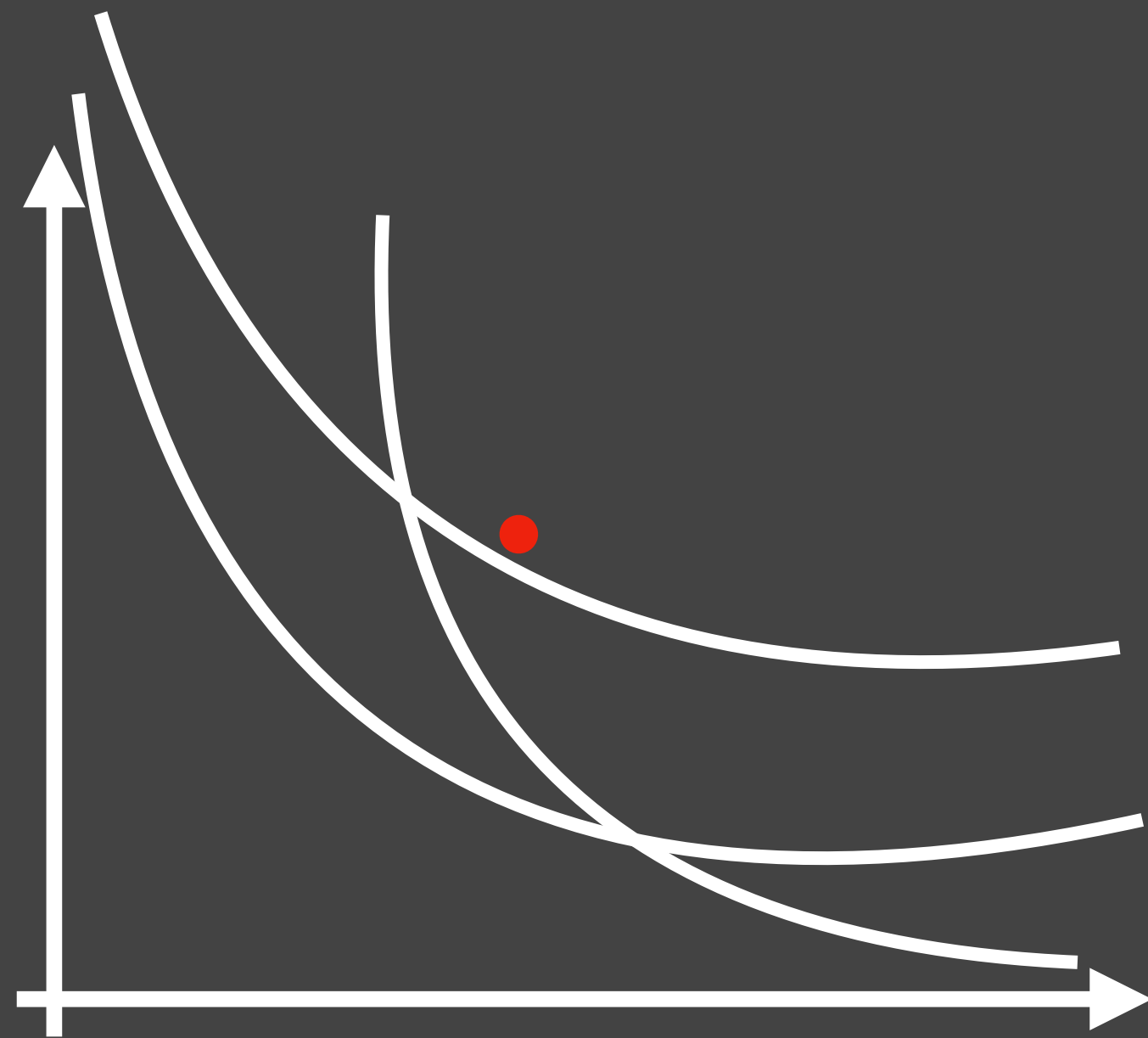
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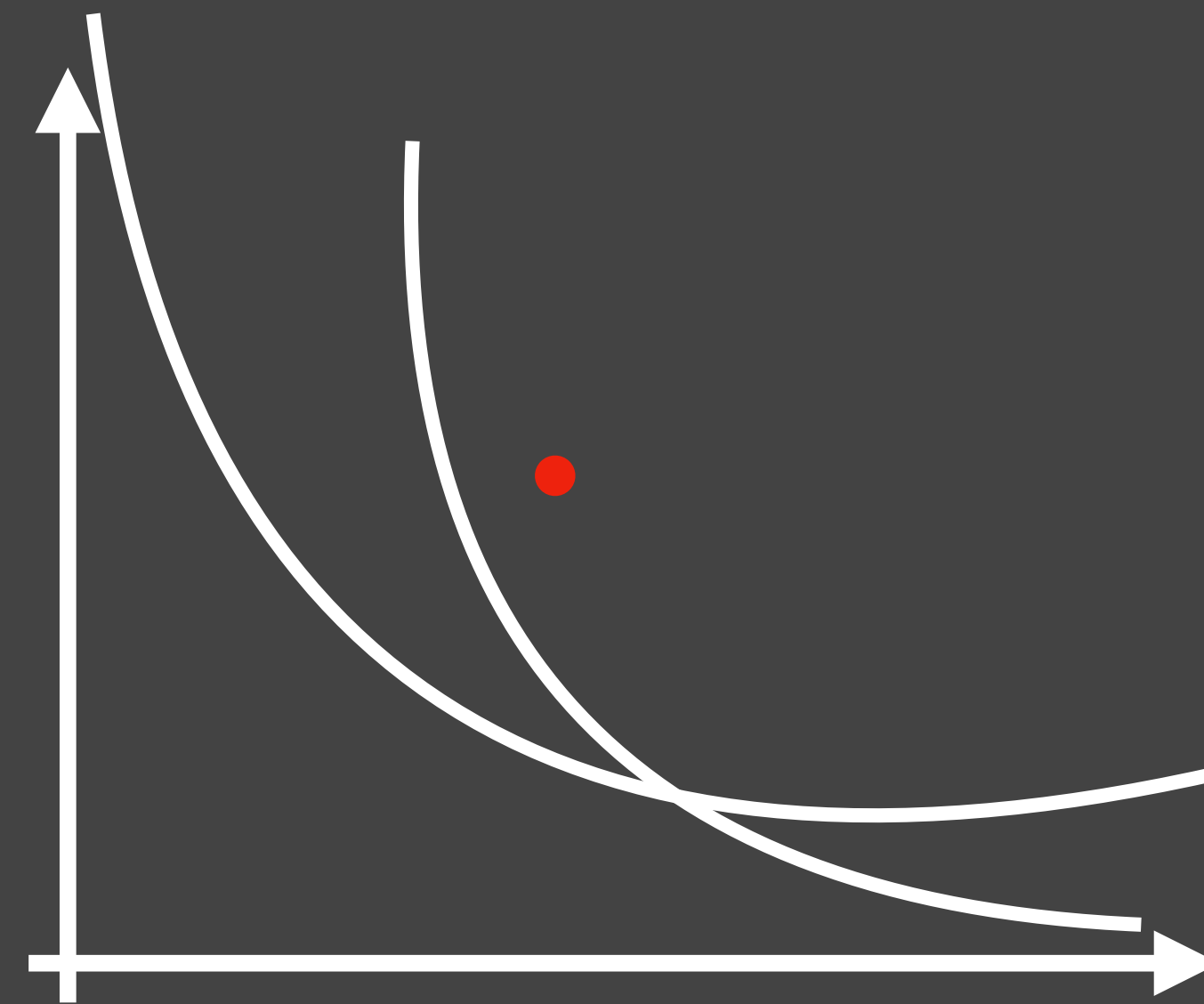
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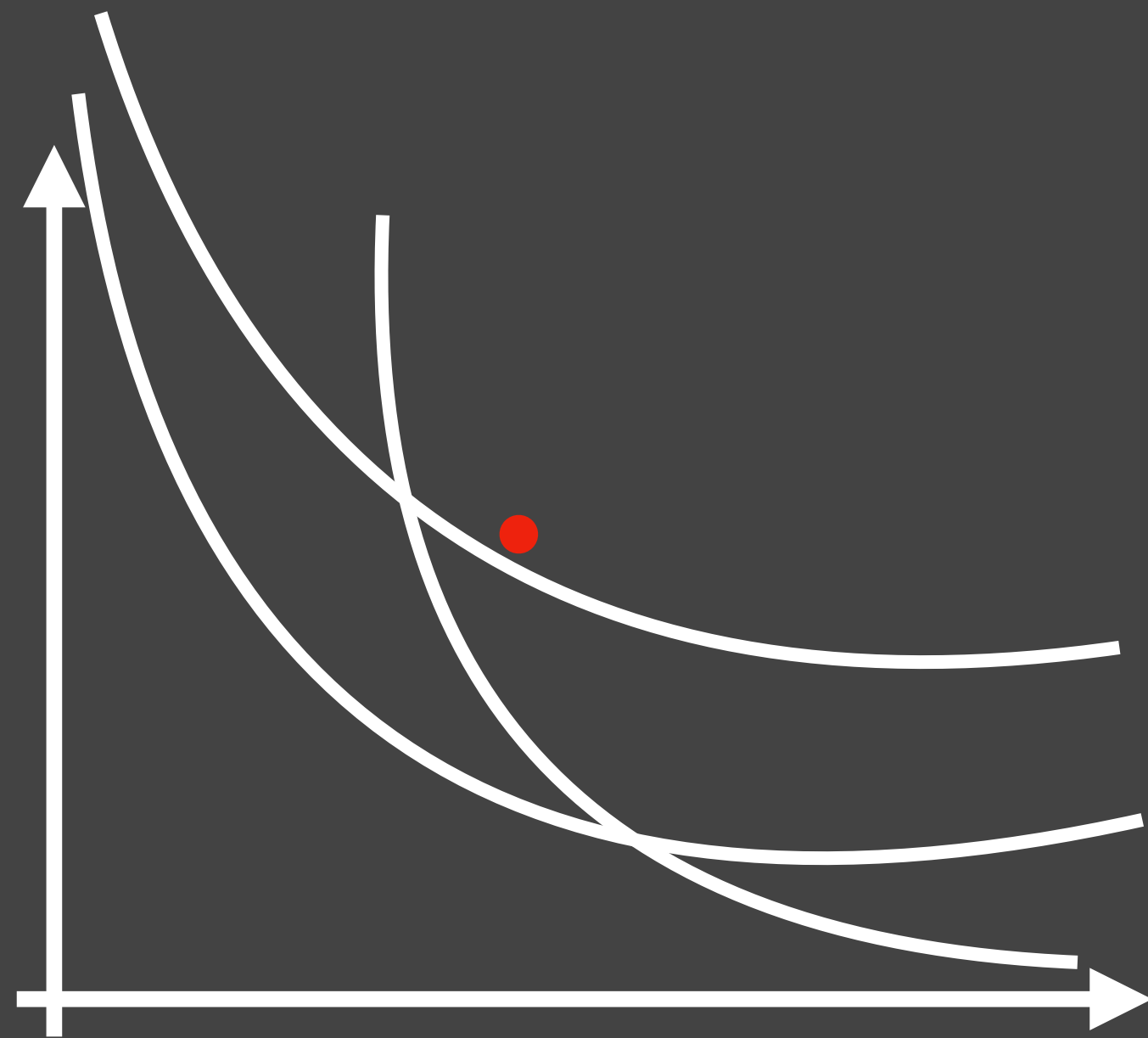
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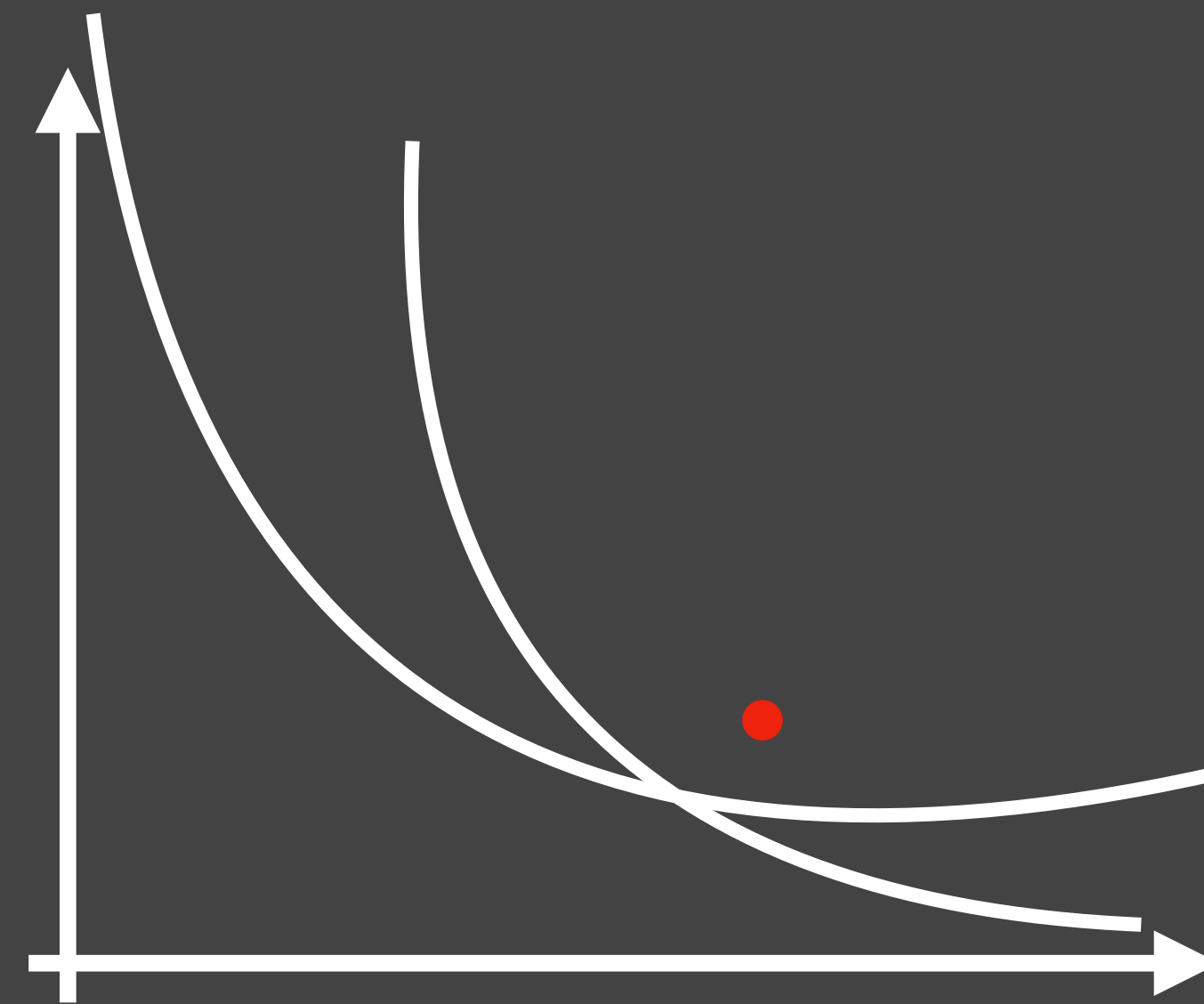
Online

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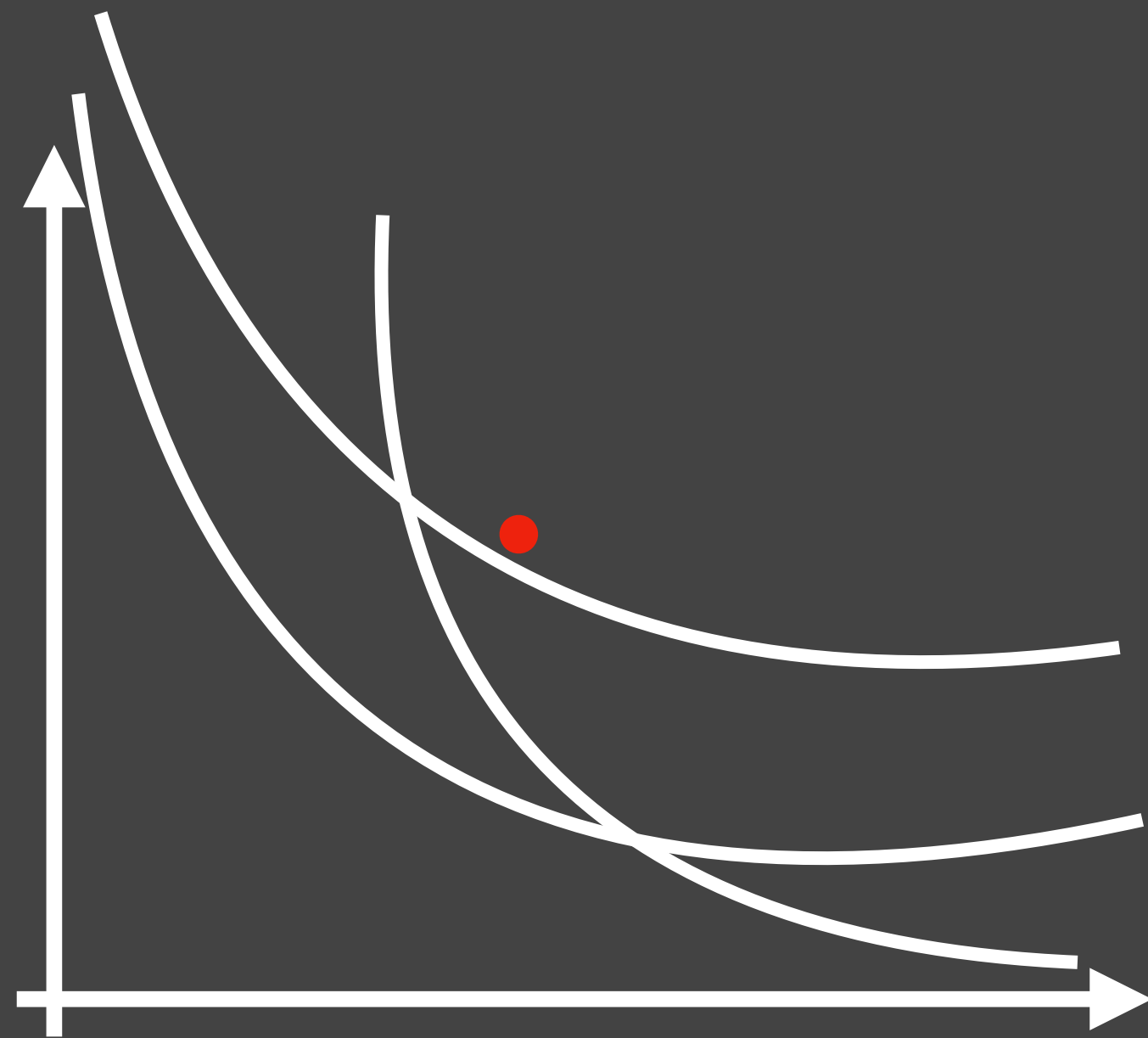
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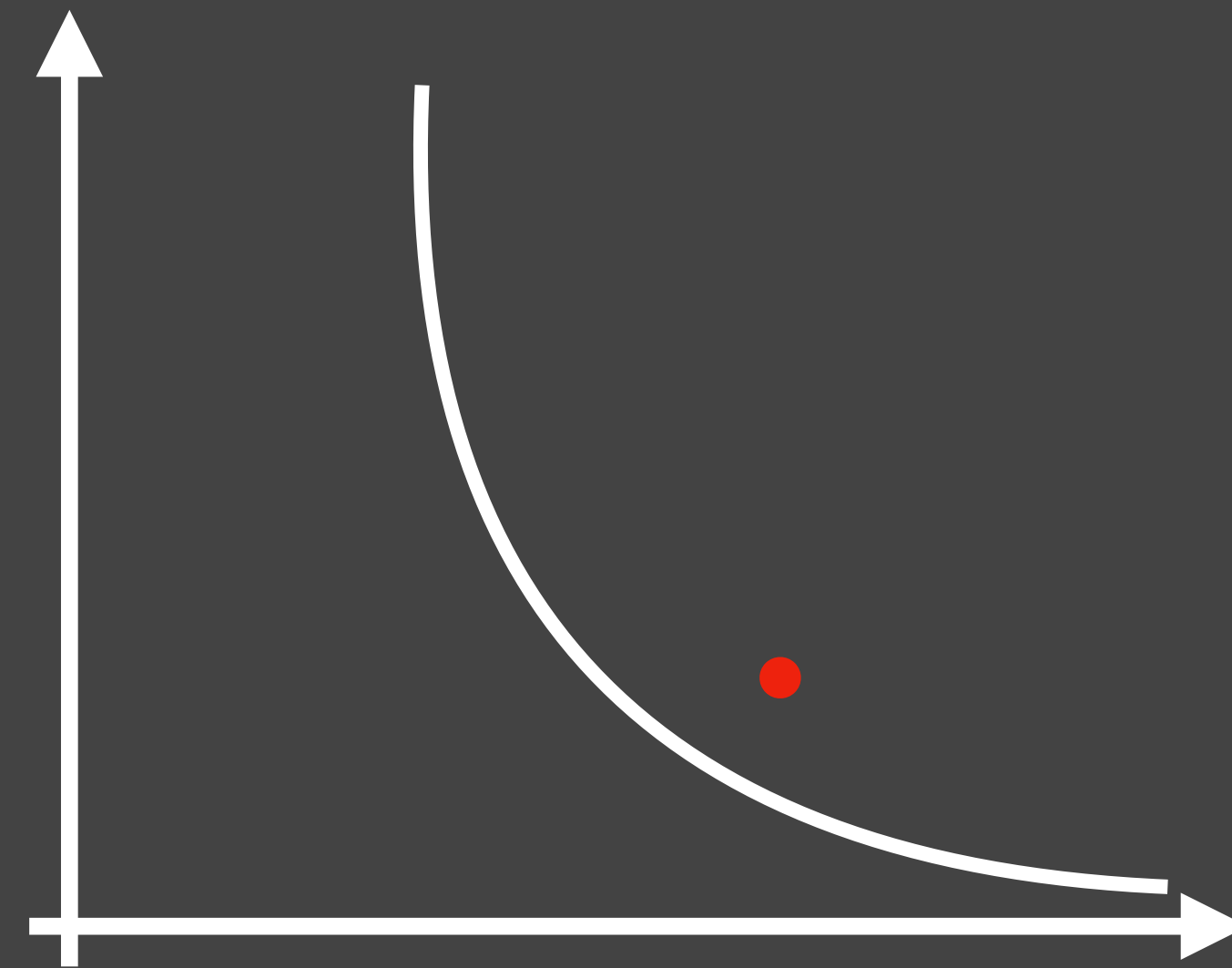
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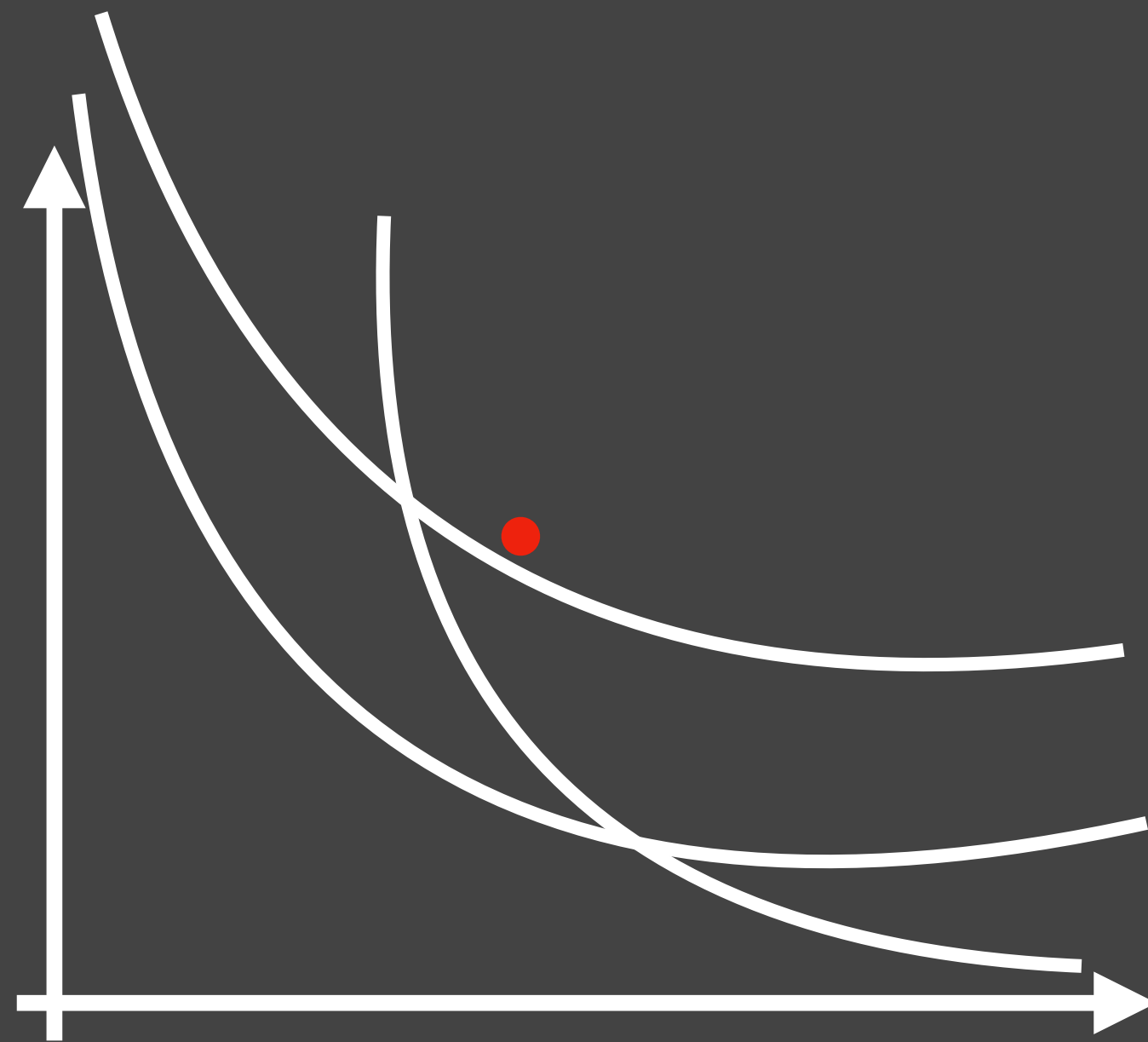
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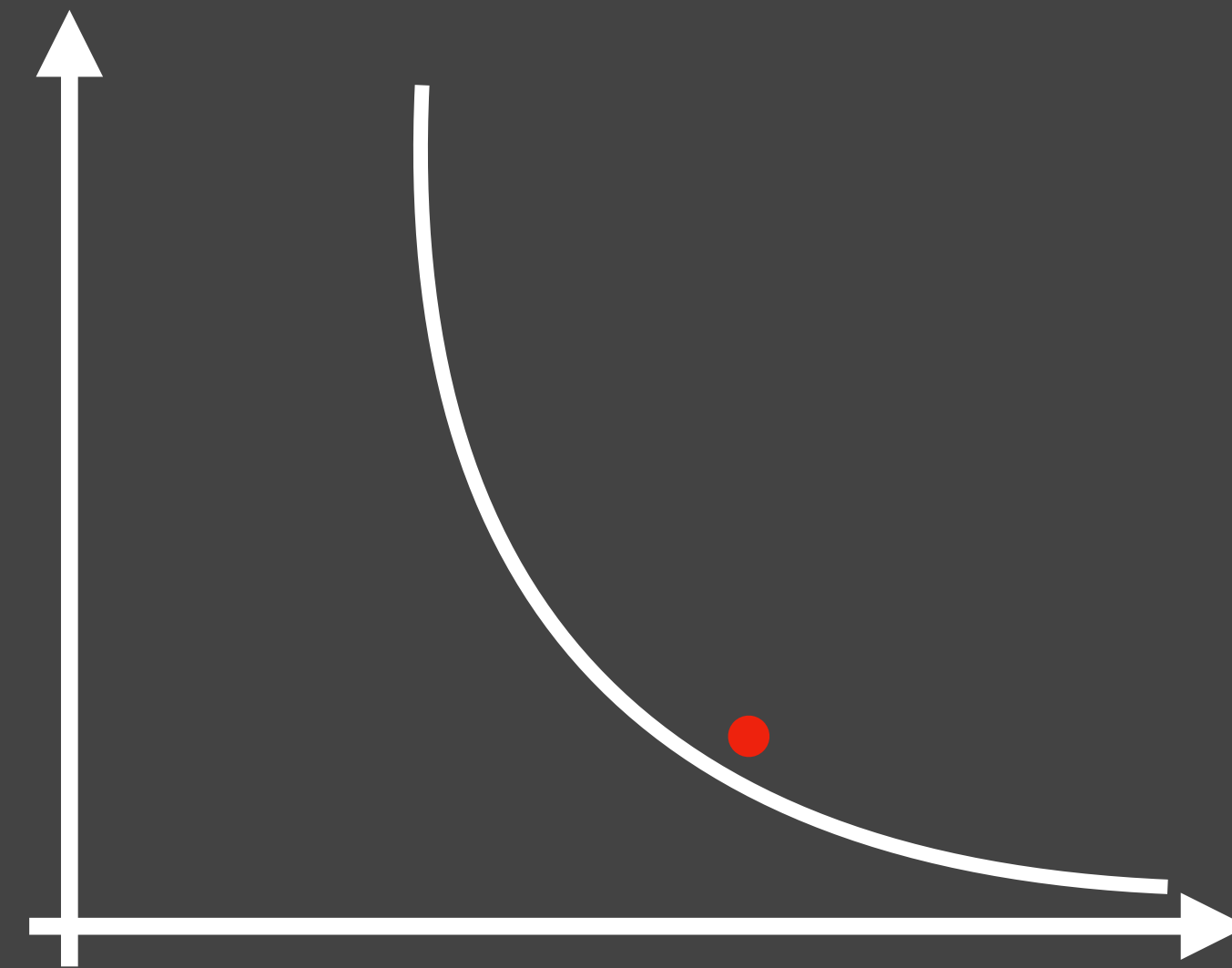
Online

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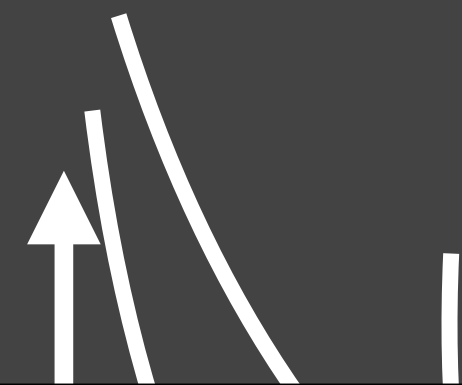
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Comparison

Online

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Theorem (Online) [Gupta L. SODA 20]:

Approximation $O(\log^2 n)$.



Dynamic

- Inserts + Deletes
- Want minimum # edits, a.k.a. recourse.



Theorem (Dynamic) [Gupta L. FOCS 20]:

(i) Approximation $O(\log n)$.

(ii) Recourse $\tilde{O}(1)$.



Dynamic Submodular Cover [Gupta L. FOCS 20]

Dynamic Set Cover

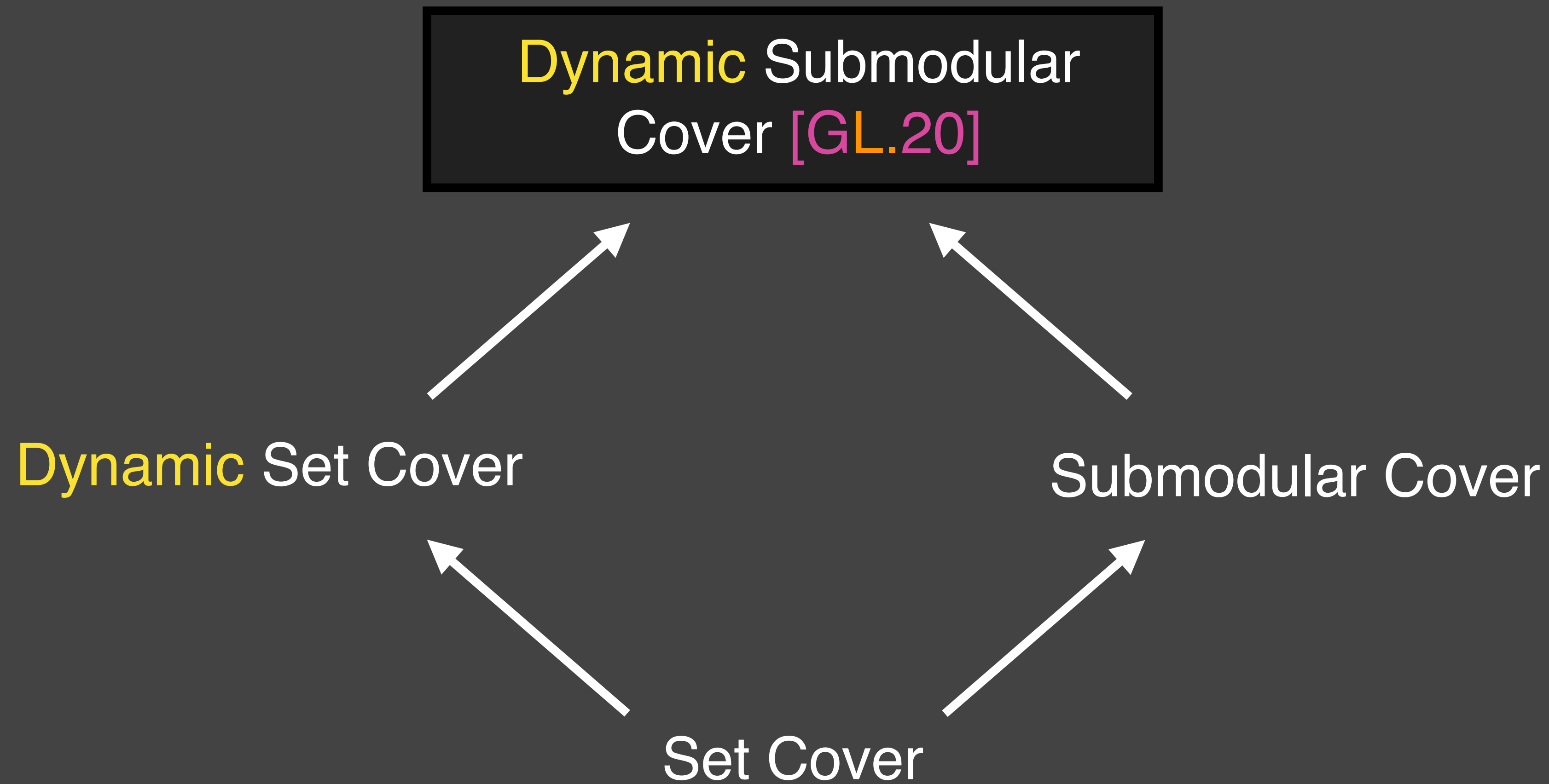
Submodular Cover

Set Cover

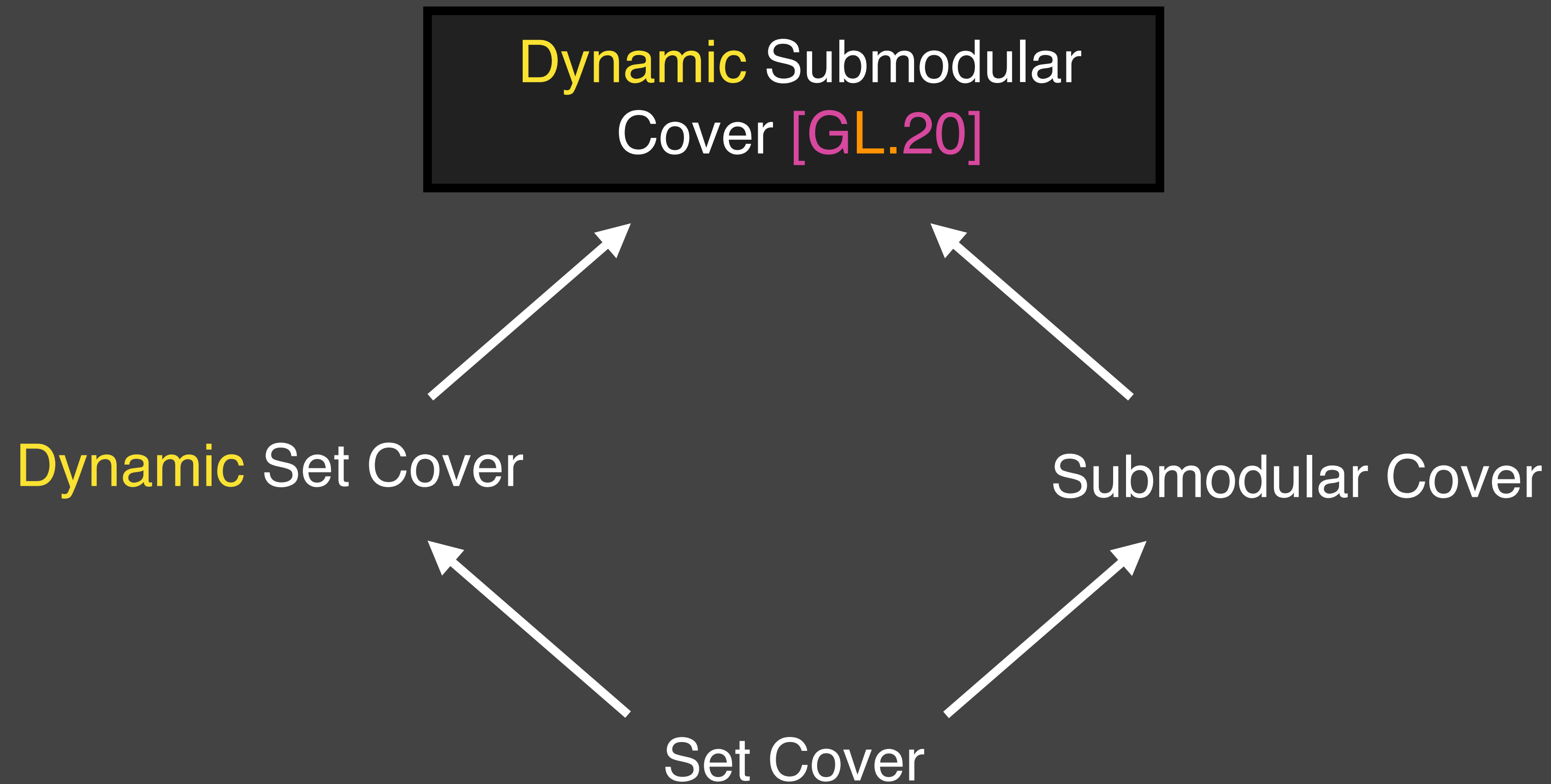
```
graph BT; SC[Set Cover] --> DSC[Dynamic Set Cover]; SC --> SubC[Submodular Cover]
```

The diagram illustrates the relationship between three concepts: Set Cover, Dynamic Set Cover, and Submodular Cover. At the bottom center is the text 'Set Cover'. Two white arrows originate from this text: one points diagonally up and to the left towards the text 'Dynamic Set Cover', and the other points diagonally up and to the right towards the text 'Submodular Cover'. This visualizes 'Set Cover' as a generalization of both 'Dynamic Set Cover' and 'Submodular Cover'.

Dynamic Submodular Cover [Gupta L. FOCS 20]



Dynamic Submodular Cover [Gupta L. FOCS 20]



Modeling power of Submodular Cover + Dynamic.

Is There a **Theory** to Build?

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Most work (mine included!) based on 1-off combinatorial insights.


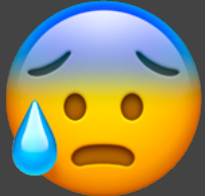
Is There a **Theory** to Build?

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- Difficult to come up with. 


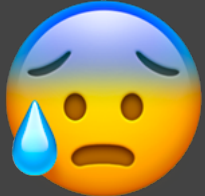
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- Difficult to generalize. 

General recipe for designing stable algorithms?

Is There a **Theory** to Build?

(Yes)

[Bhattacharya,
Buchbinder, **L.**,
Saranurak, In
submission]

Is There a **Theory** to Build?

(Yes)

Theorem [Bhattacharya,
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Dynamic Linear
Programming with
movement $O(\log n) \cdot \mathbf{OPT}$.

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$$A_1 x \geq 1$$

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[Bhattacharya,
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Dynamic Linear Programming with movement $O(\log n) \cdot \text{OPT}$.

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Require *Mixed Packing/Covering* LPs, i.e. constraints have *positive* coefficients.

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Rounding gives **improved results** for **Dynamic** Set Cover, Load Balancing, Matching, Minimum Spanning Tree.

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[Bhattacharya, Buchbinder, L., Saranurak, In submission]

Technical Ingredient:
Max Entropy Principle.

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Is There a **Theory** to Build?

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Theorem [Bhattacharya, Buchbinder, L., Saranurak, In submission]:

Dynamic Linear Programming with movement $O(\log n) \cdot \text{OPT}$.

Optimal!

Rounding gives improved results for **Dynamic** Set Cover, Load Balancing, Matching, Minimum Spanning Tree.

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Technical Ingredient:
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Take Away II

[Gupta L. FOCS 20]

[Bhattacharya, Buchbinder, L.,
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Q: Can we understand
recourse/approximation
tradeoffs?

Take Away II

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Q: Can we understand
recourse/approximation
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A1: Get optimal tradeoff for
Submodular Cover class.

Take Away II

[Gupta L. FOCS 20]

[Bhattacharya, Buchbinder, L.,
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Q: Can we understand
recourse/approximation
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A1: Get optimal tradeoff for
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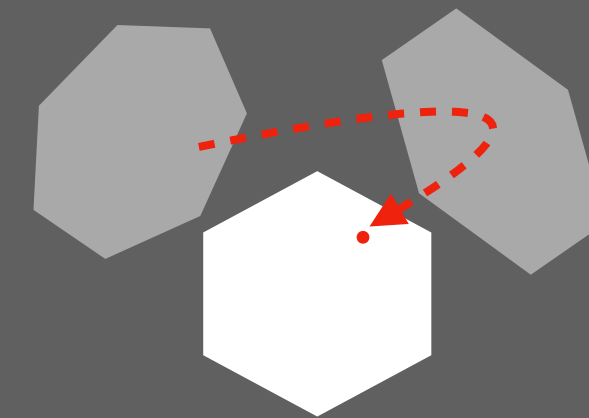
A2: Get stable Dynamic
analog of fundamental
algorithmic primitive,
Linear Programming.

Outline

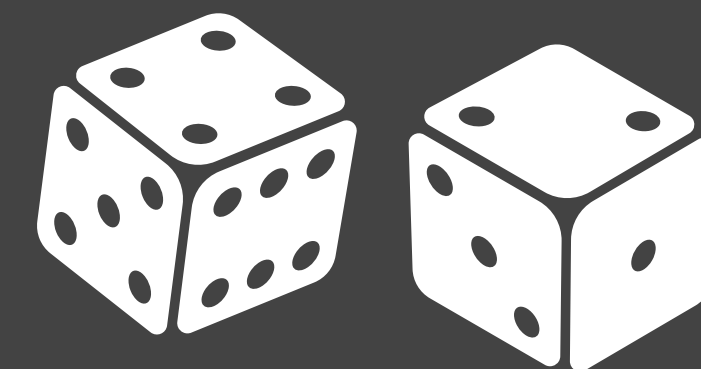
Theme I — Submodular Optimization

$$f(\text{🍕} \mid \text{🥕}) \geq f(\text{🍕} \mid \text{🥕}, \text{🍩})$$

Theme II — Stable Algorithms



Theme III — Beyond Worst-Case Analysis



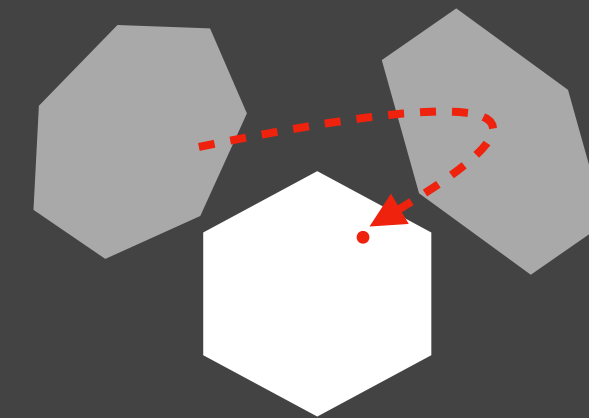
Conclusion

Outline

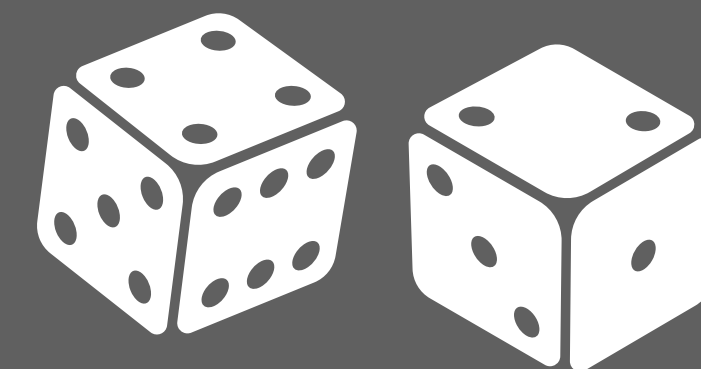
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Theme II — Stable Algorithms



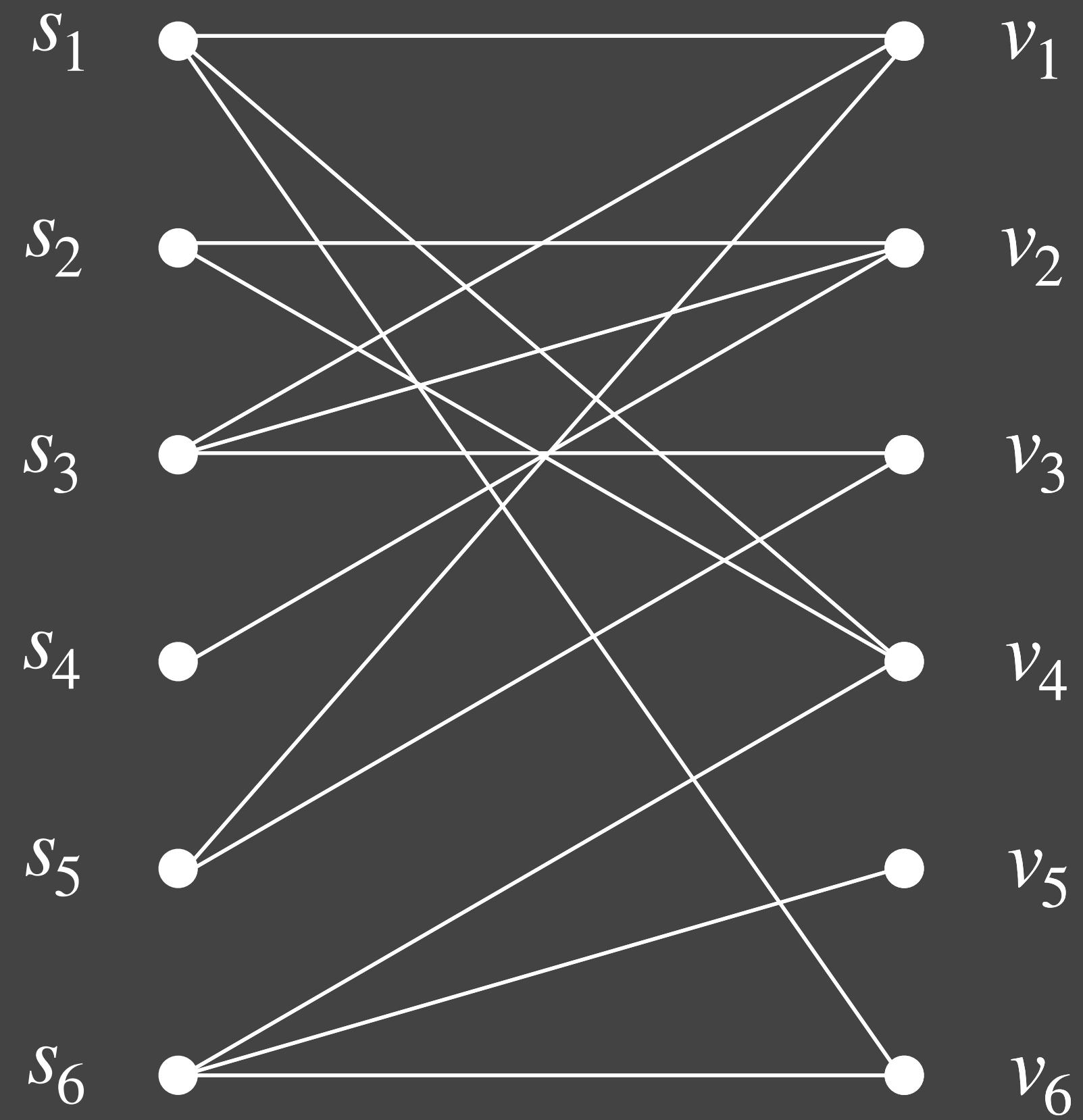
Theme III — Beyond Worst-Case Analysis



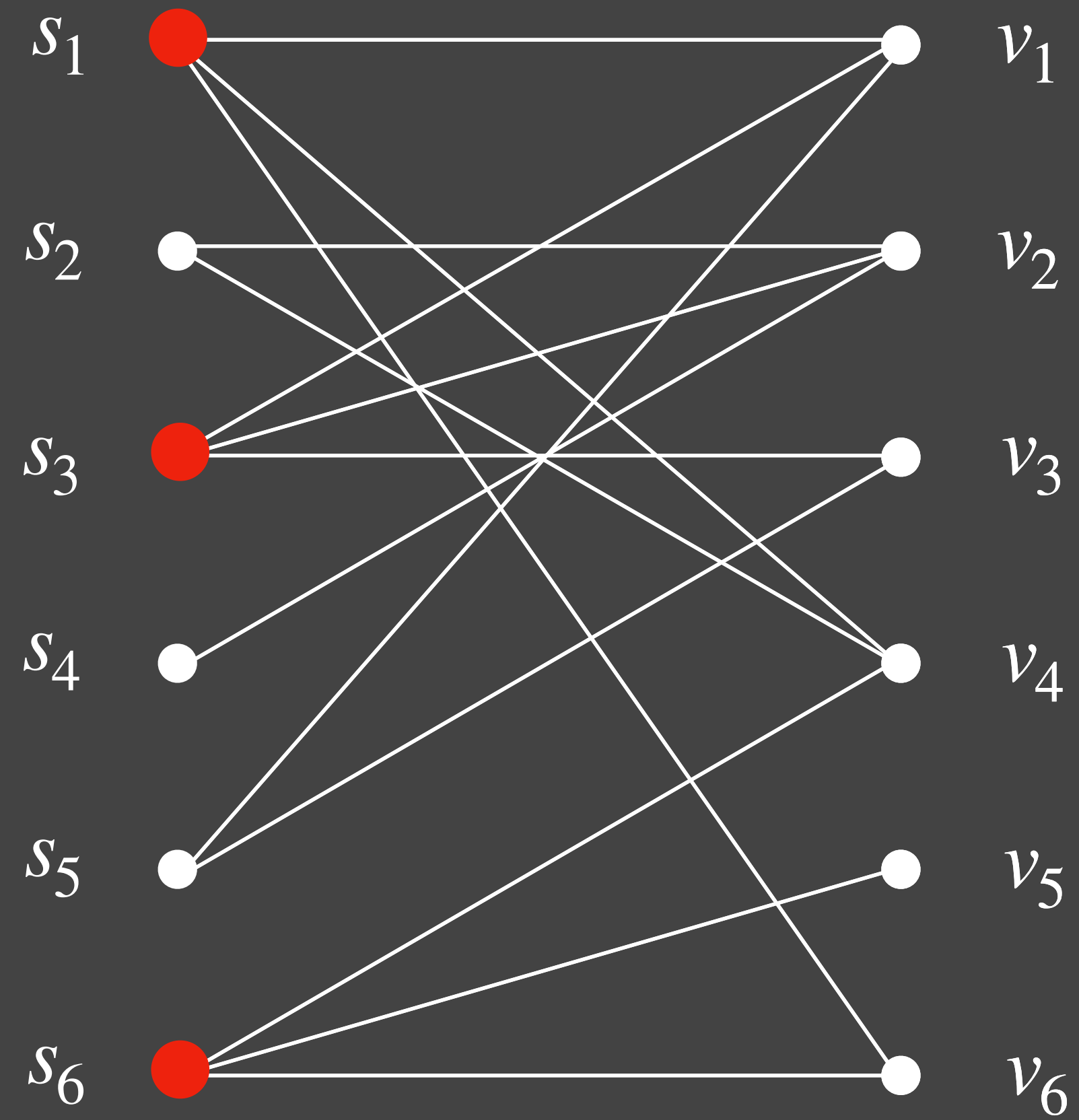
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Theme III — Beyond Worst-Case Analysis

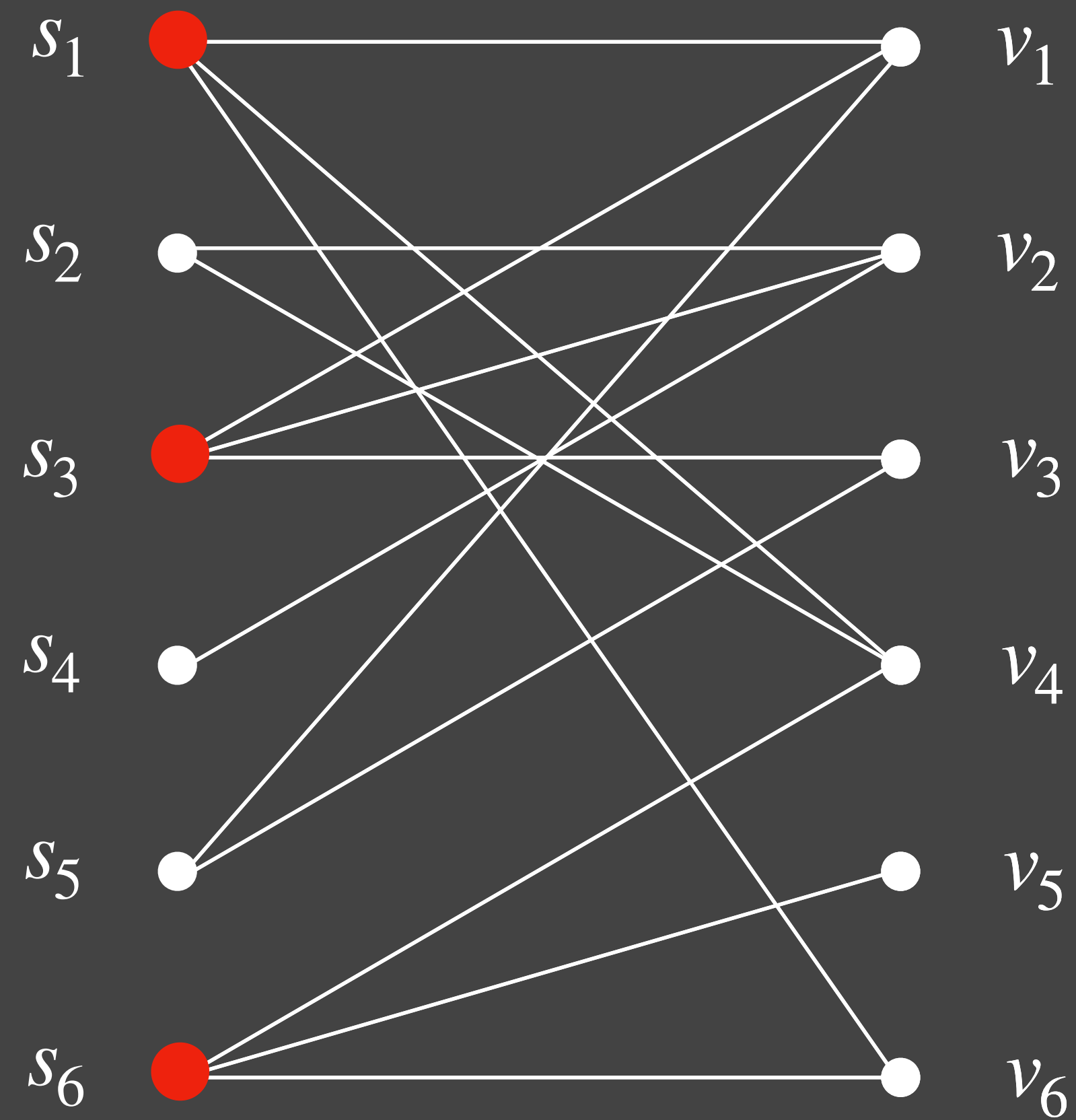
Set Cover



Set Cover

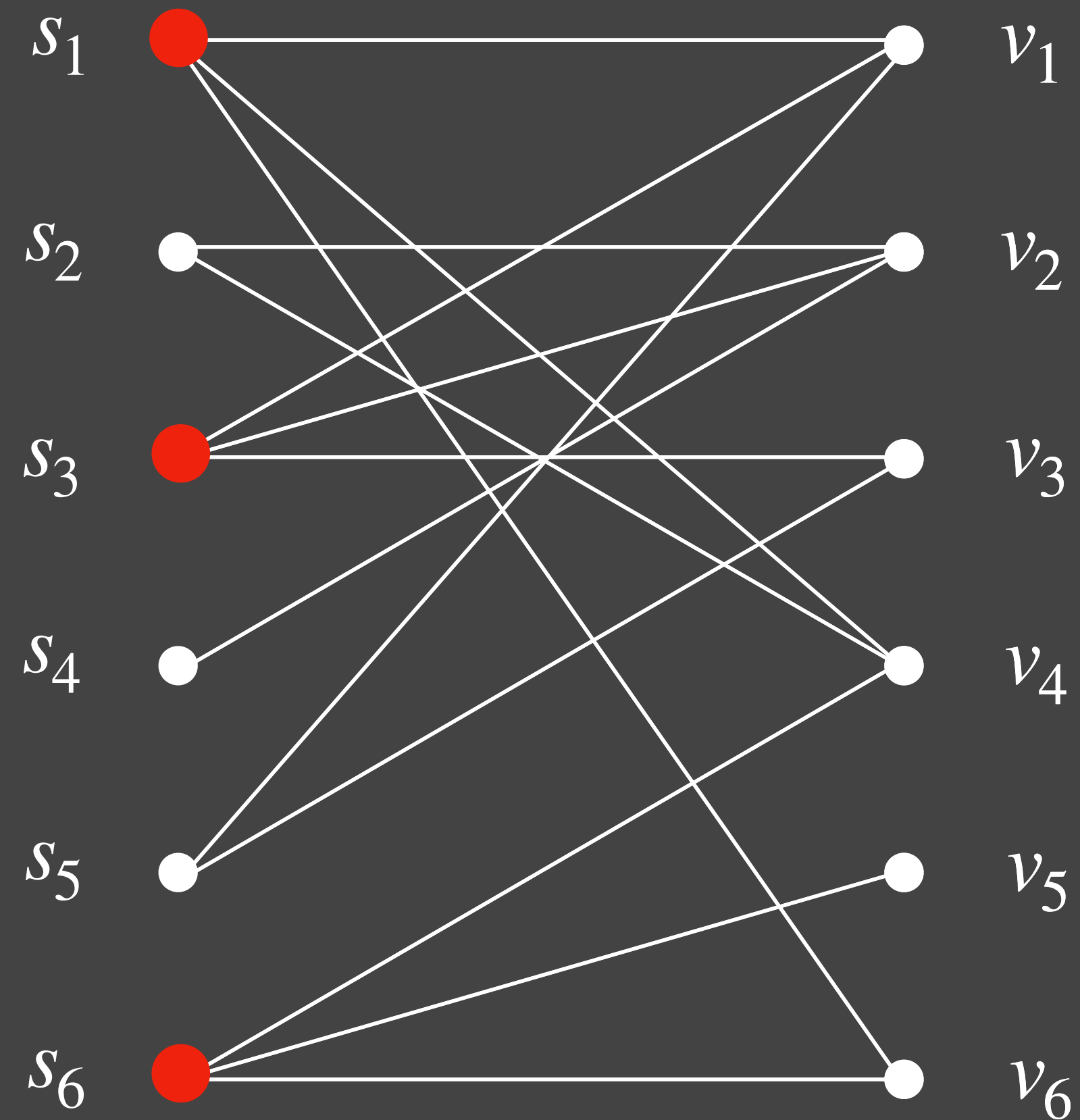


Set Cover



Approximation:
 $O(\log n)$
[Johnson 74],
[Lovasz 75],
[Chvatal 79]

Set Cover



Approximation:
 $O(\log n)$
[Johnson 74],
[Lovasz 75],
[Chvatal 79]

Optimal!
(in poly time)

Online Set Cover [Alon Awerbuch Azar Buchbinder Naor 03]

s_1 ●

s_2 ●

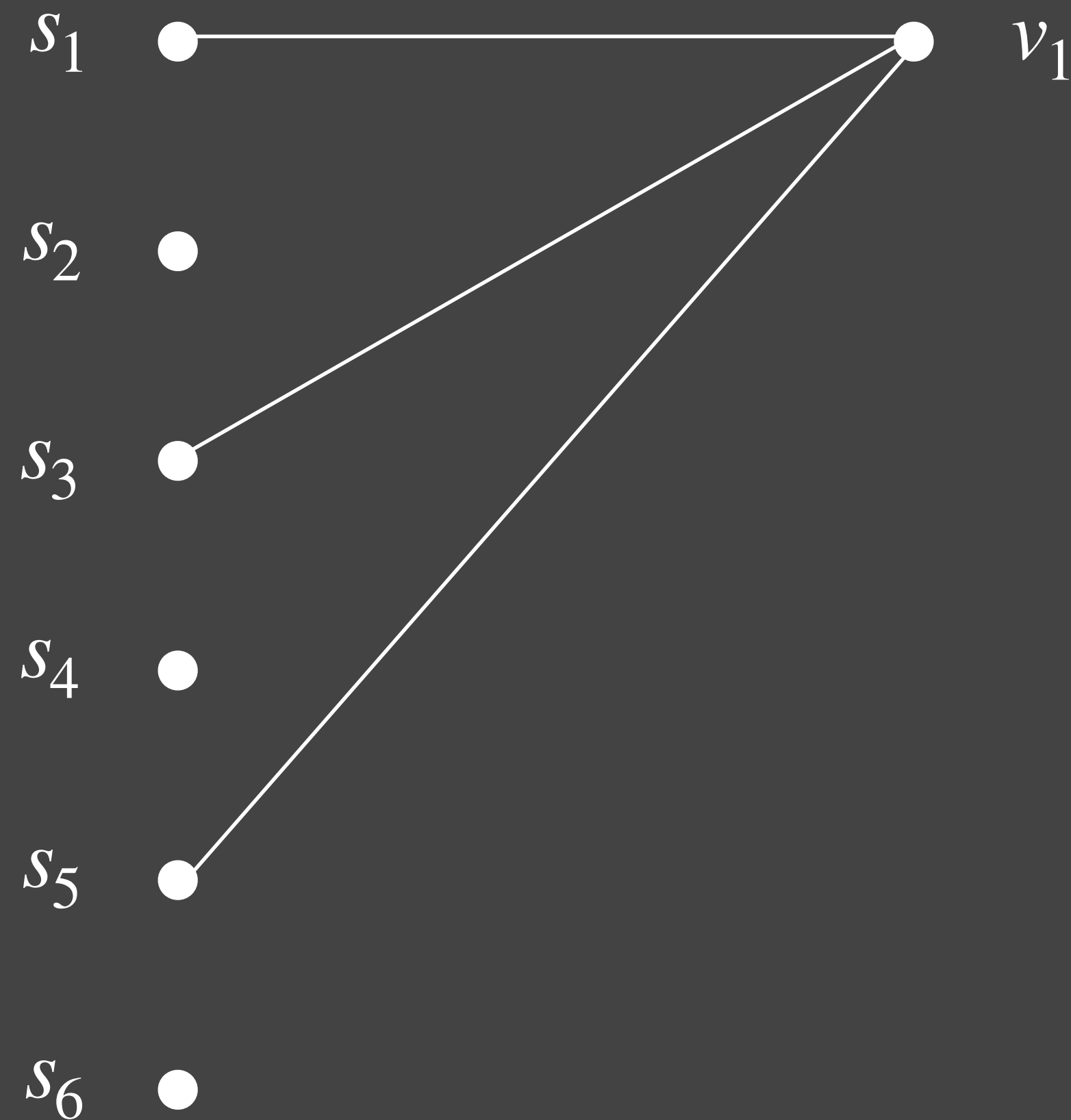
s_3 ●

s_4 ●

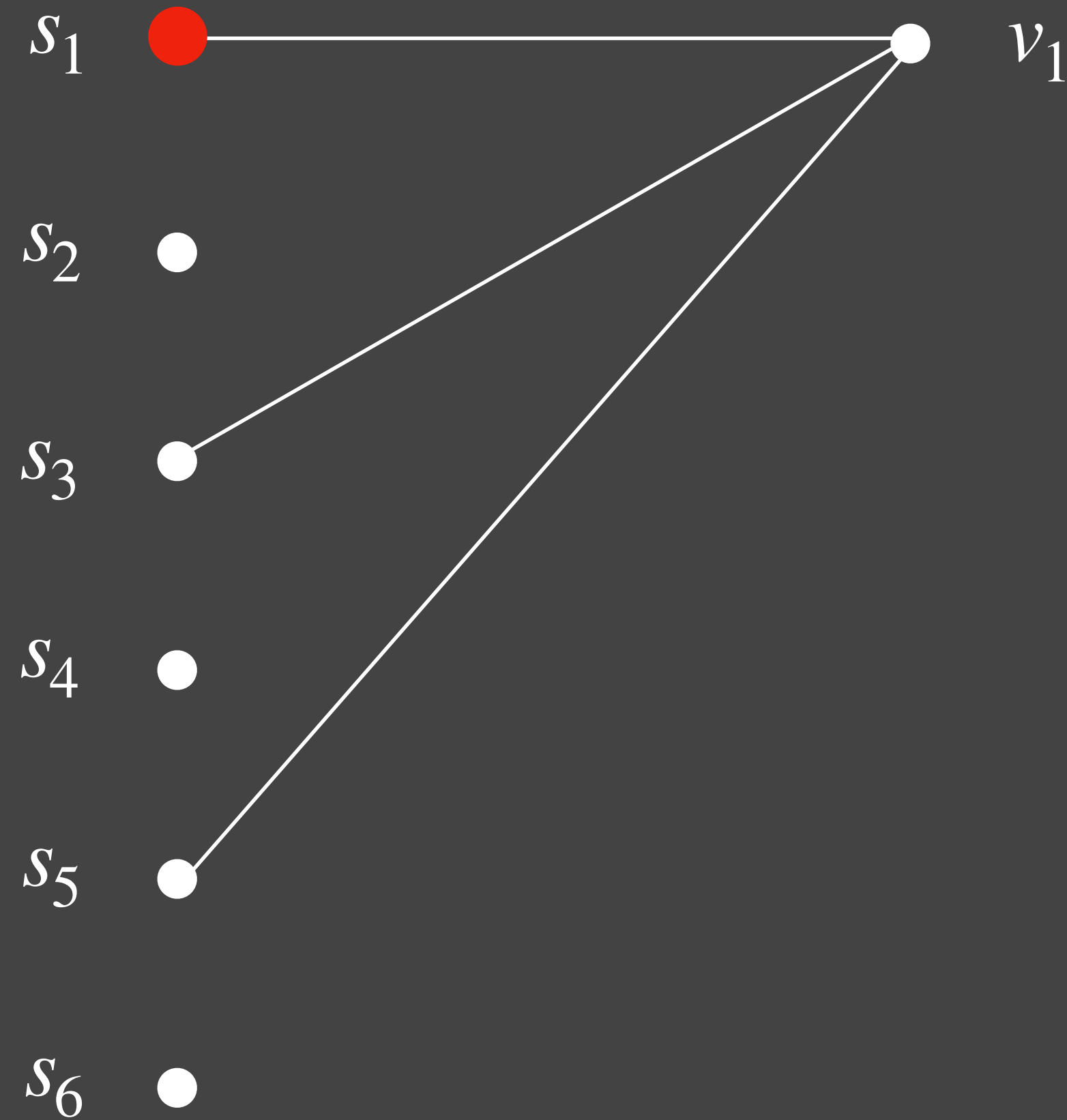
s_5 ●

s_6 ●

Online Set Cover [Alon Awerbuch Azar Buchbinder Naor 03]

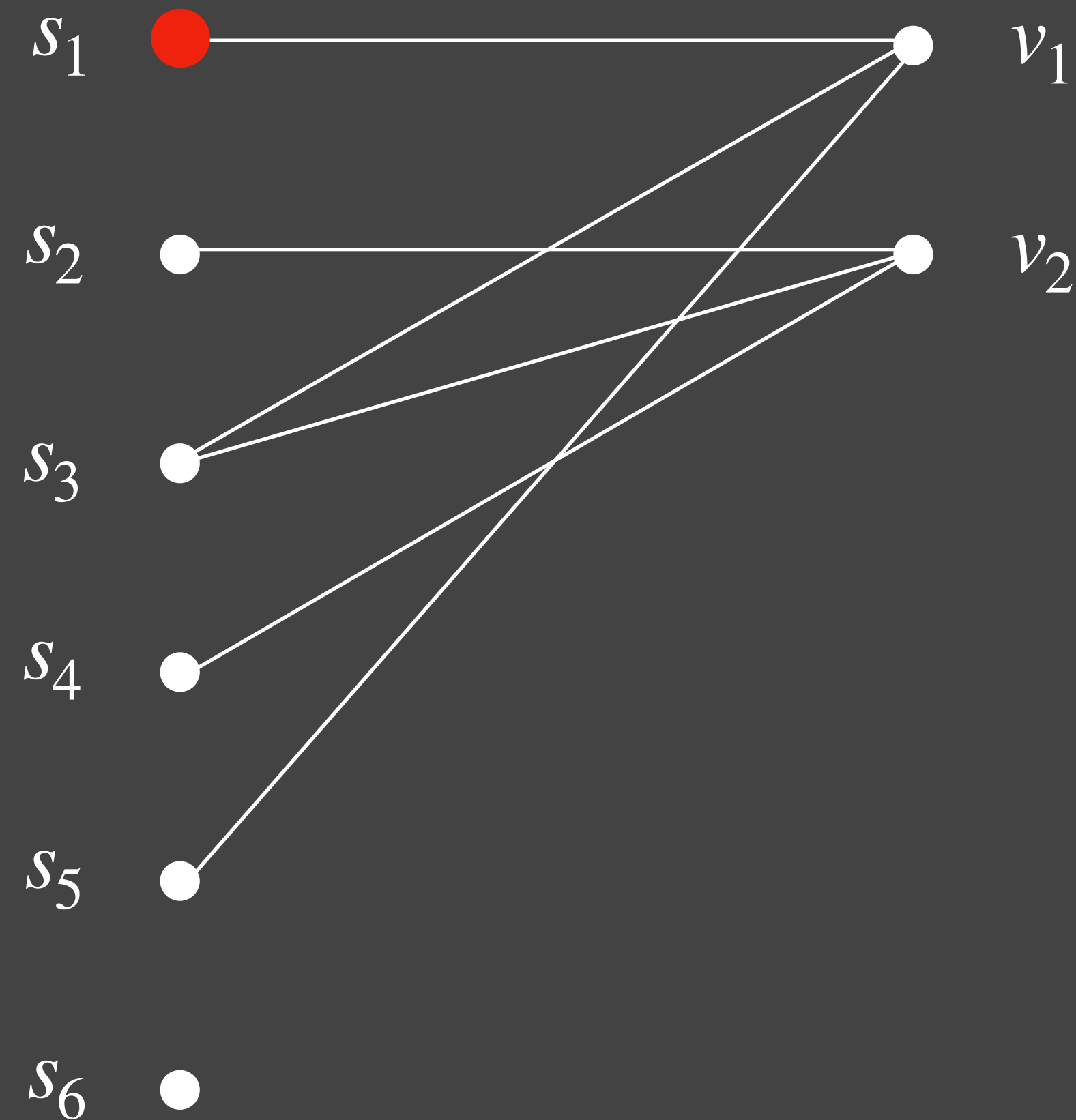


Online Set Cover [Alon Awerbuch Azar Buchbinder Naor 03]



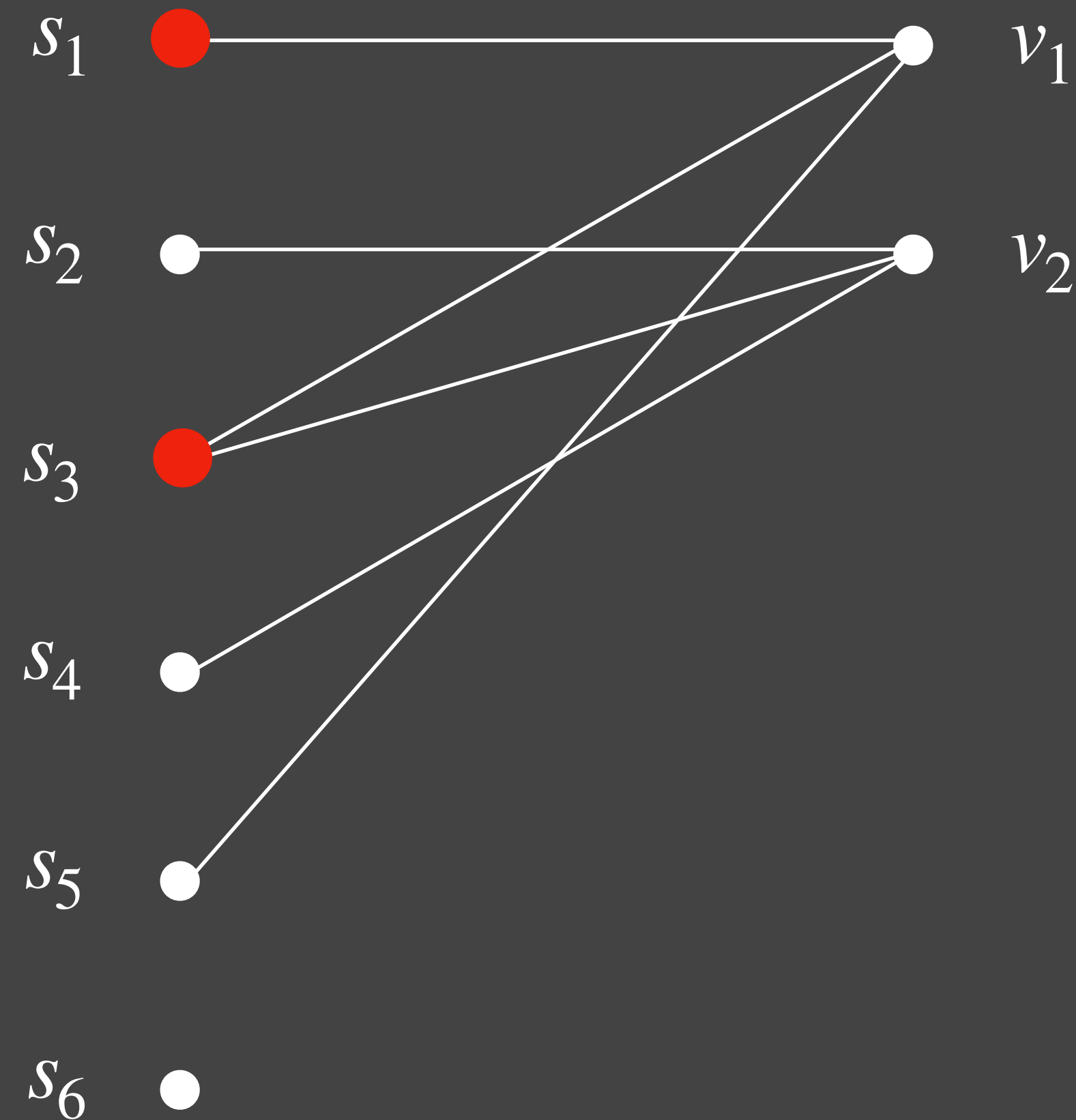
Online Set Cover

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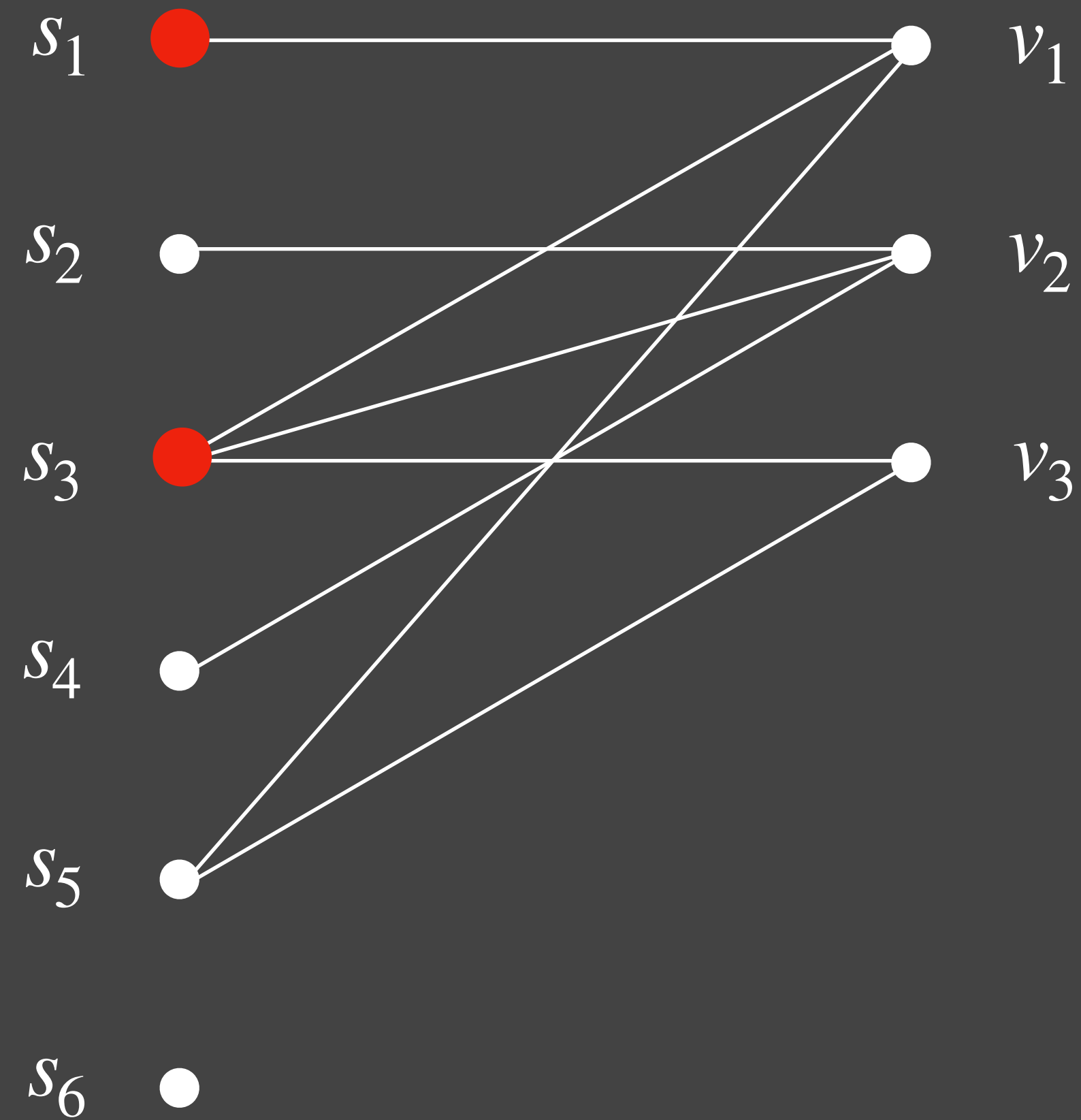


Online Set Cover

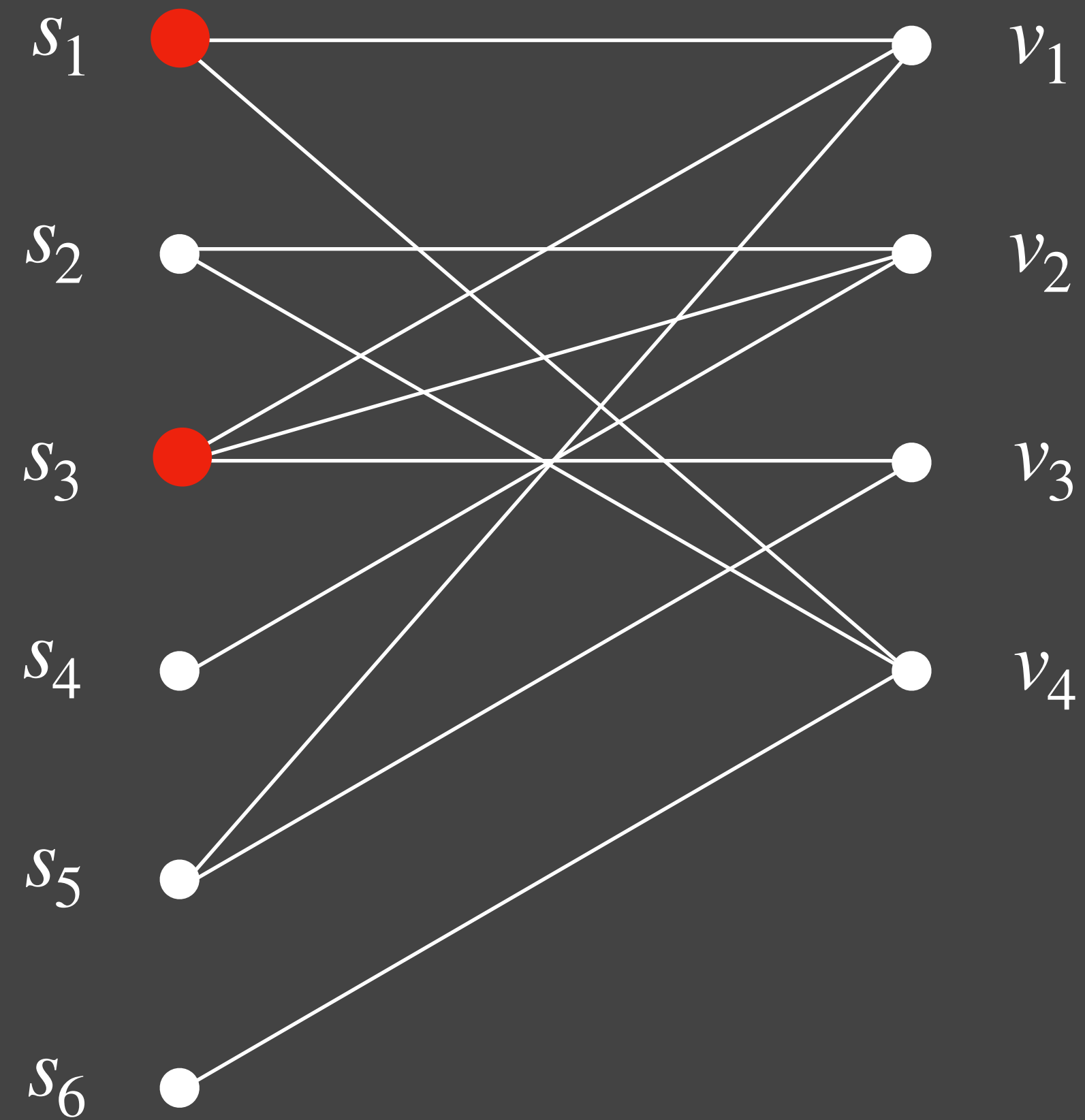
[Alon Awerbuch Azar Buchbinder Naor 03]



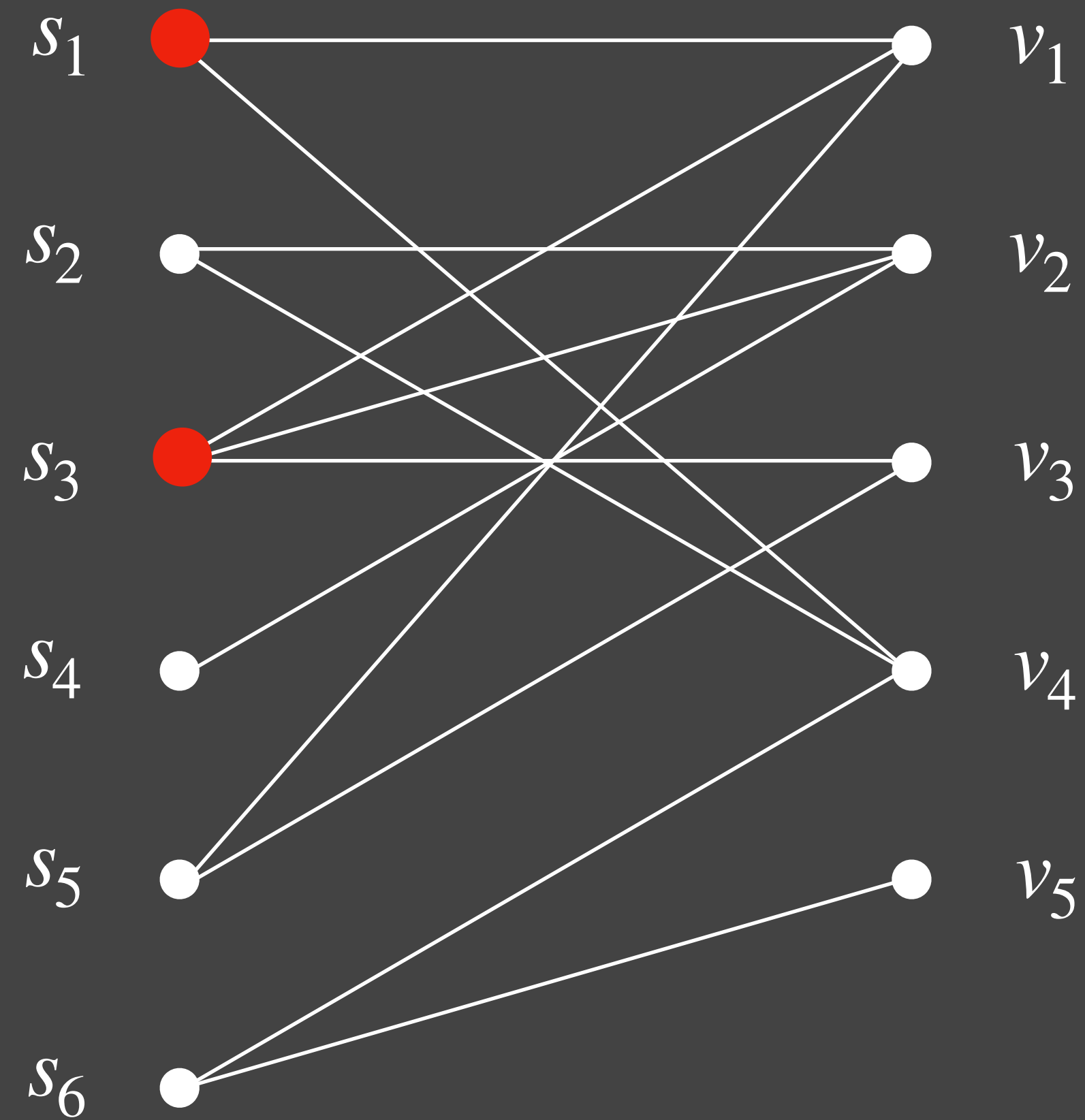
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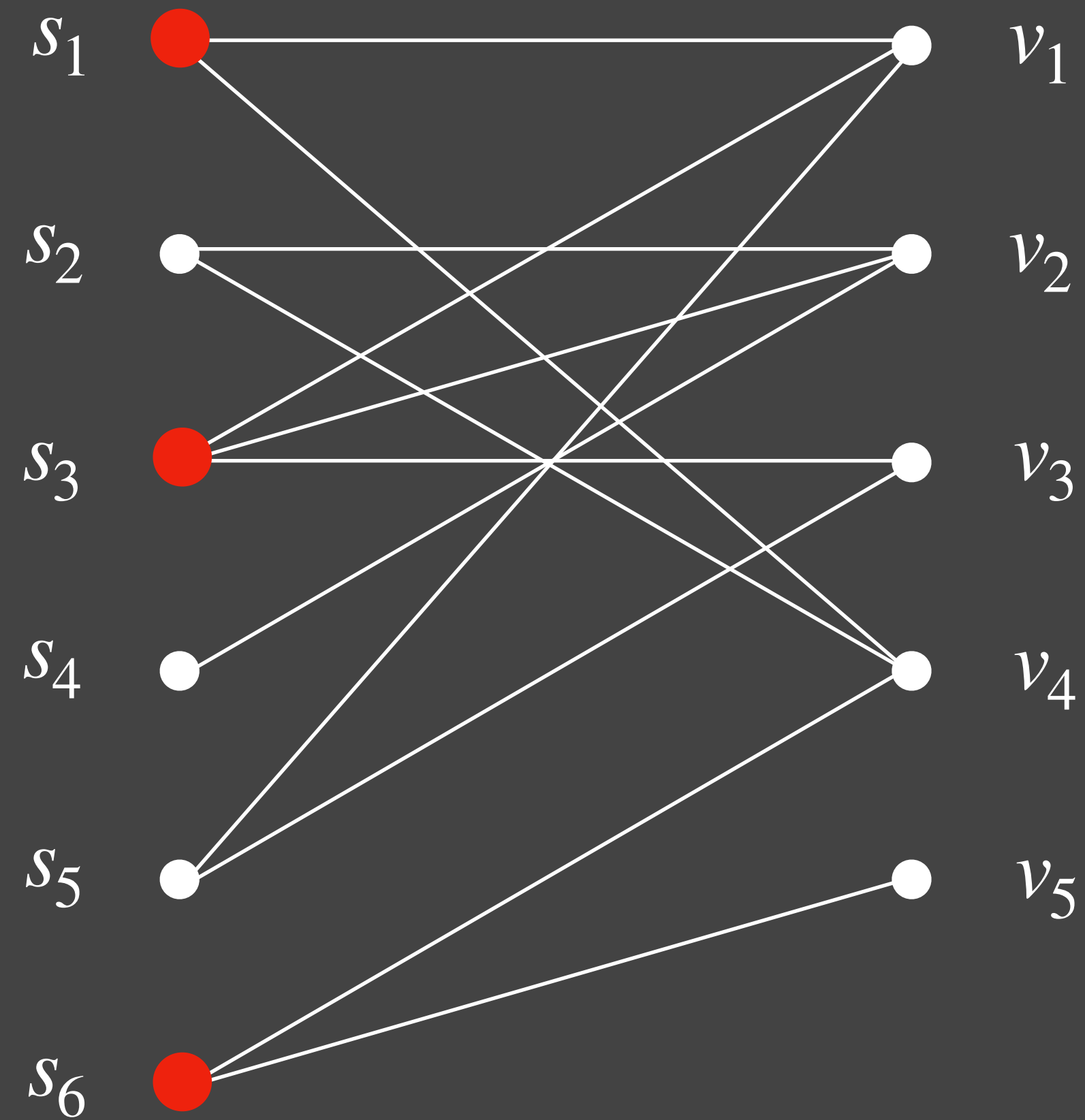
Online Set Cover [Alon Awerbuch Azar Buchbinder Naor 03]



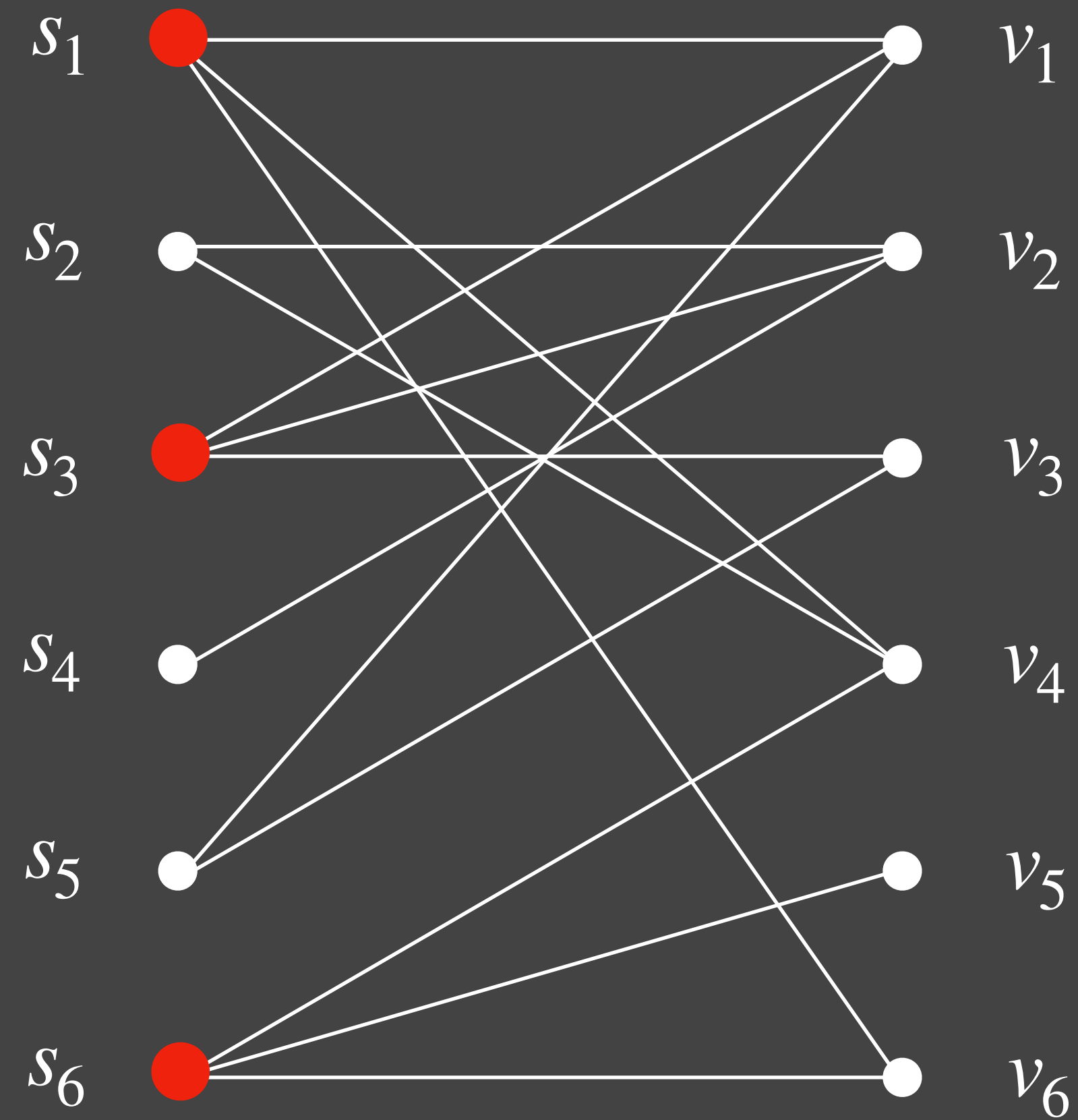
Online Set Cover [Alon Awerbuch Azar Buchbinder Naor 03]



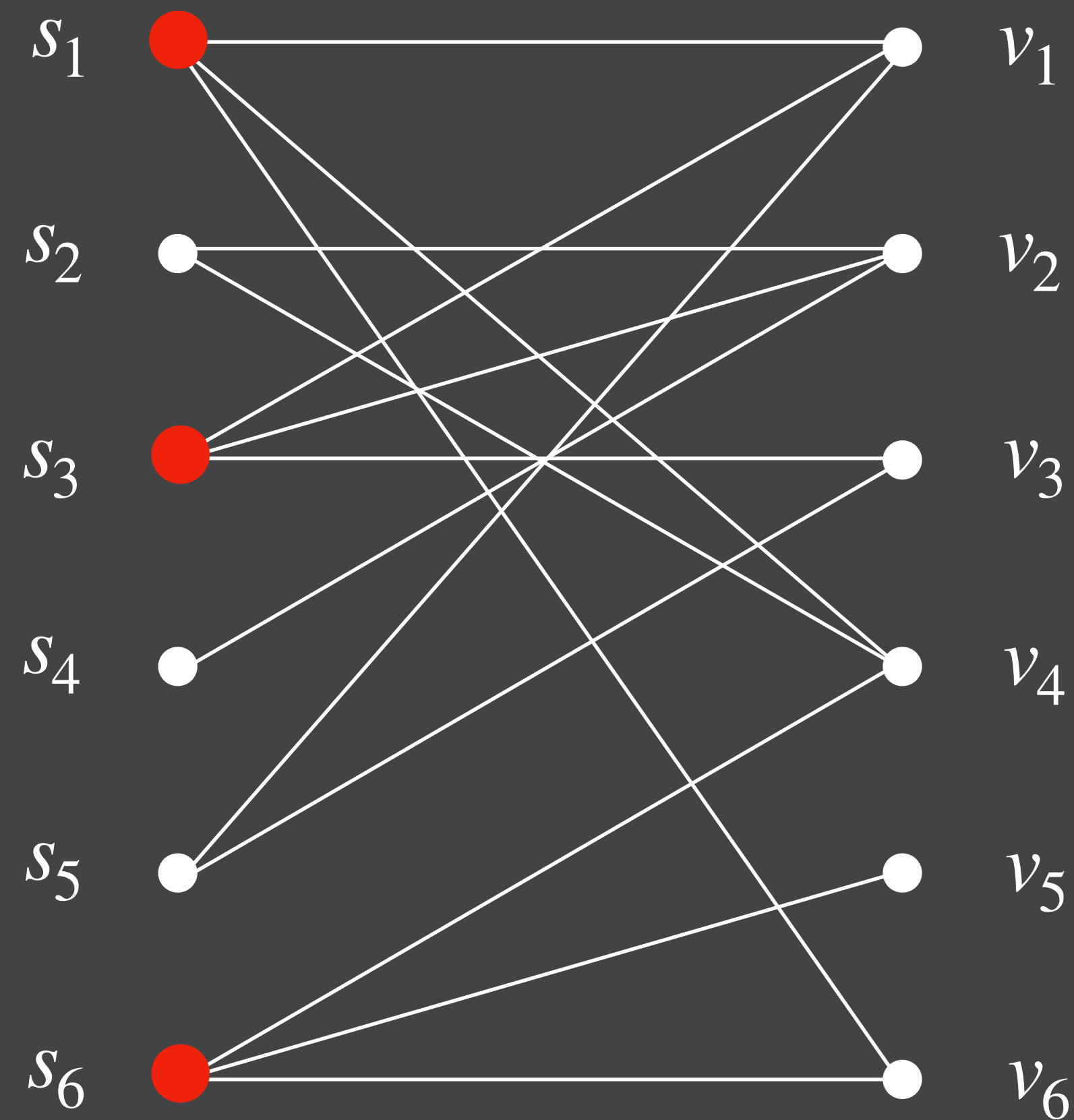
Online Set Cover [Alon Awerbuch Azar Buchbinder Naor 03]



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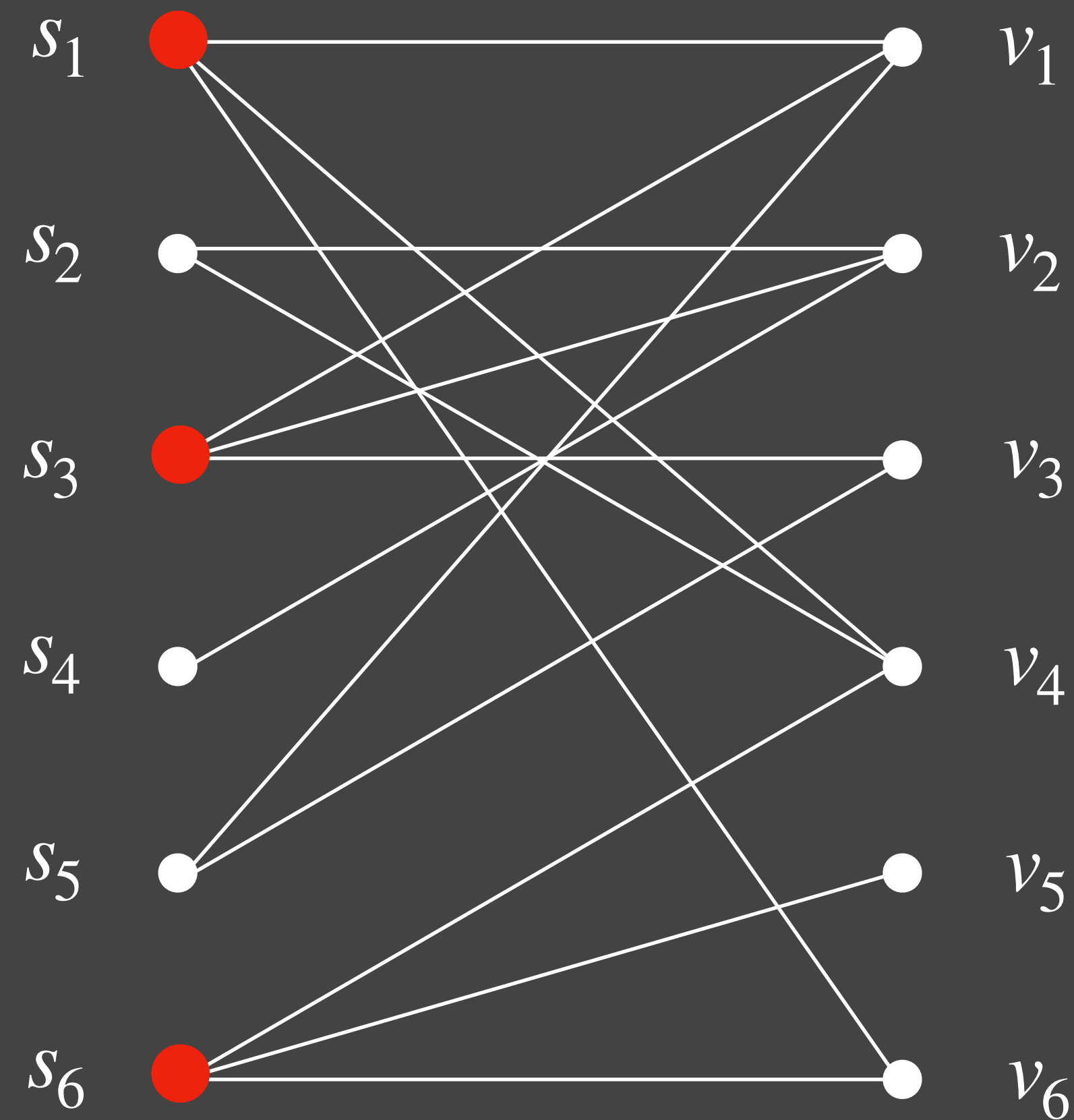
Approximation:

$$O(\log^2 n)$$

[Alon+ 03]

[Buchbinder
Naor 09]

Online Set Cover [Alon Awerbuch Azar Buchbinder Naor 03]



Approximation:

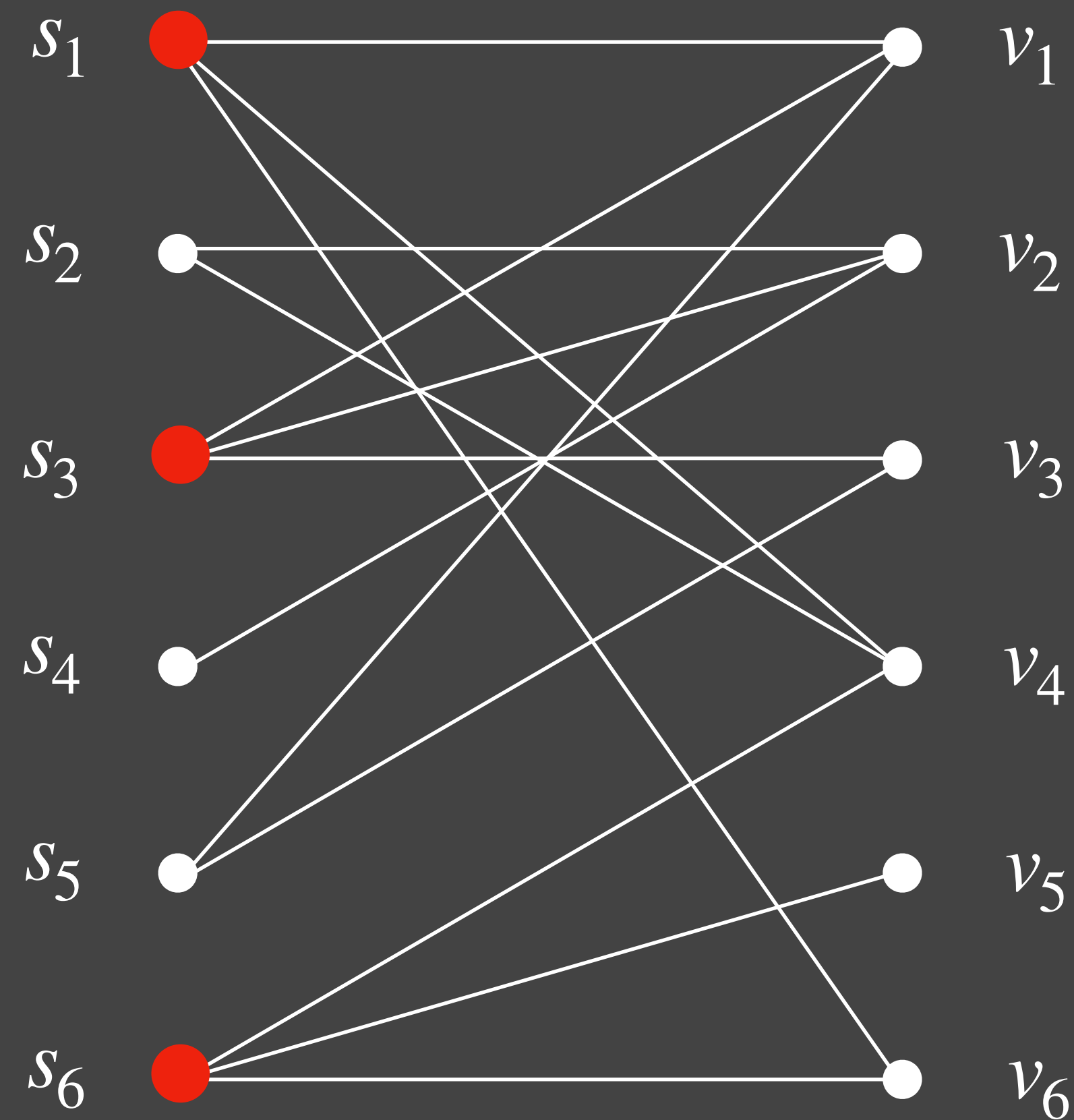
$O(\log^2 n)$

[Alon+ 03]

[Buchbinder
Naor 09]

Optimal!
(in poly time)

Online Set Cover [Alon Awerbuch Azar Buchbinder Naor 03]



Approximation:

$O(\log^2 n)$

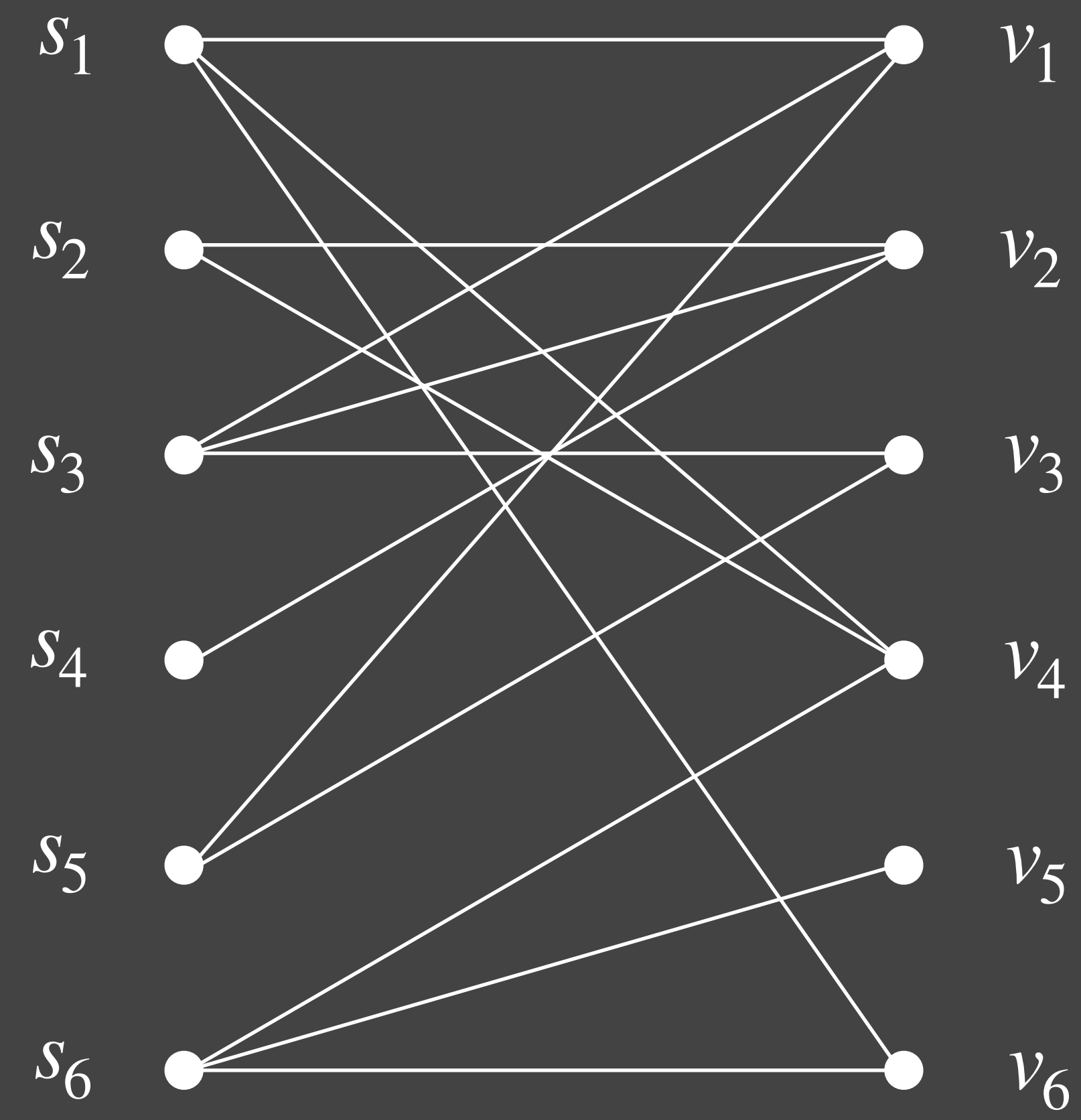
[Alon+ 03]

[Buchbinder
Naor 09]

Optimal!
(in poly time)

Q: What happens beyond the worst case?

Relaxation 1: Random Order (RO)



Relaxation 1: Random Order (RO)

s_1 ●

s_2 ●

s_3 ●

s_4 ●

s_5 ●

s_6 ●

Relaxation 1: Random Order (RO)

s_1 ●

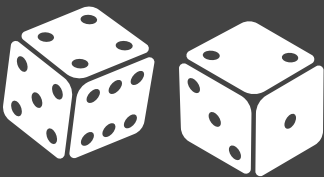
s_2 ●

s_3 ●

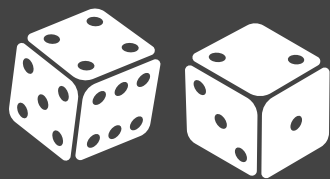
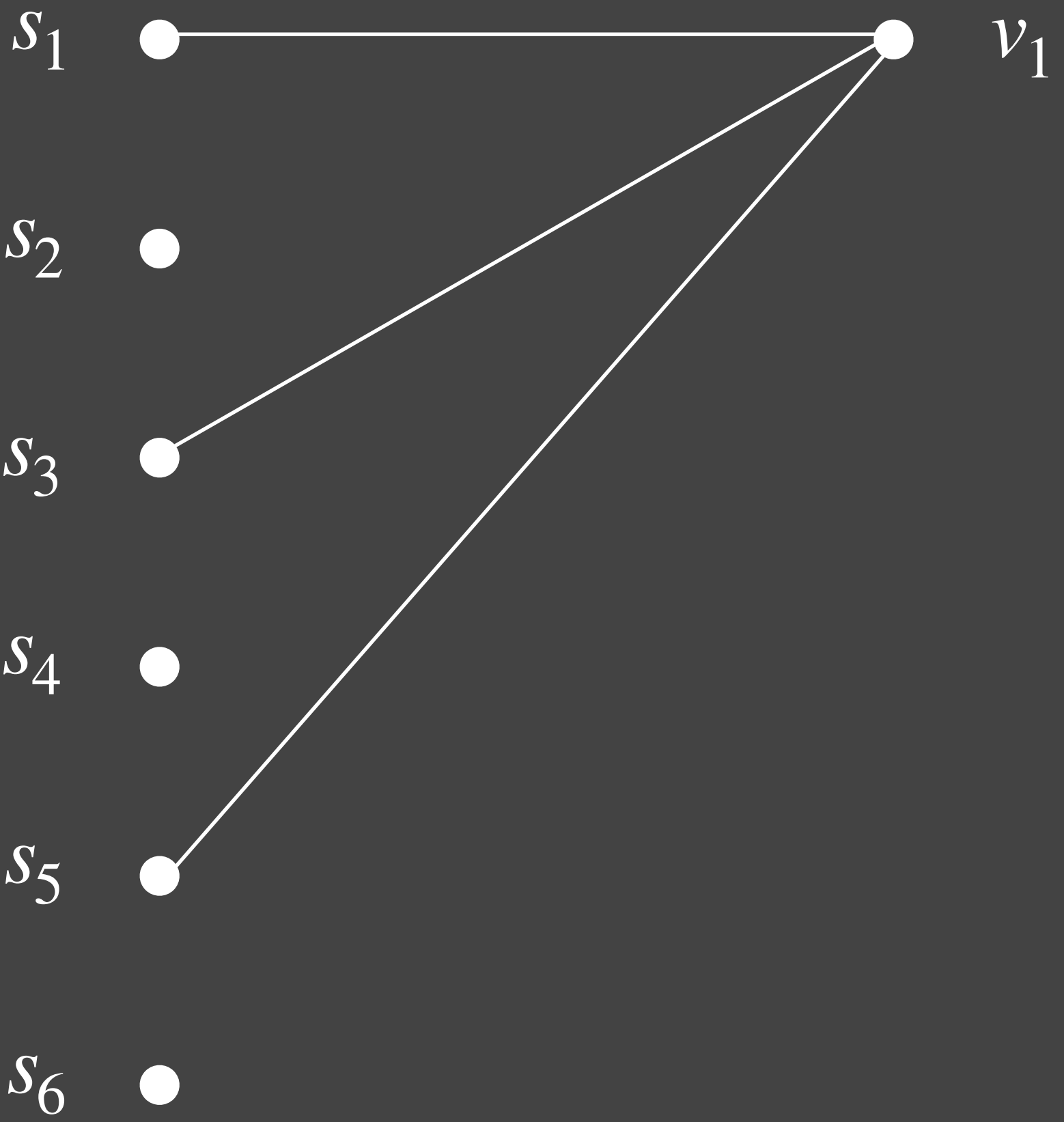
s_4 ●

s_5 ●

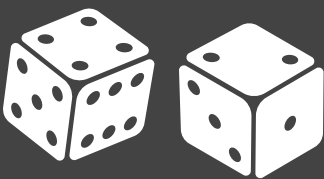
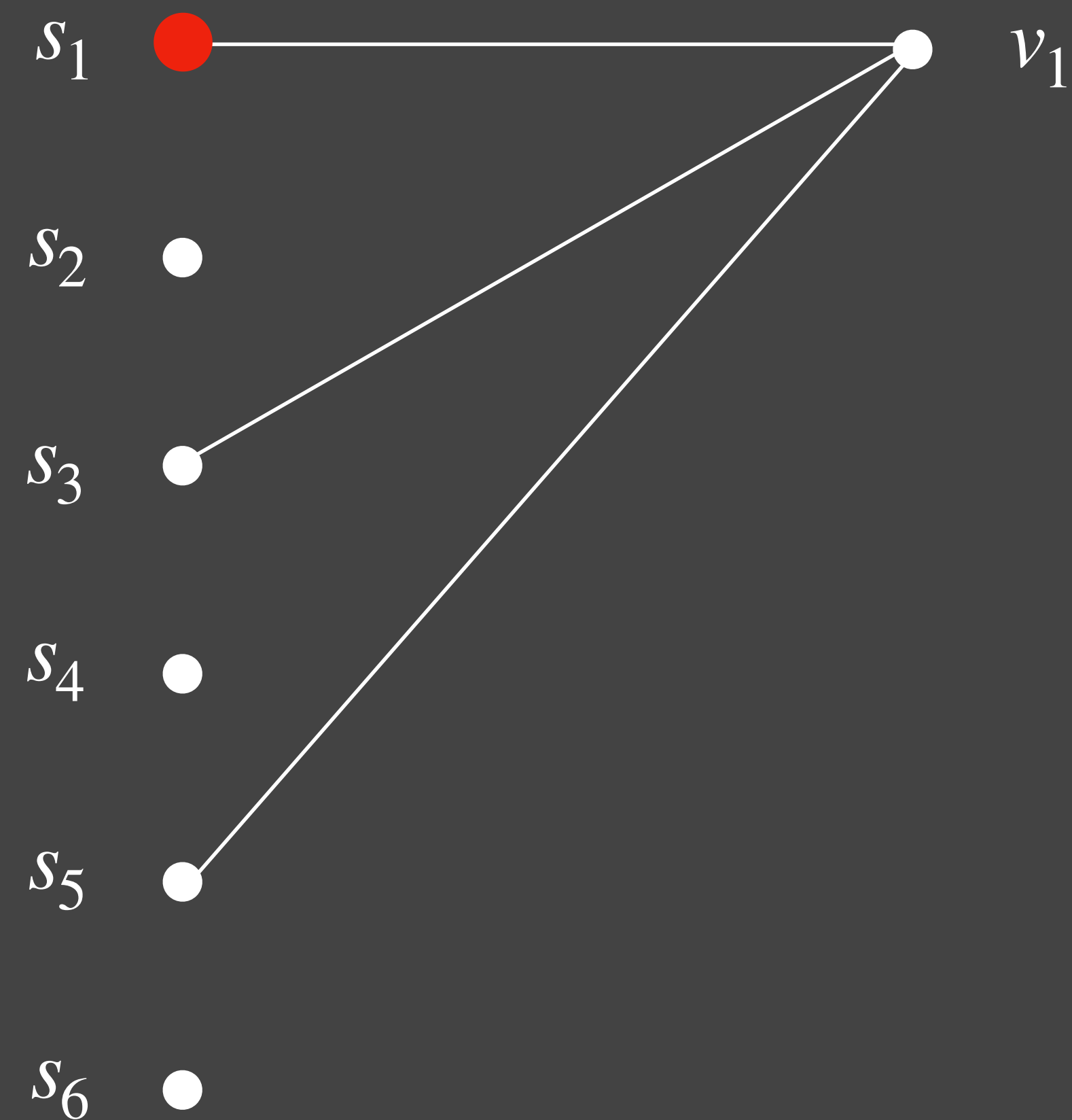
s_6 ●



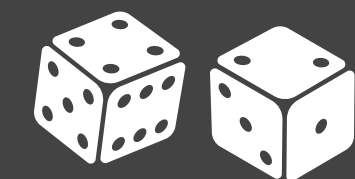
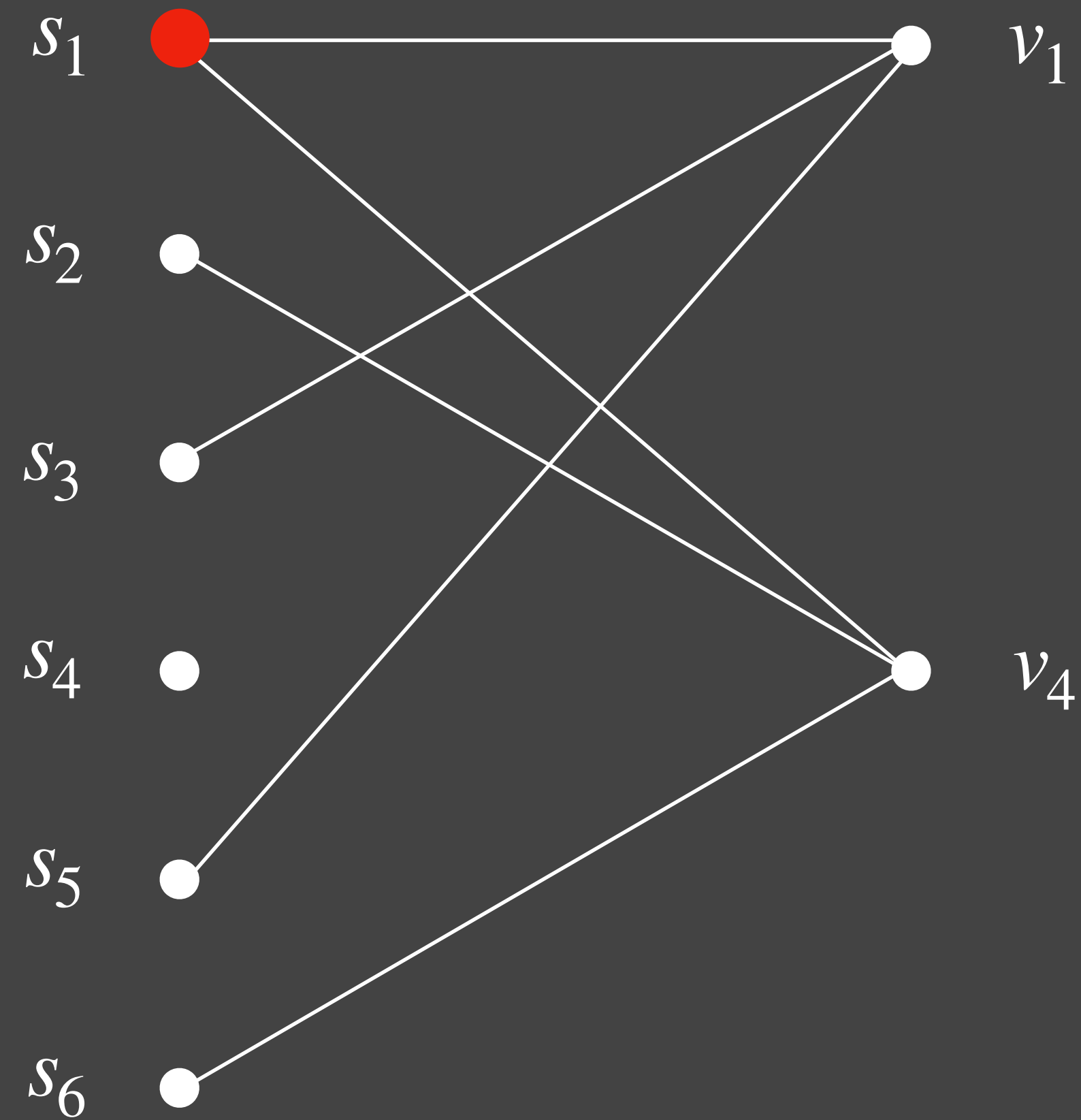
Relaxation 1: Random Order (RO)



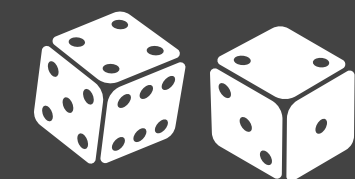
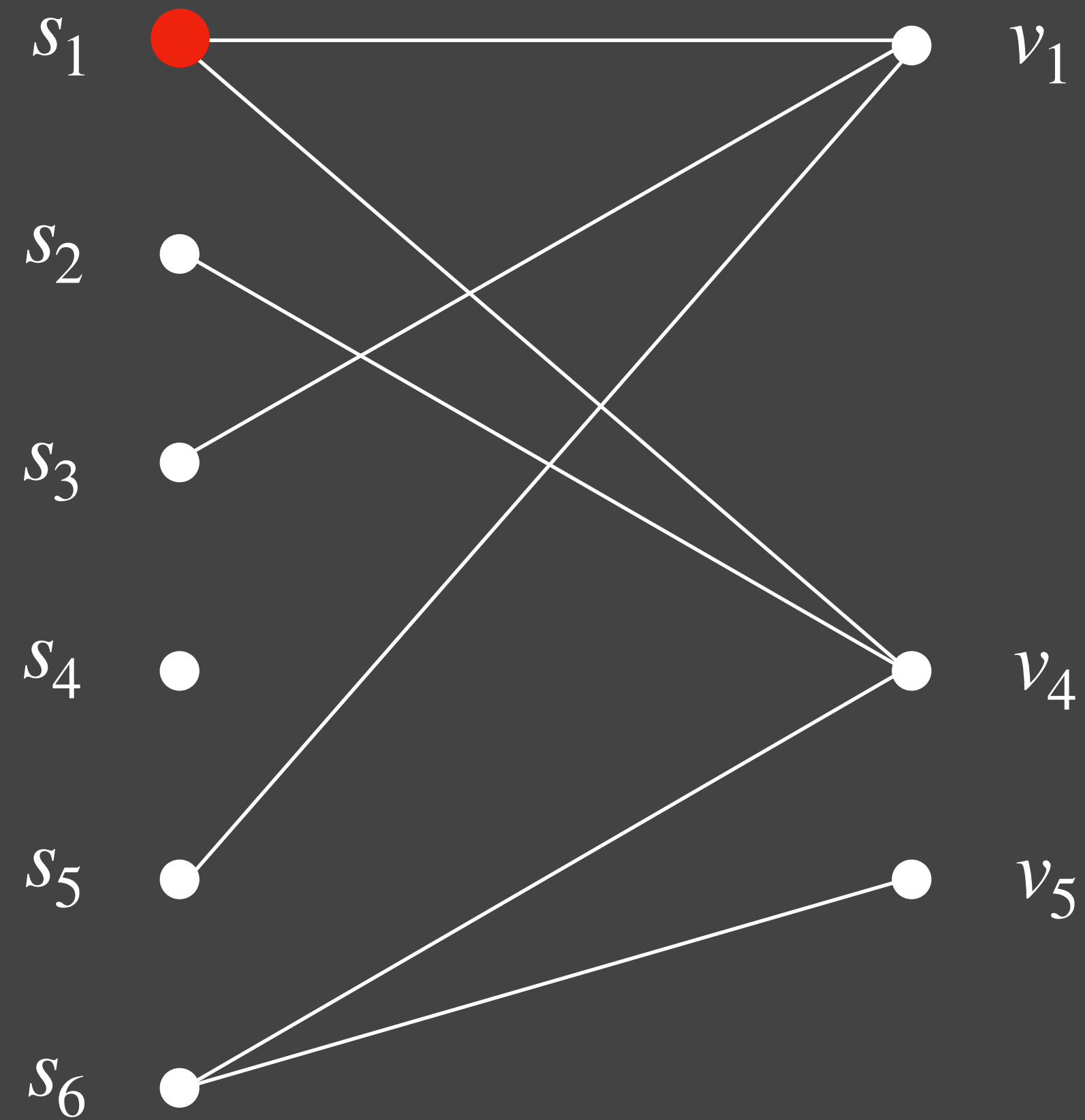
Relaxation 1: Random Order (RO)



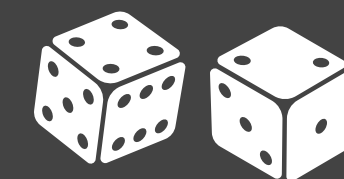
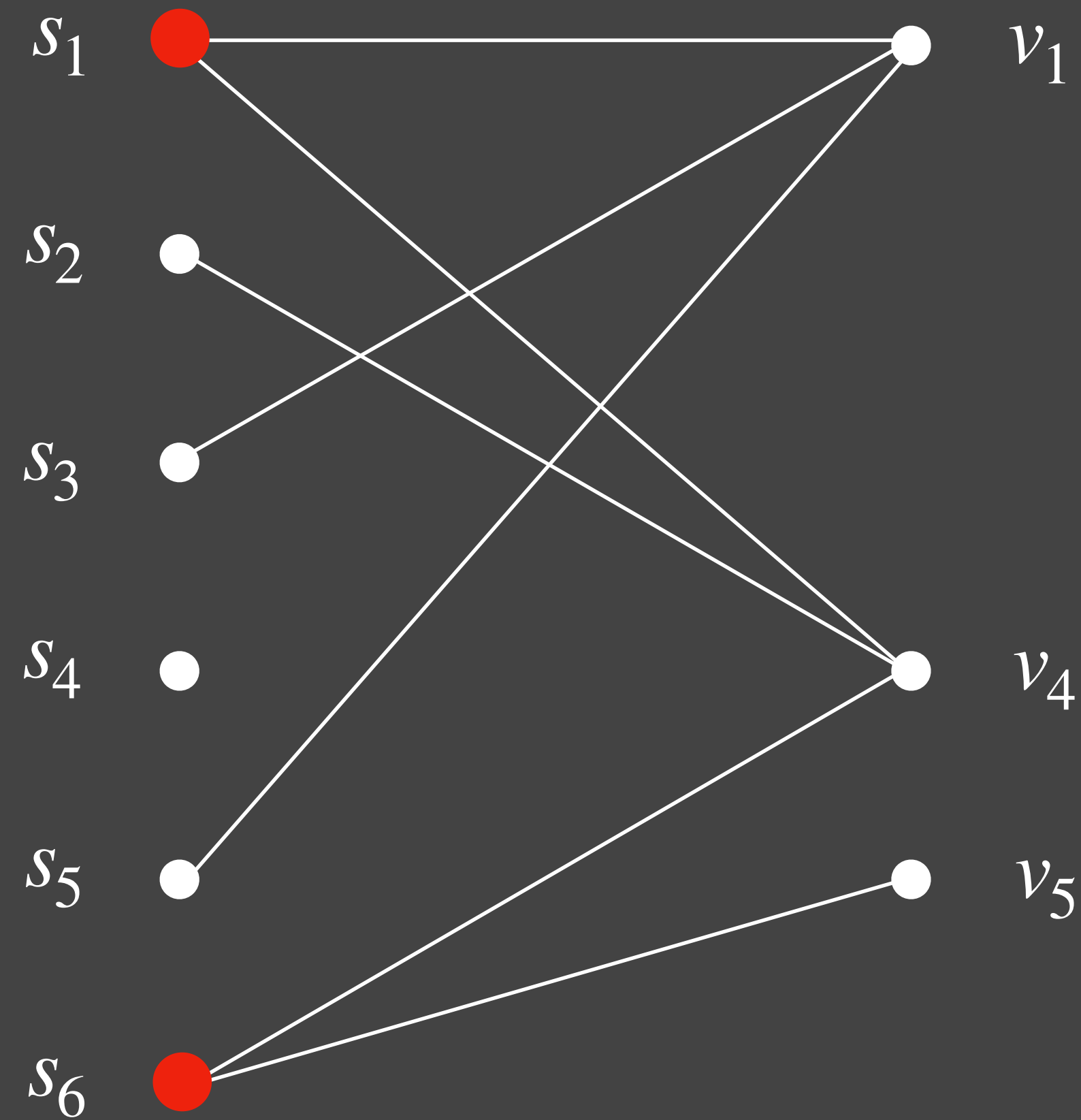
Relaxation 1: Random Order (RO)



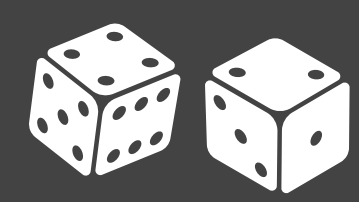
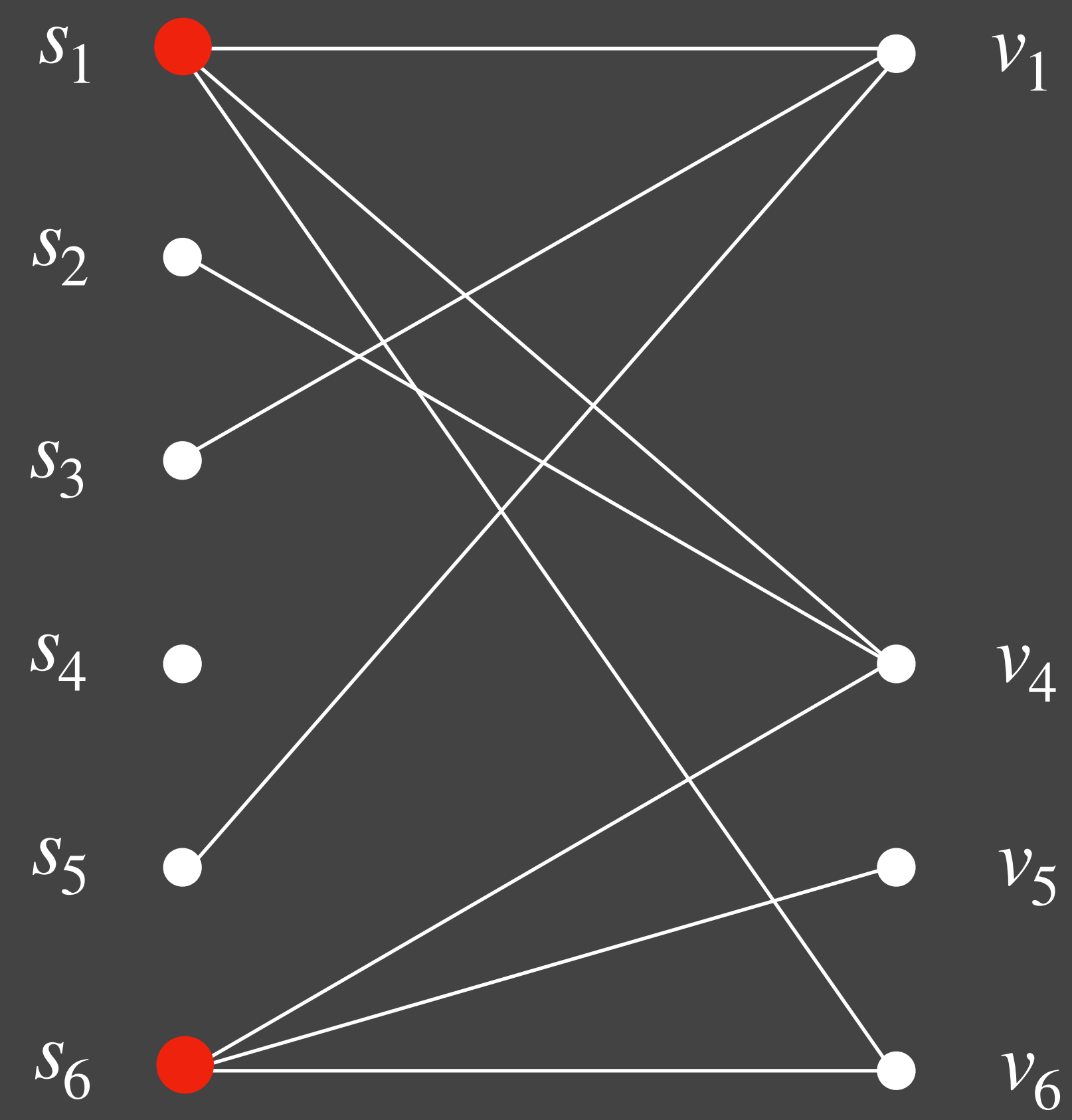
Relaxation 1: Random Order (RO)



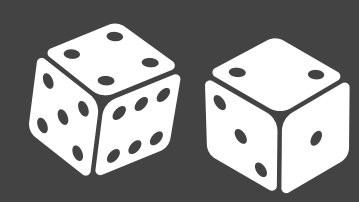
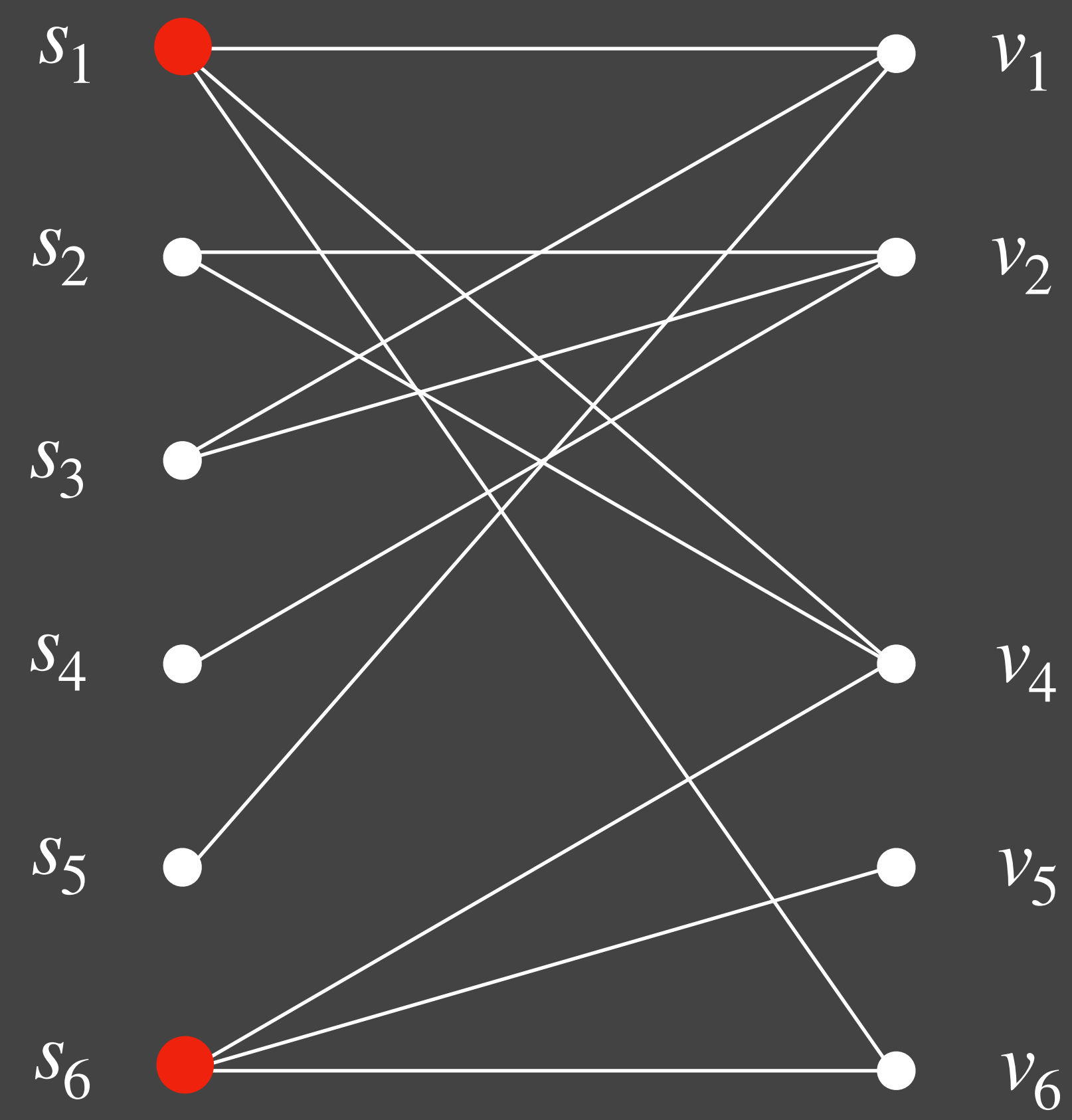
Relaxation 1: Random Order (RO)



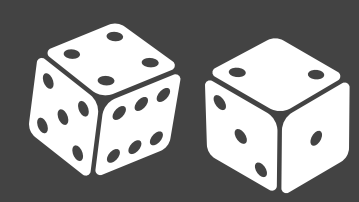
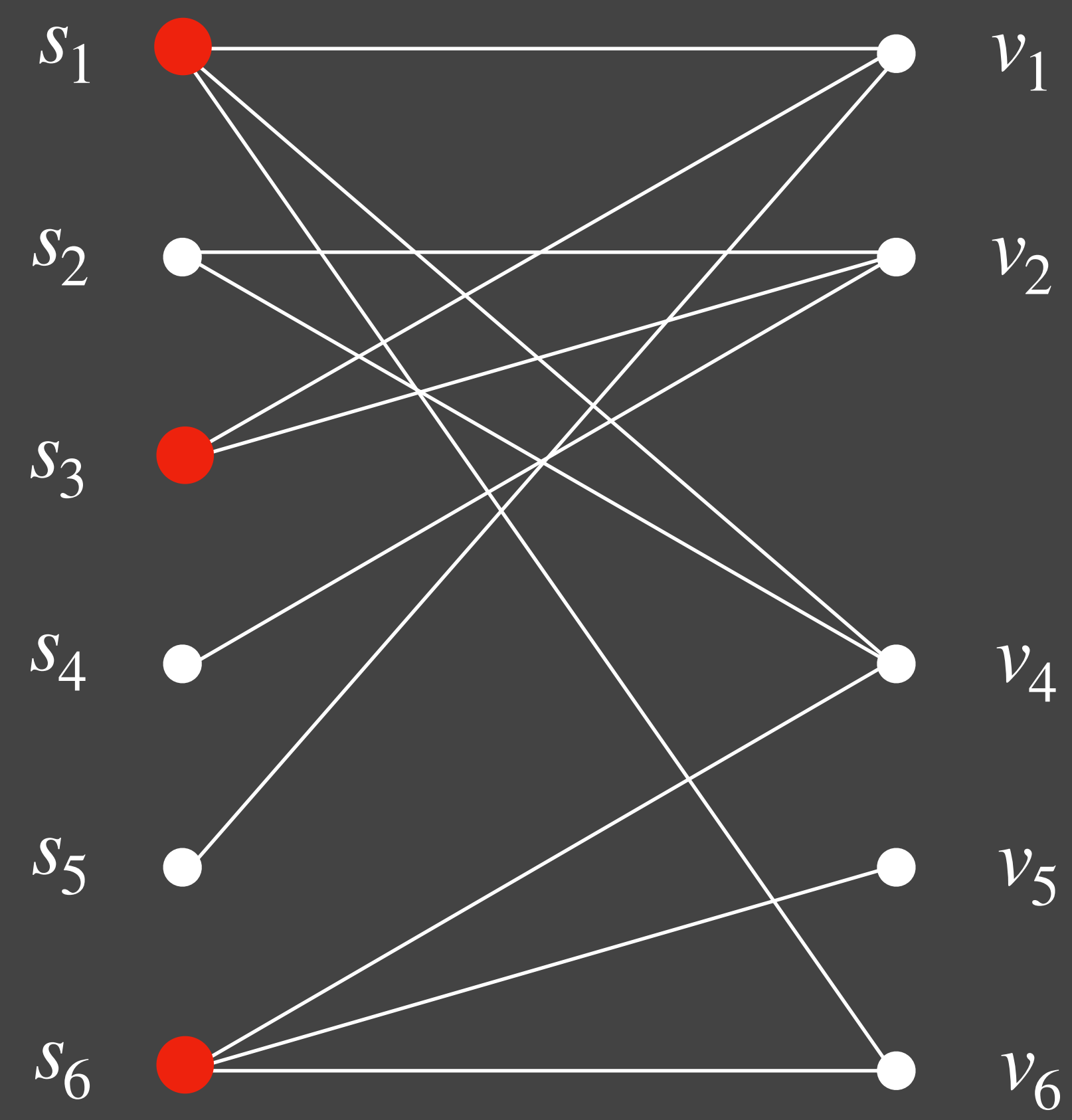
Relaxation 1: Random Order (RO)



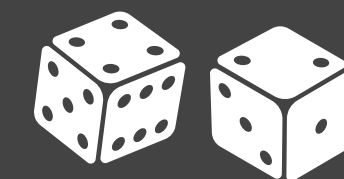
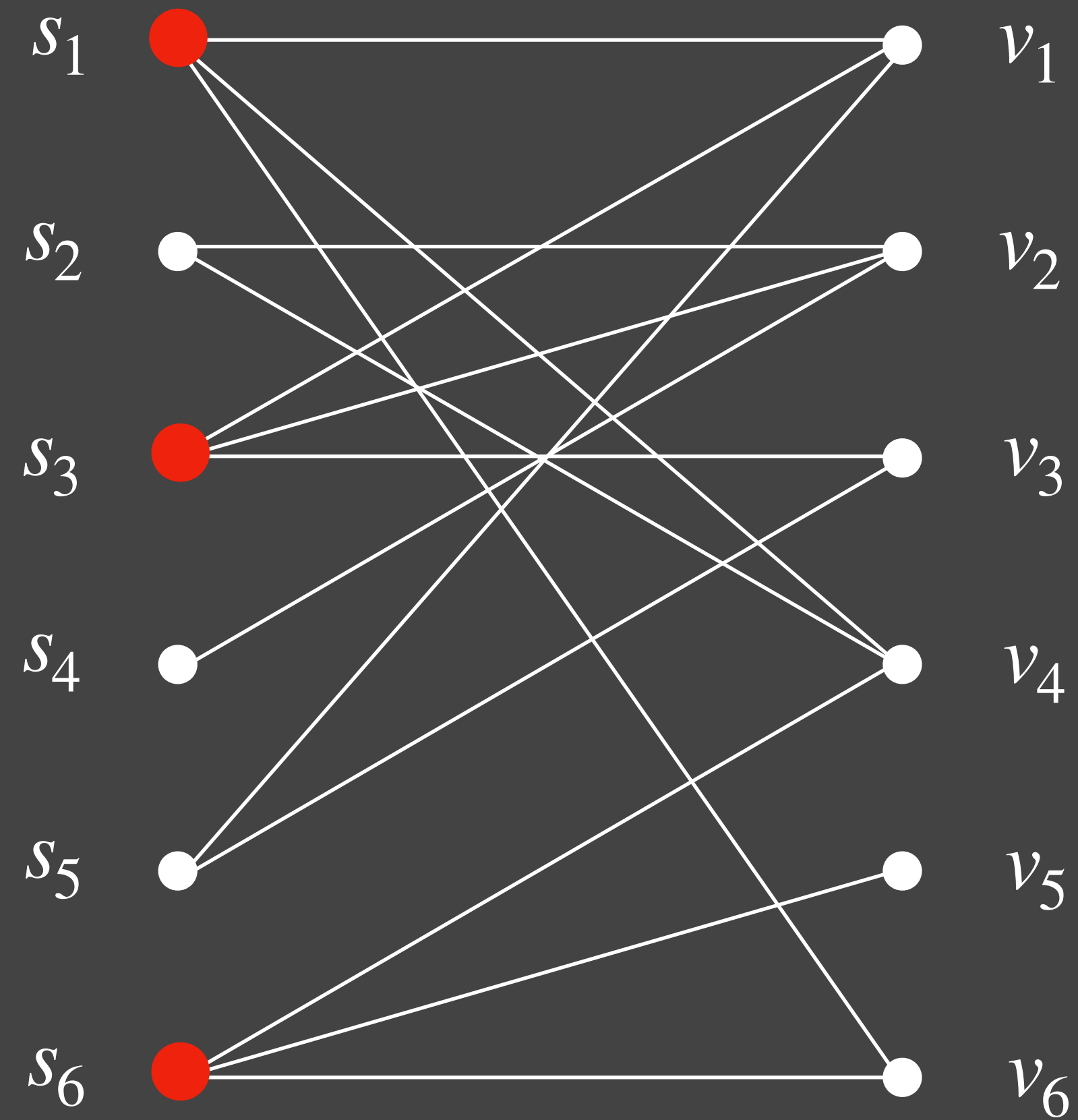
Relaxation 1: Random Order (RO)



Relaxation 1: Random Order (RO)



Relaxation 1: Random Order (RO)



Relaxation 2: Random Instance

s_1 ●

s_2 ●

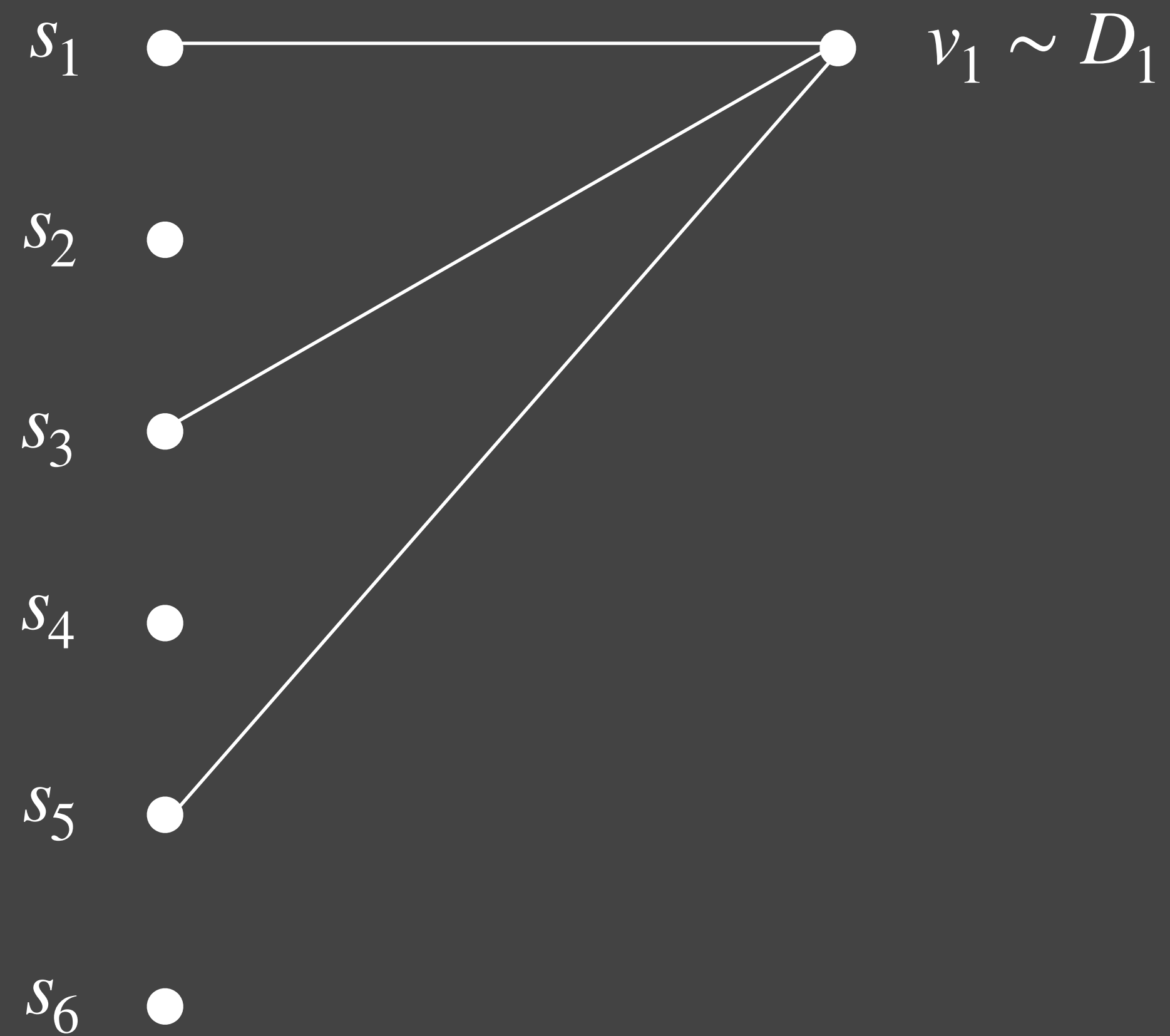
s_3 ●

s_4 ●

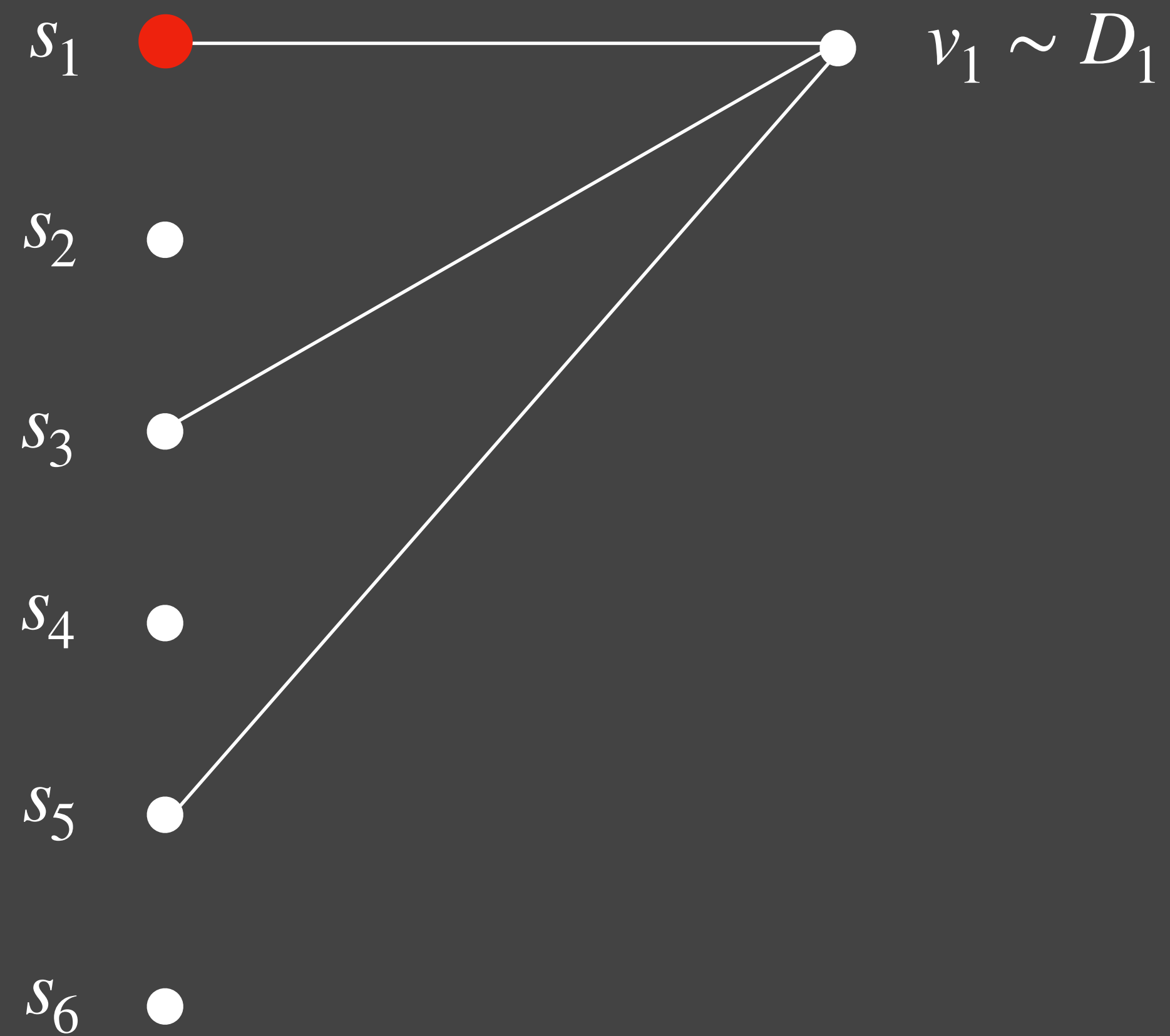
s_5 ●

s_6 ●

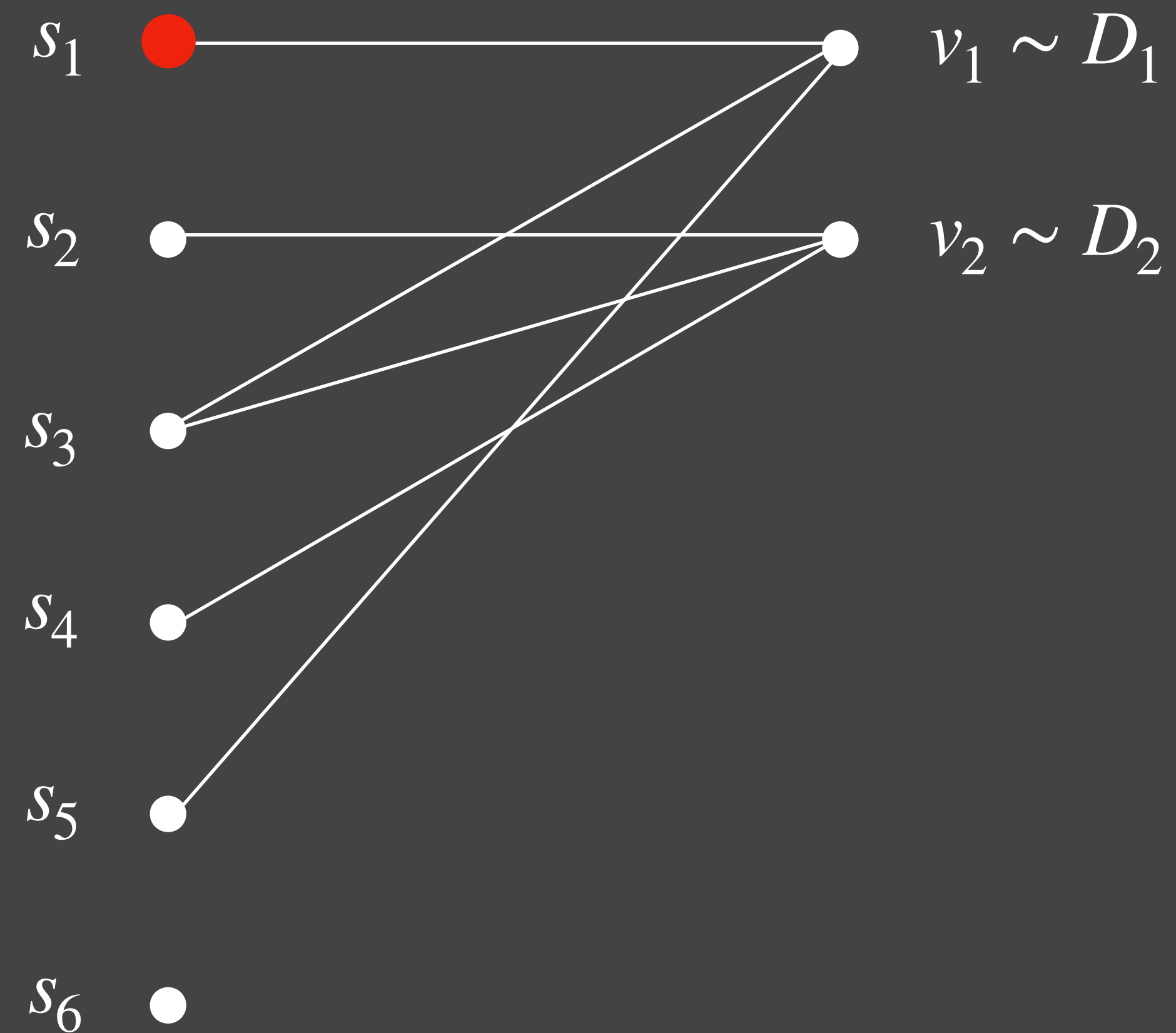
Relaxation 2: Random Instance



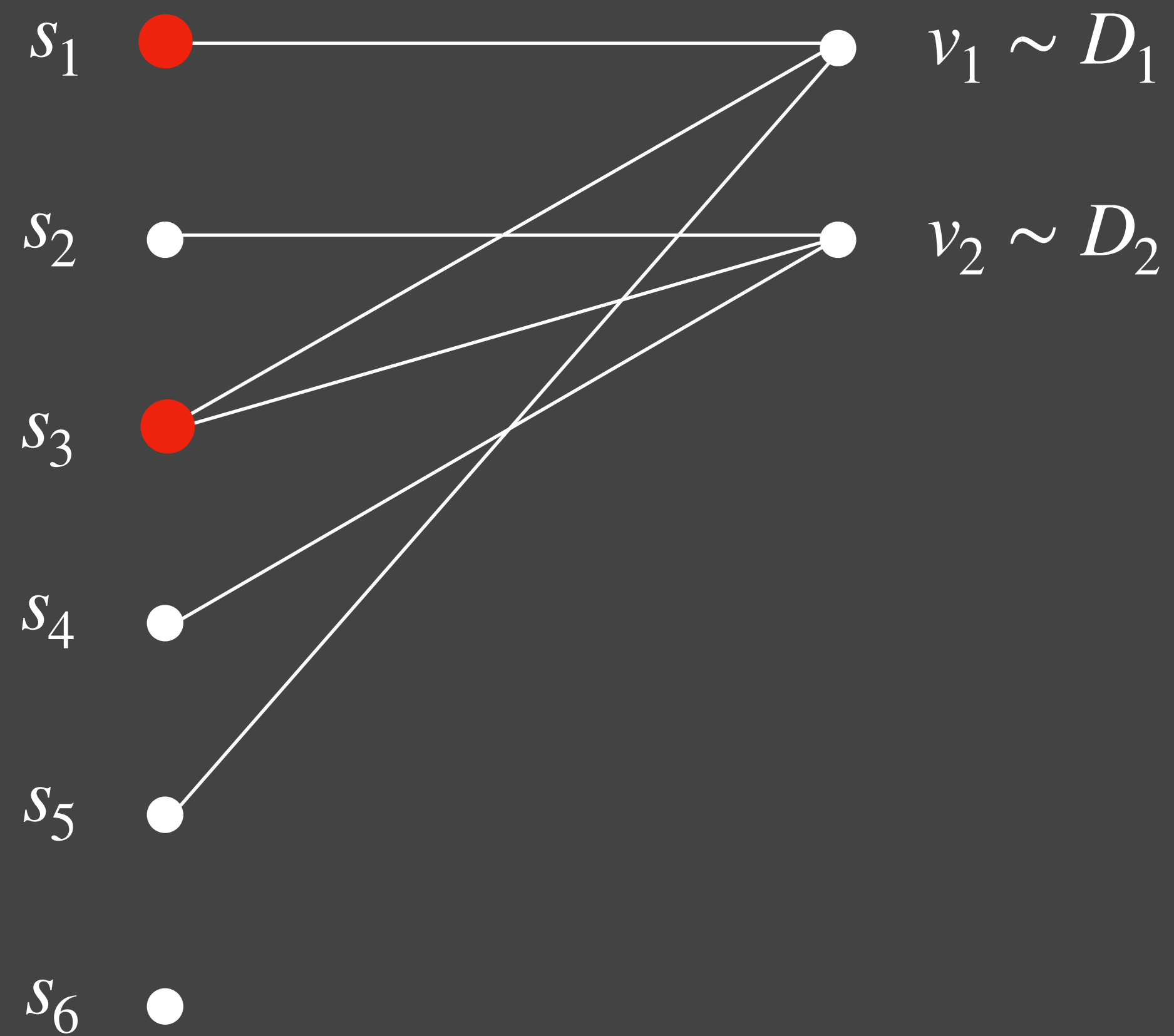
Relaxation 2: Random Instance



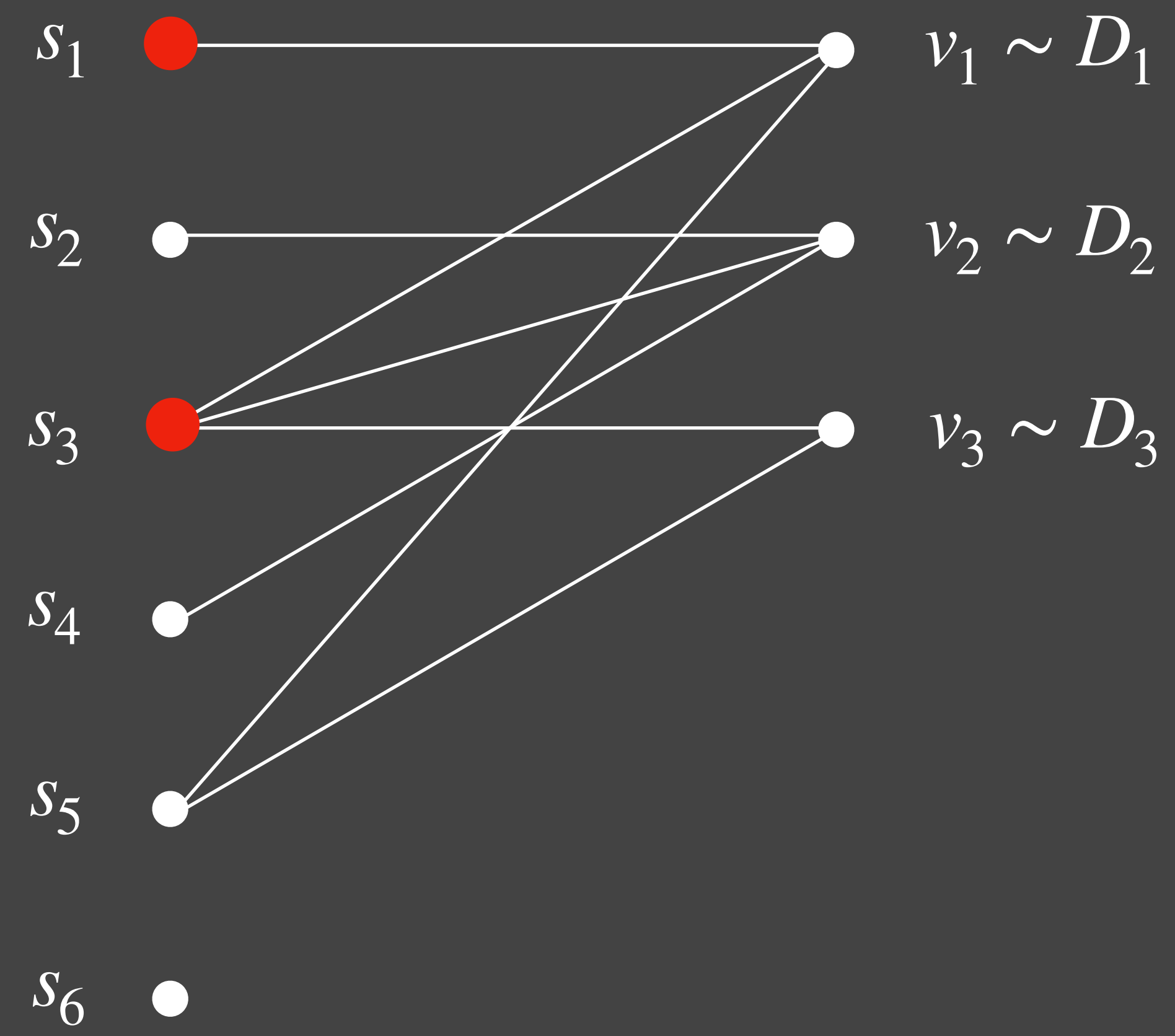
Relaxation 2: Random Instance



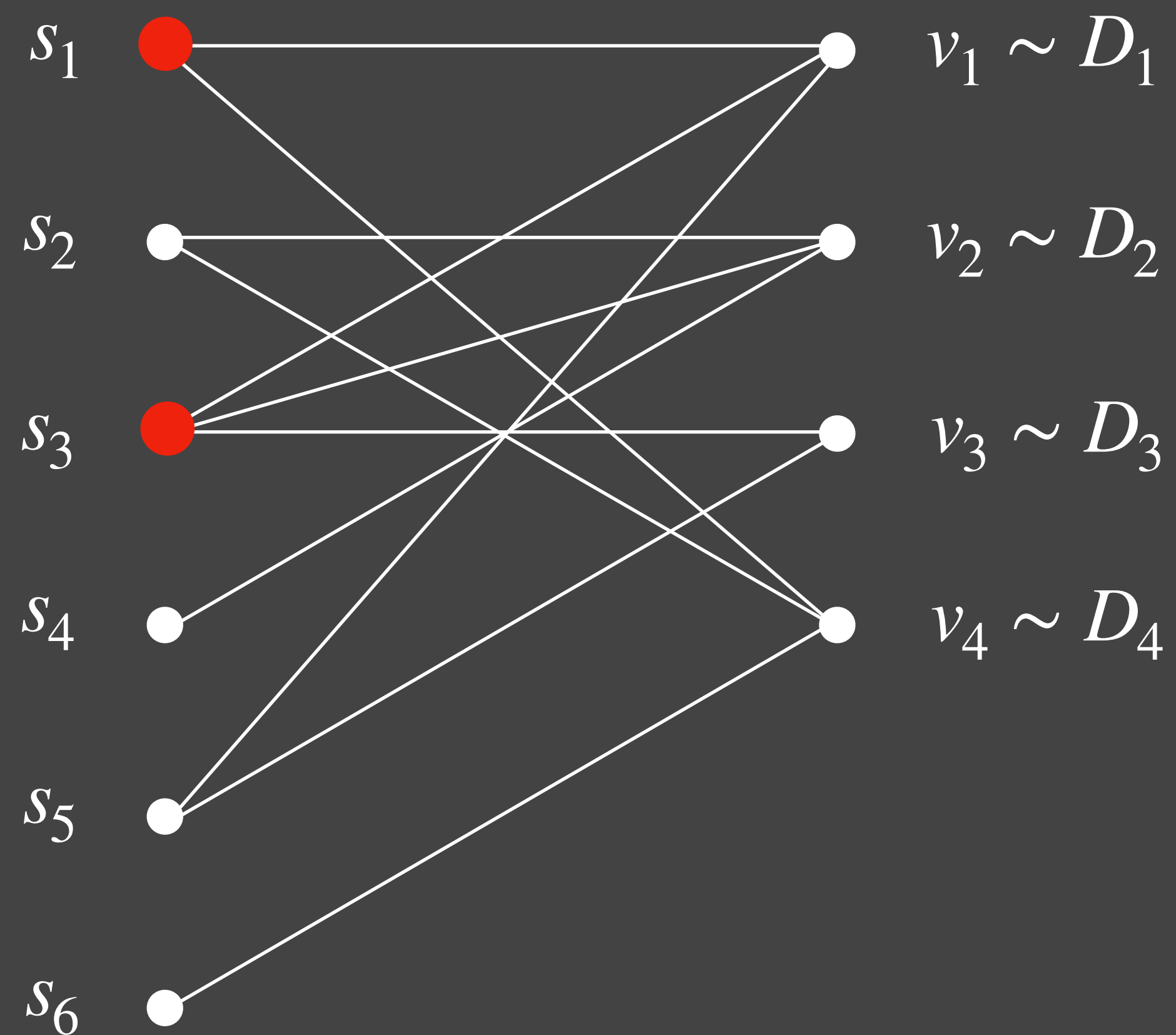
Relaxation 2: Random Instance



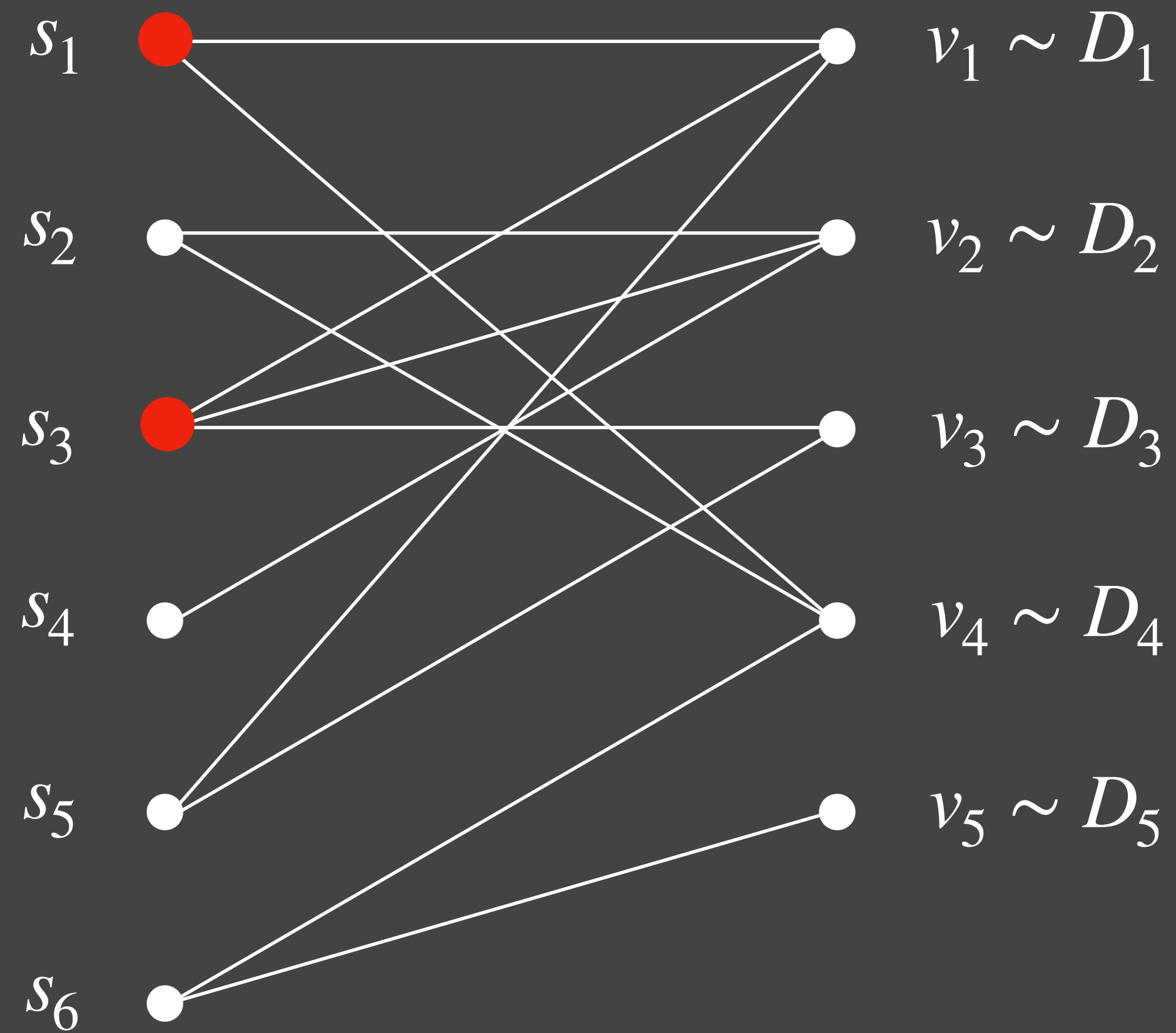
Relaxation 2: Random Instance



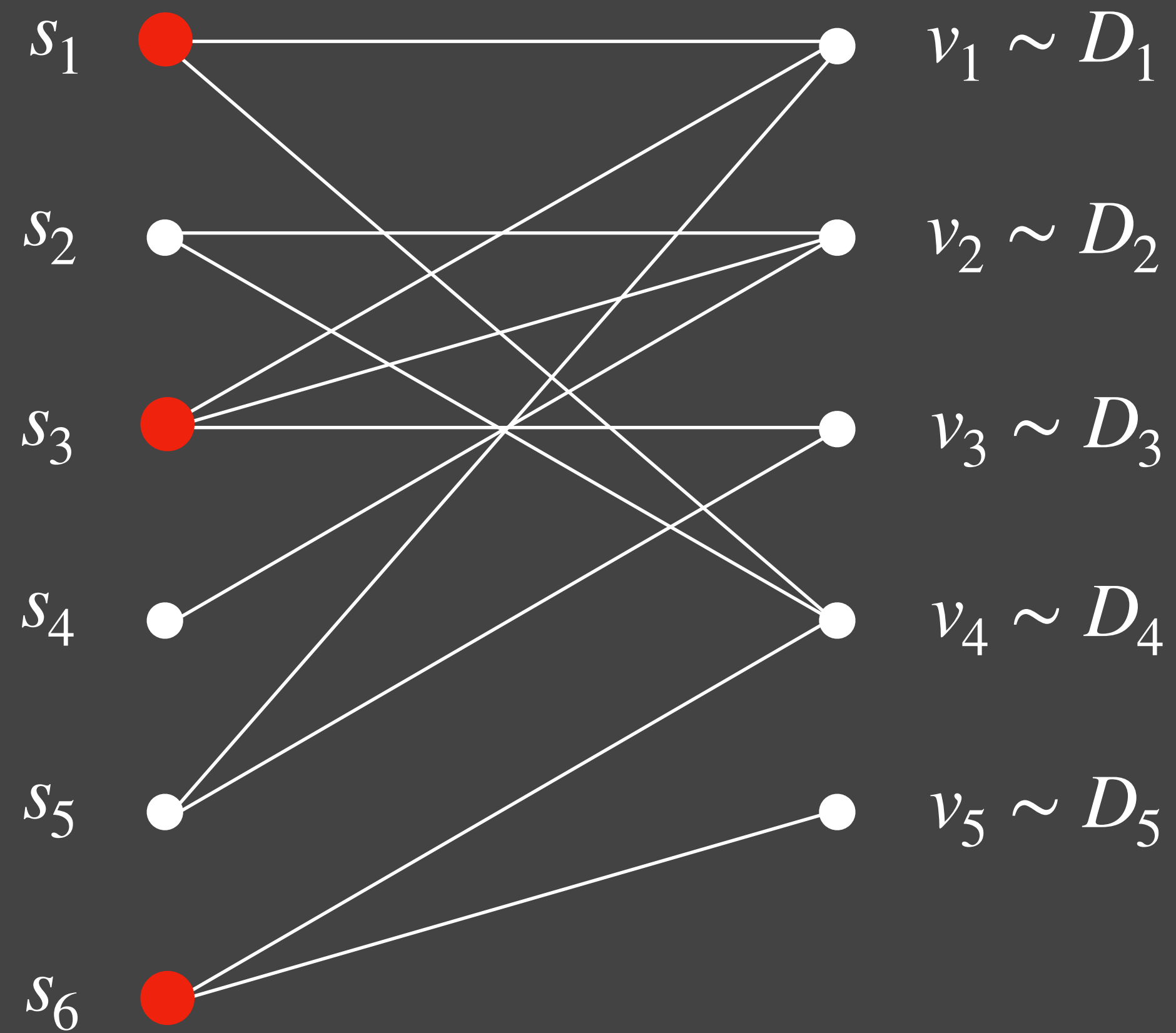
Relaxation 2: Random Instance



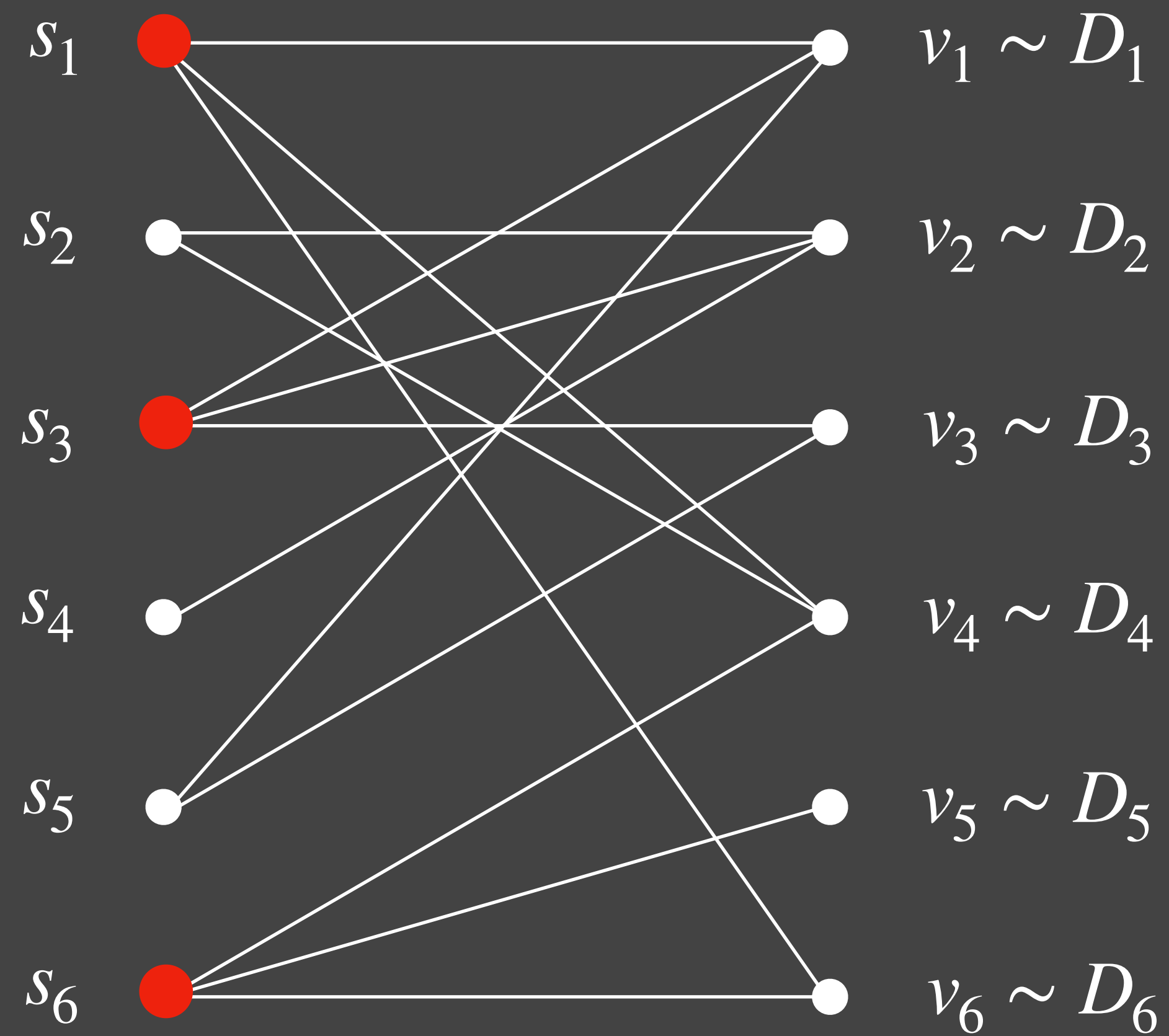
Relaxation 2: Random Instance



Relaxation 2: Random Instance



Relaxation 2: Random Instance



The Landscape

The Landscape

		Instance	
		Random	Adversarial
Arrival Order	Random		
	Adversarial		$O(\log^2 n)$ [Alon+ 03] [Buchbinder Naor 09]

The Landscape

		Instance	
		Random	Adversarial
Arrival Order	Random	$O(\log(n \text{ [support size]}))$ [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	
	Adversarial		$O(\log^2 n)$ [Alon+ 03] [Buchbinder Naor 09]

The Landscape

		Instance	
		Random	Adversarial
Arrival Order	Random	$O(\log(n \text{ [support size]}))$ [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	<div>Secretary</div> <div></div>
	Adversarial		$O(\log^2 n)$ [Alon+ 03] [Buchbinder Naor 09]

The Landscape

		Instance	
		Random	Adversarial
Arrival Order	Random	$O(\log(n \text{ [support size]}))$ [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	<div>Secretary</div>
	Adversarial	<div>Prophet</div>	$O(\log^2 n)$ [Alon+ 03] [Buchbinder Naor 09]

The Landscape

		Instance	
		Random	Adversarial
Arrival Order	Random	$O(\log(n \text{ [support size]}))$ [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	<div>Secretary</div>
	Adversarial	<div>Prophet</div>	$O(\log^2 n)$ [Alon+ 03] [Buchbinder Naor 09]

Was believed $O(\log^2 n)$
best possible [Gupta+ 09]...

The Landscape

		Instance	
		Random	Adversarial
Arrival Order	Random	$O(\log(n \text{ [support size]}))$ [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	$O(\log n)$ Our work
	Adversarial		$O(\log^2 n)$ [Alon+ 03] [Buchbinder Naor 09]

Secretary

Prophet

Theorem [Gupta Kehne L.
FOCS 21]:

Polynomial time algo for
secretary Covering IP with
 approximation $O(\log n)$.

The Landscape

		Instance	
		Random	Adversarial
Arrival Order	Random	$O(\log(n \text{ [support size]}))$ [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	$O(\log n)$ Our work
	Adversarial	$O(\log^2 n)$ [Alon+ 03] [Buchbinder Naor 09]	$O(\log^2 n)$ [Alon+ 03] [Buchbinder Naor 09]

Secretary

Prophet

Theorem [Gupta Kehne L.
FOCS 21]:

Polynomial time algo for
secretary Covering IP with
 approximation $O(\log n)$.

New algorithm, LearnOrCover!
 Not just new analysis of old
 algorithm.

The Landscape

		Instance	
		Random	Adversarial
Arrival Order	Random	$O(\log(n \text{ [support size]}))$ [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	$O(\log n)$ Our work <i>Secretary</i>
	Adversarial	$O(\log n)$ Our work <i>Prophet</i>	$O(\log^2 n)$ [Alon+ 03] [Buchbinder Naor 09]

Theorem [Gupta Kehne L.
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Theorem [Gupta Kehne L.
In submission]:

Polynomial time algo for
prophet Covering IPs with
approximation $O(\log n)$.

The Landscape

		Instance	
		Random	Adversarial
Arrival Order	Random	$O(\log(n \text{ [support size]}))$ [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	$O(\log n)$ Our work Secretary
	Adversarial	$O(\log n)$ Our work Prophet	$O(\log^2 n)$ [Alon+ 03] [Buchbinder Naor 09]

Bonus!

Only need 1 sample from each D_i !

Theorem [Gupta Kehne L.
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Polynomial time algo for
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approximation $O(\log n)$.

The Landscape

Bonus!

1-pass **Streaming** Algorithm!

Arrival Order

Instance

		Instance	
		Random	Adversarial
Arrival Order	Random	$O(\log(n \text{ [support size]}))$ [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	$O(\log n)$ Our work
	Adversarial	$O(\log n)$ Our work	$O(\log^2 n)$ [Alon+ 03] [Buchbinder Naor 09]

Secretary

Prophet

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Polynomial time algo for **secretary** Covering IP with approximation $O(\log n)$.

Theorem [Gupta Kehne L. In submission]:

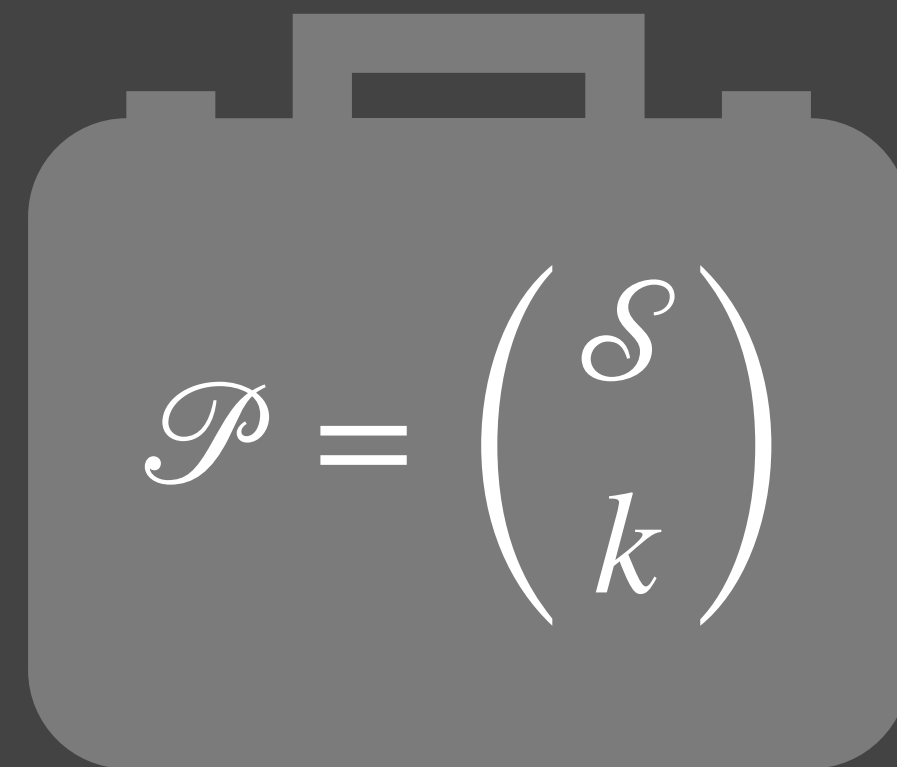
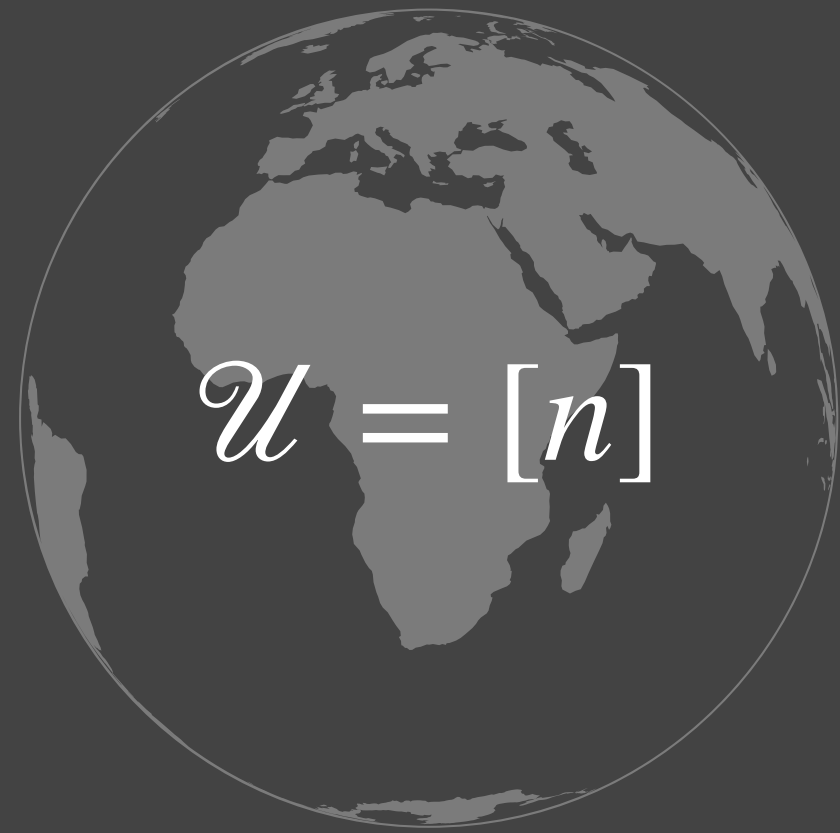
Polynomial time algo for **prophet** Covering IPs with approximation $O(\log n)$.

Bonus!

Only need 1 sample from each D_i !

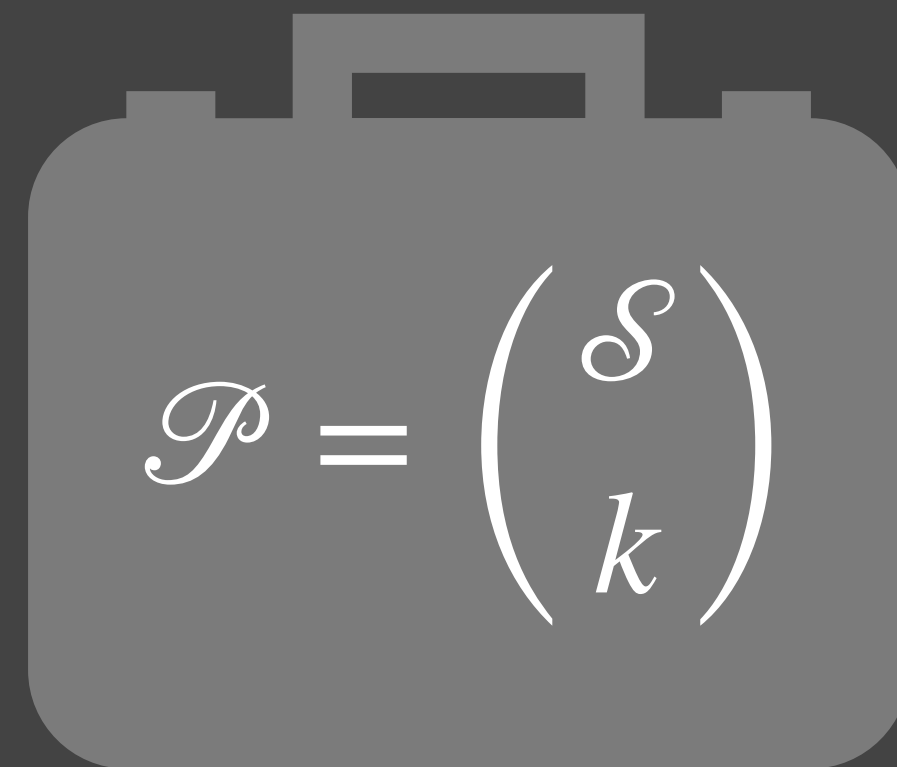
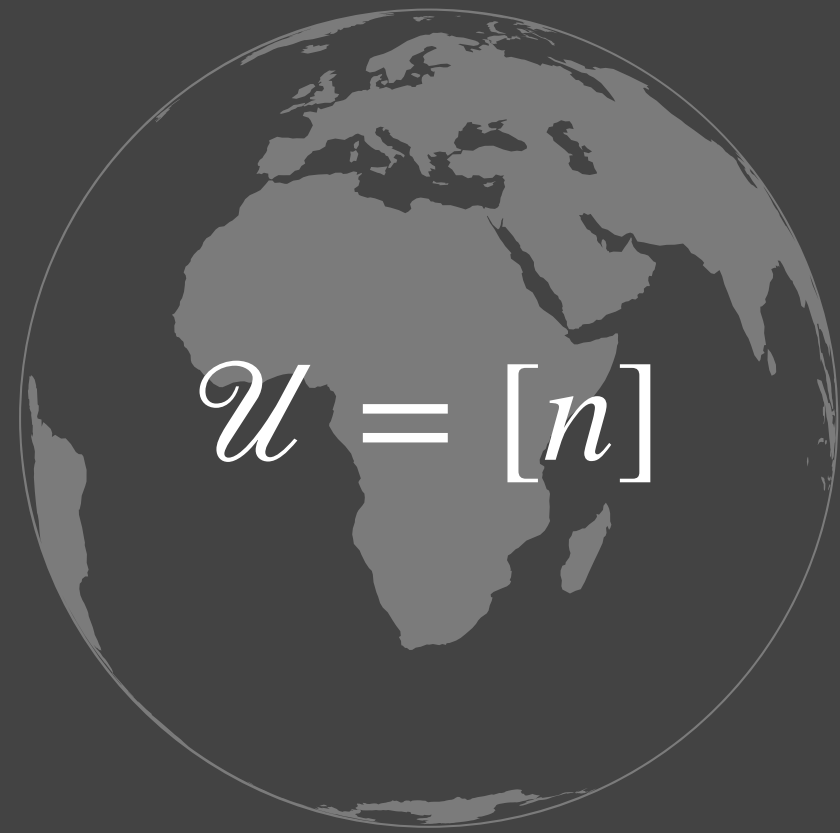
LearnOrCover

LearnOrCover



$$k := |OPT|$$

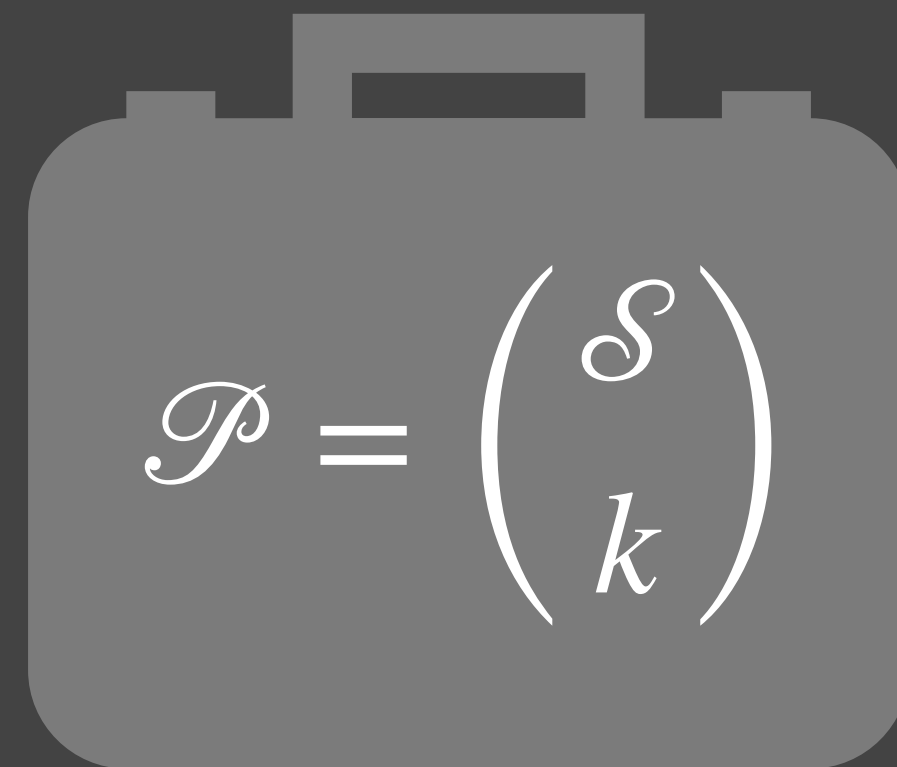
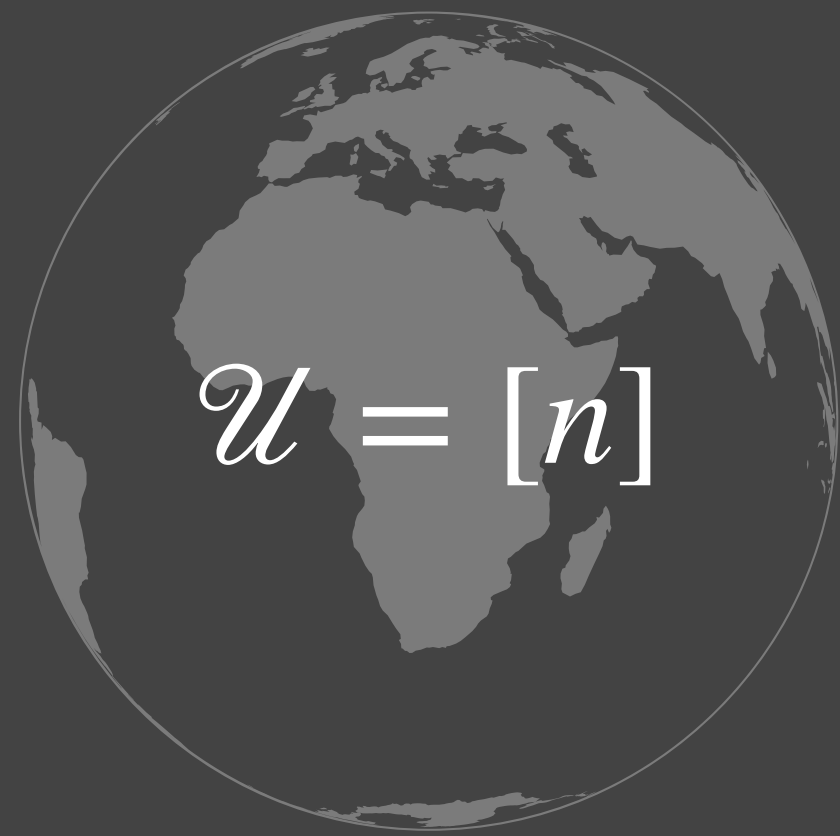
LearnOrCover



$$k := |OPT|$$

@ time t , element v arrives:

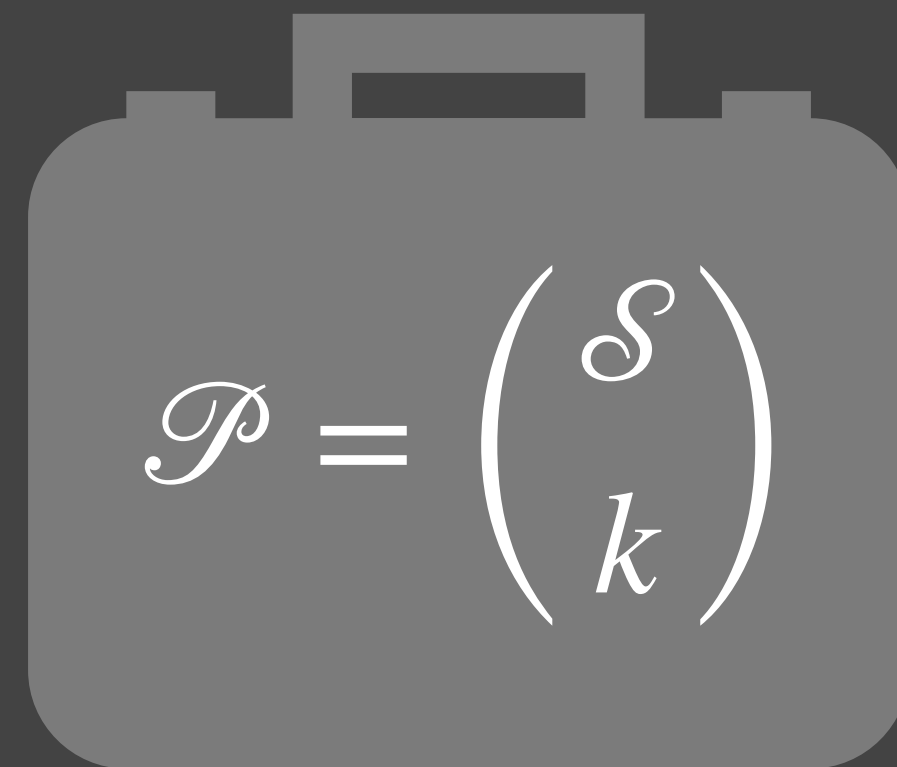
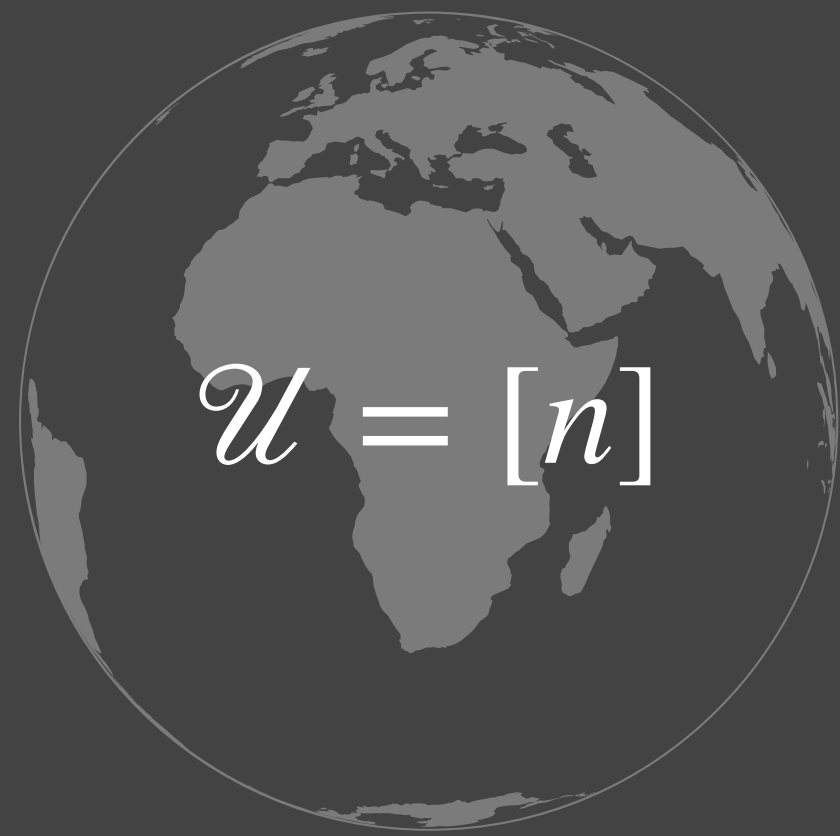
LearnOrCover



$$k := |OPT|$$

@ time t , element v arrives:
If v covered, do nothing.

LearnOrCover



$$k := |OPT|$$

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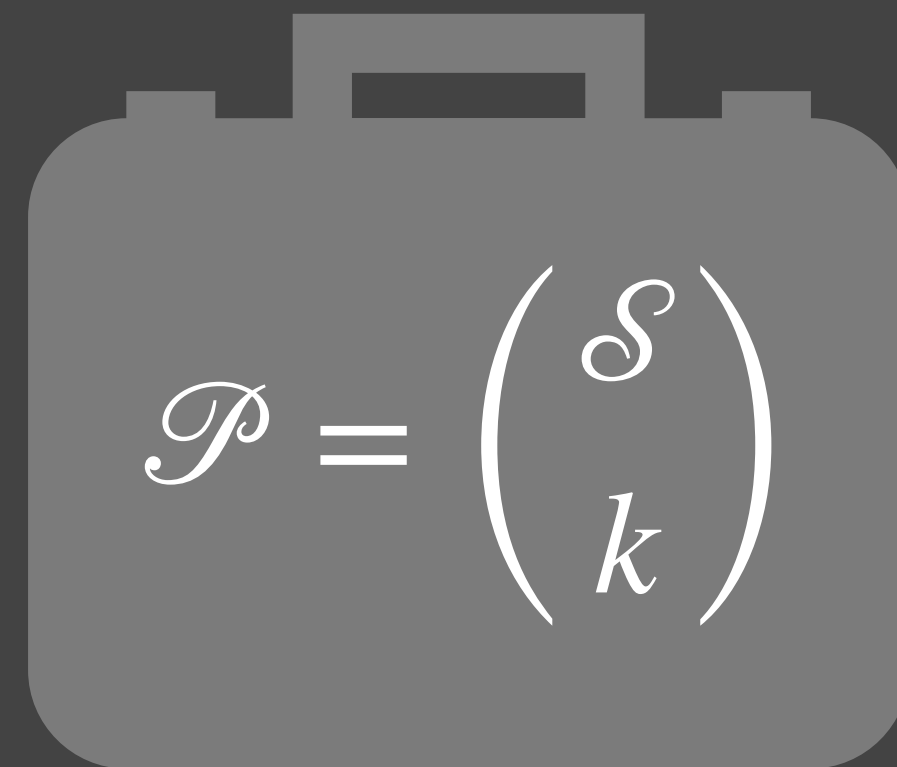
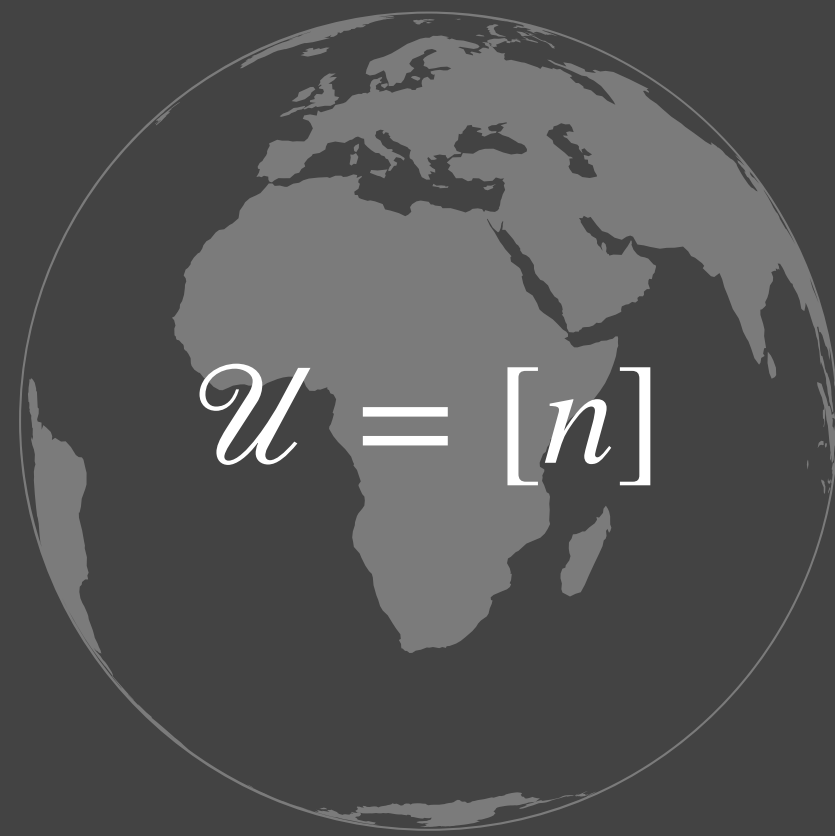
Else:

(I) Buy random set R from \mathcal{P} to cover v .

(II) “Prune” $P \not\supseteq v$ from \mathcal{P} .

LearnOrCover

Proof idea: progress **learning** or **covering**.



$$k := |\mathcal{P}|$$

@ time t , element v arrives:

If v covered, do nothing.

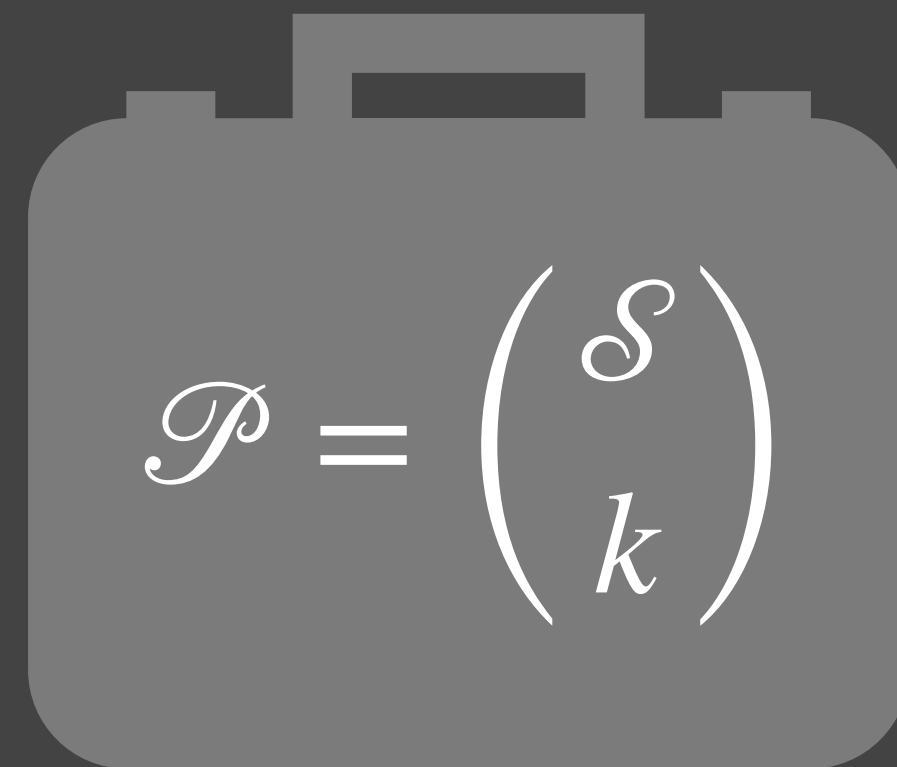
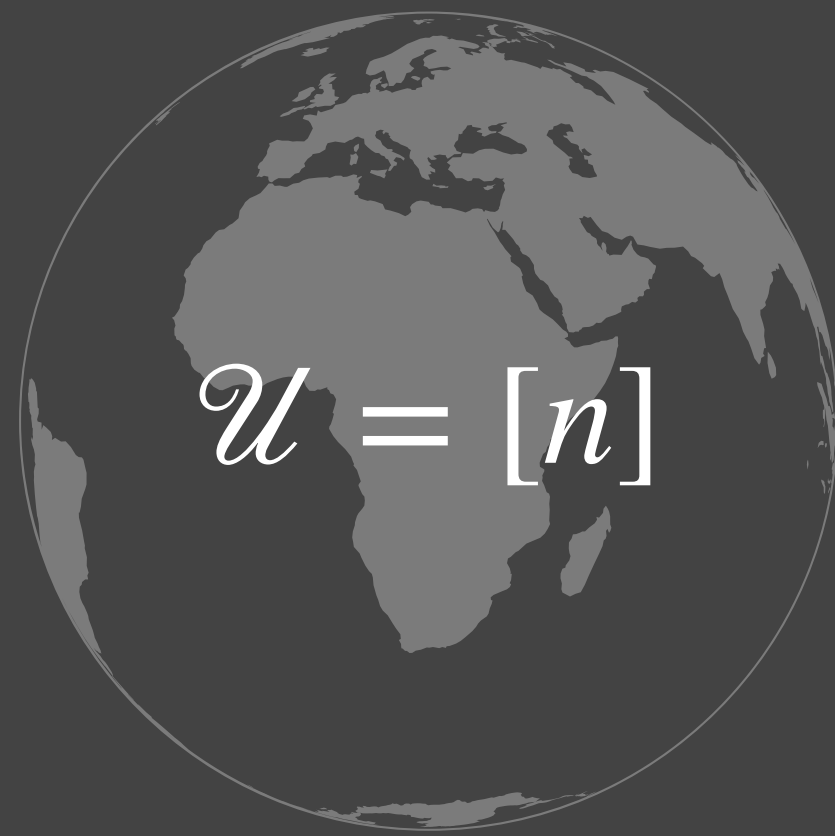
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LearnOrCover

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s_1 ●

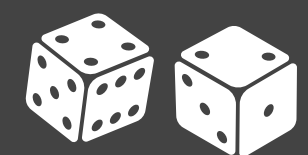
s_2 ●

s_3 ●

s_4 ●

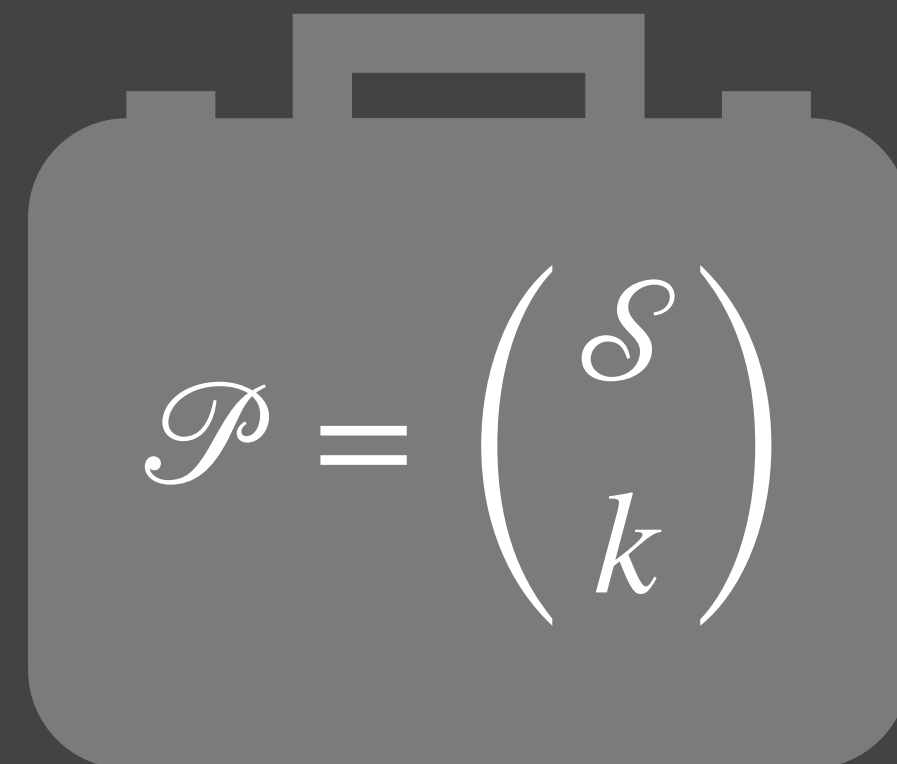
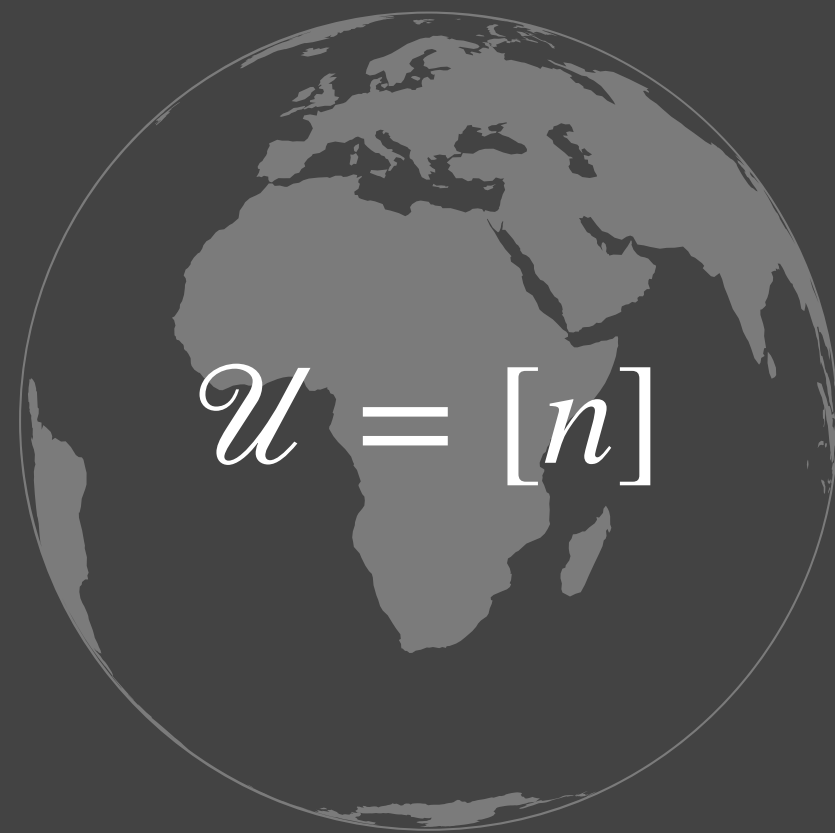
s_5 ●

s_6 ●



LearnOrCover

Proof idea: progress **learning** or **covering**.



$$k := |\text{OPT}|$$

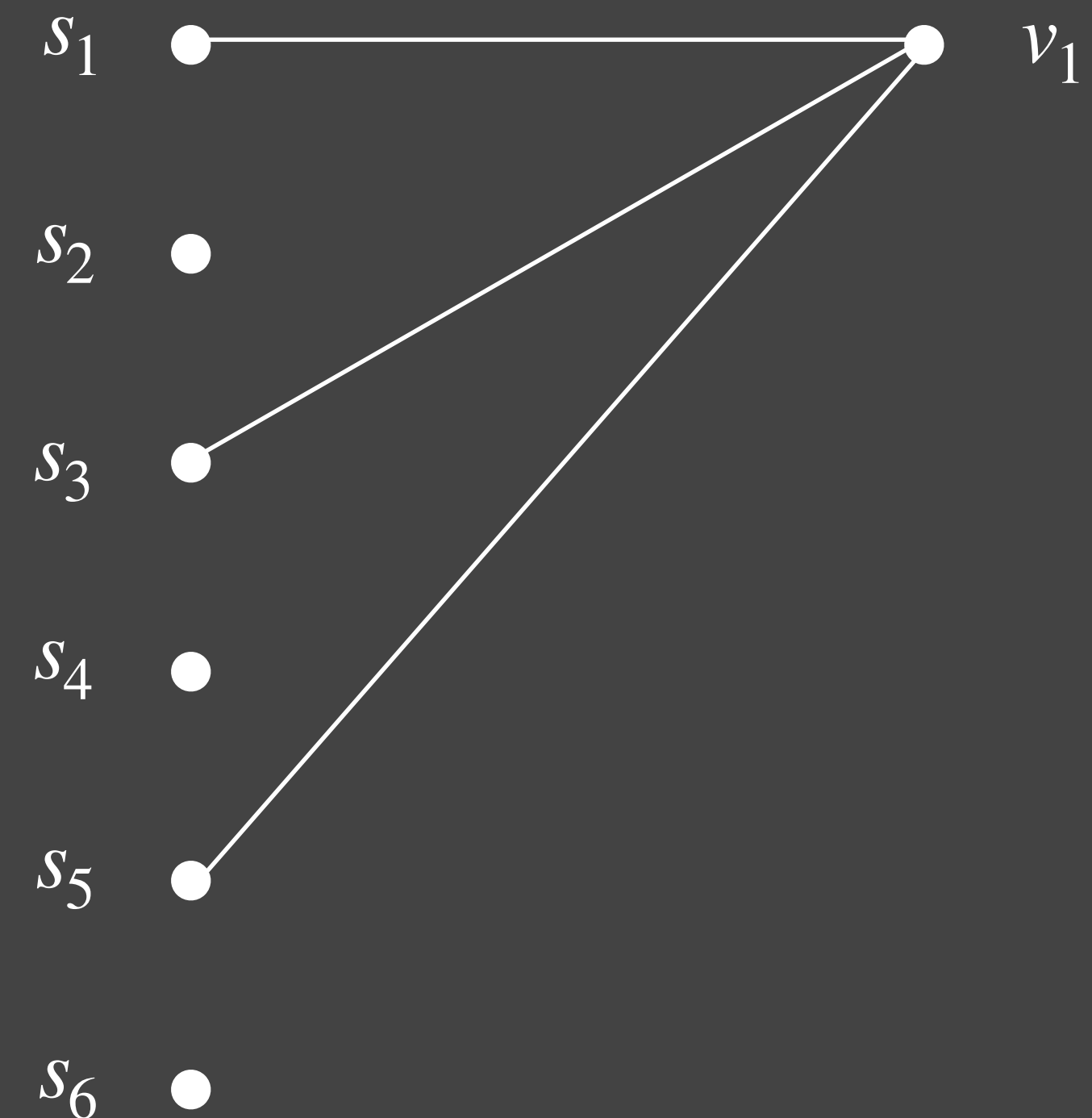
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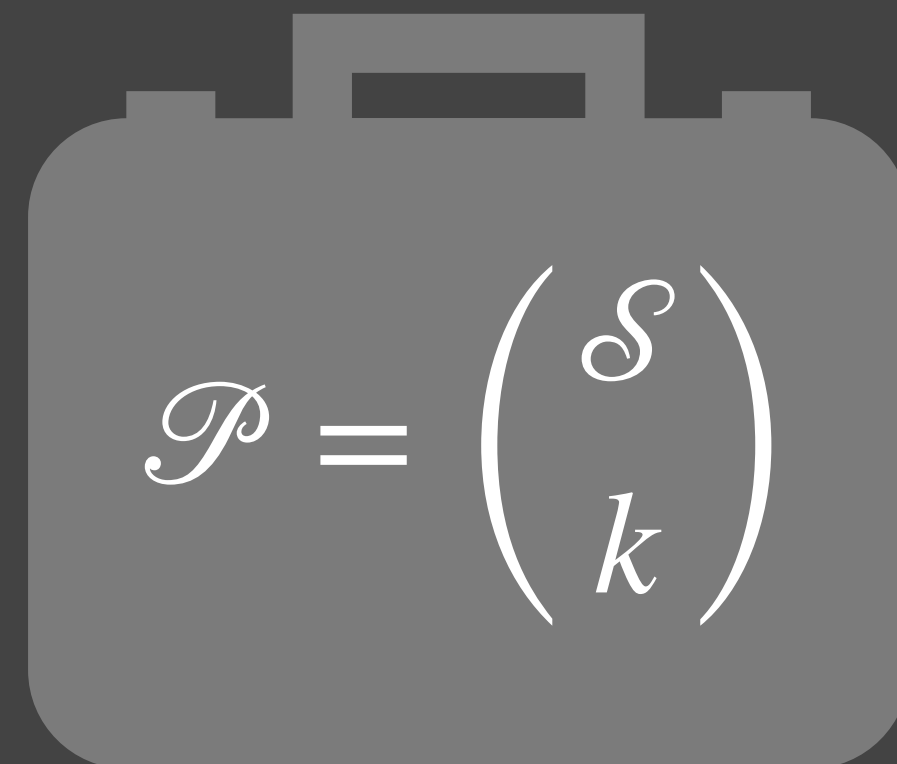
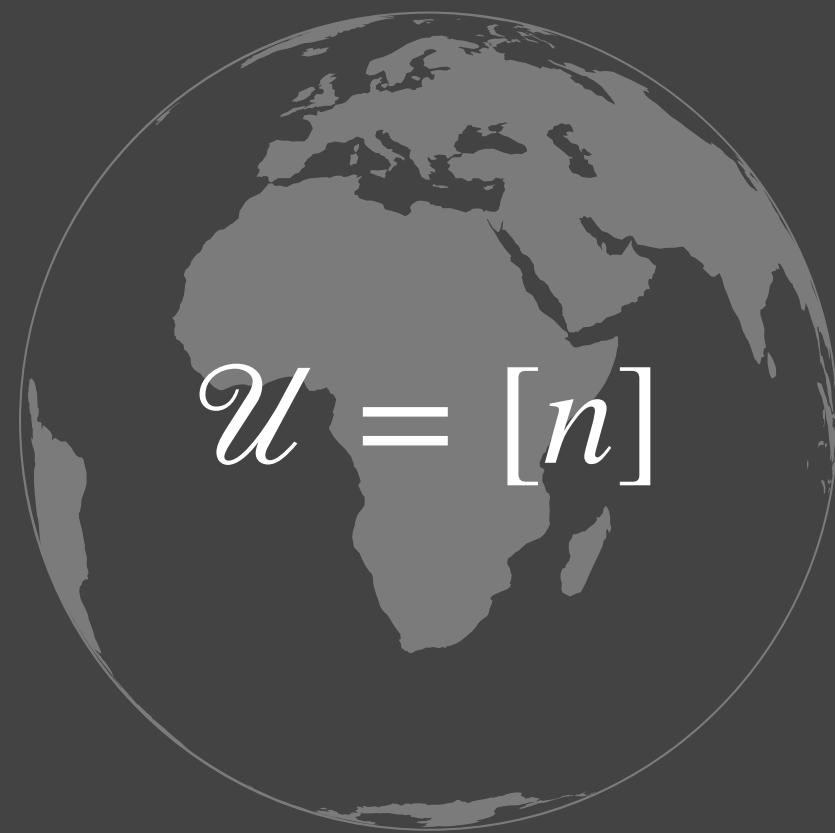
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LearnOrCover

Proof idea: progress **learning** or **covering**.



$$k := |\text{OPT}|$$

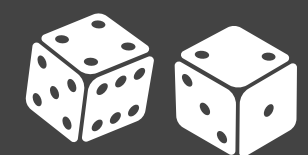
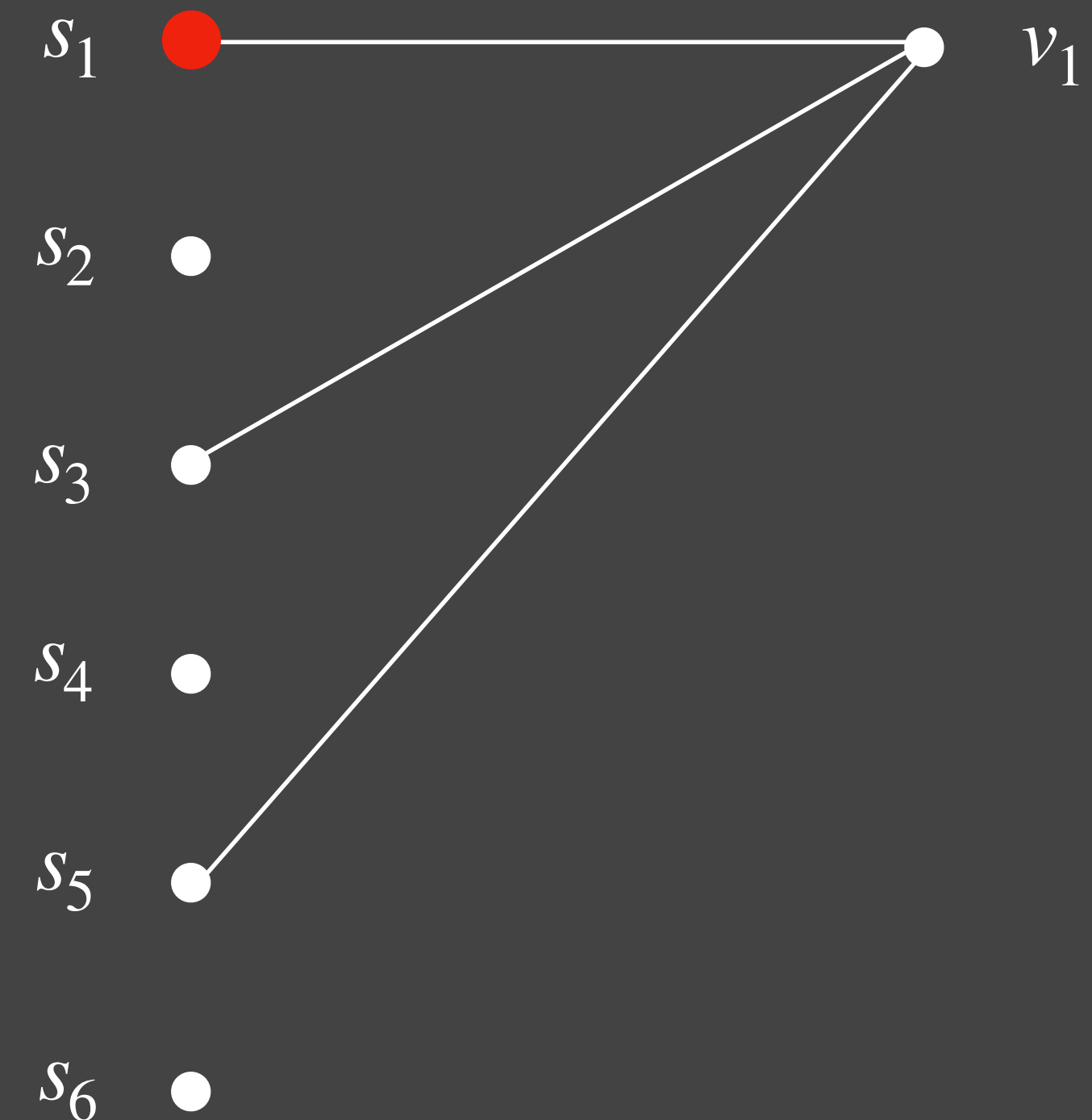
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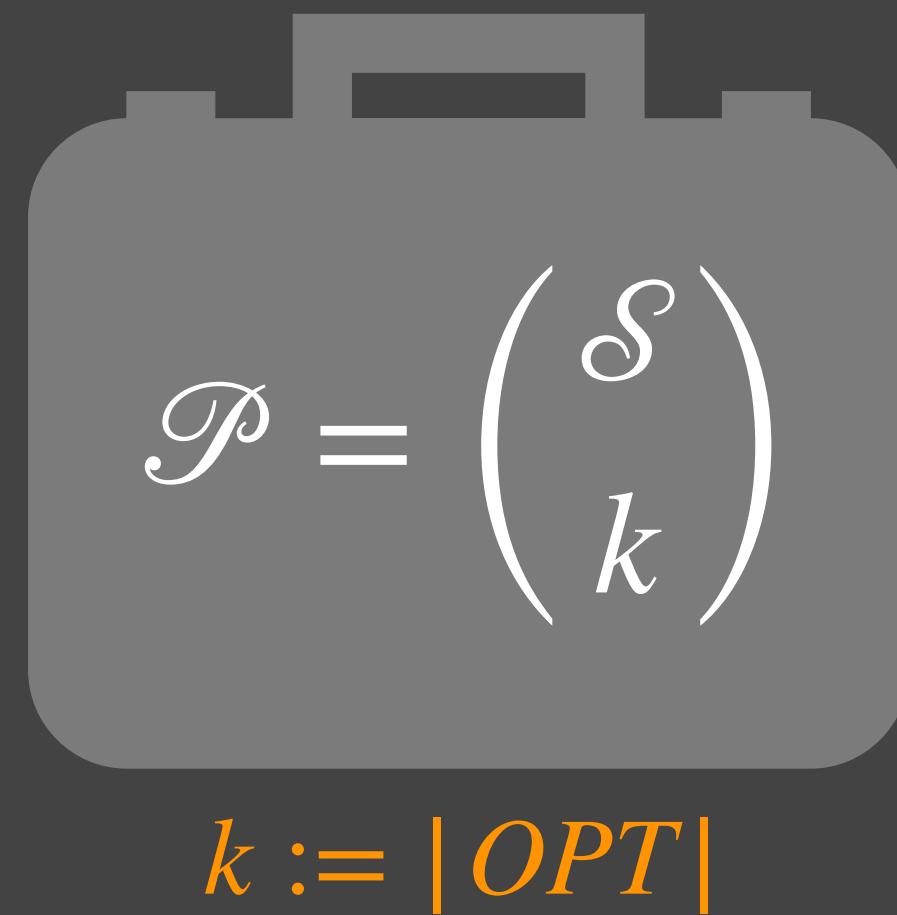
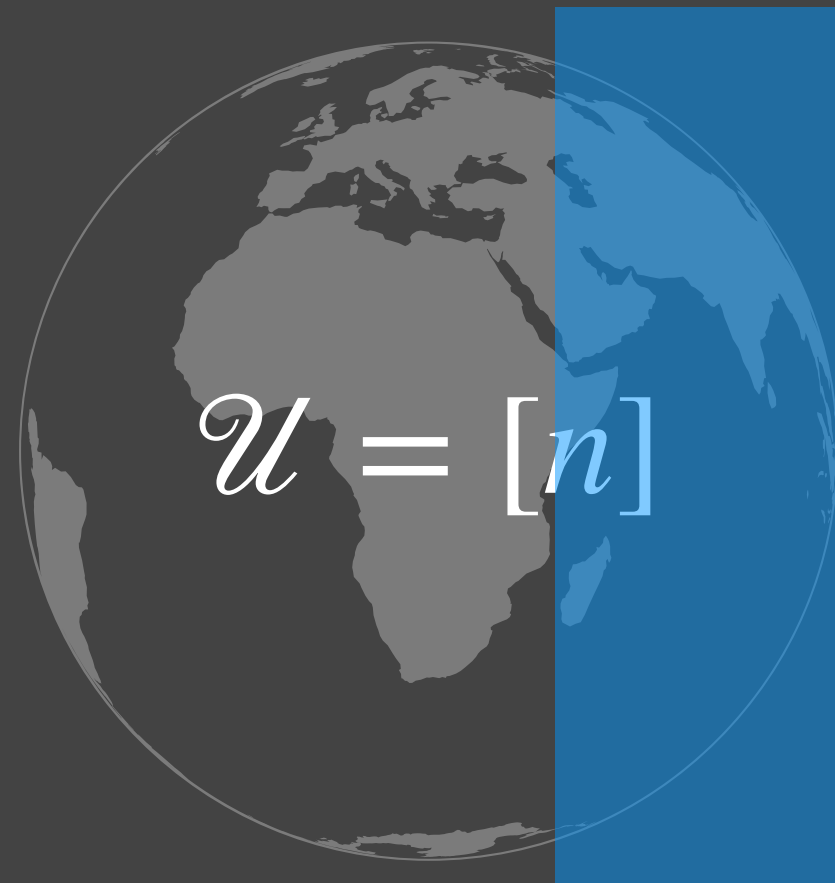
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LearnOrCover

Proof idea: progress **learning** or **covering**.



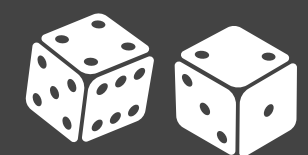
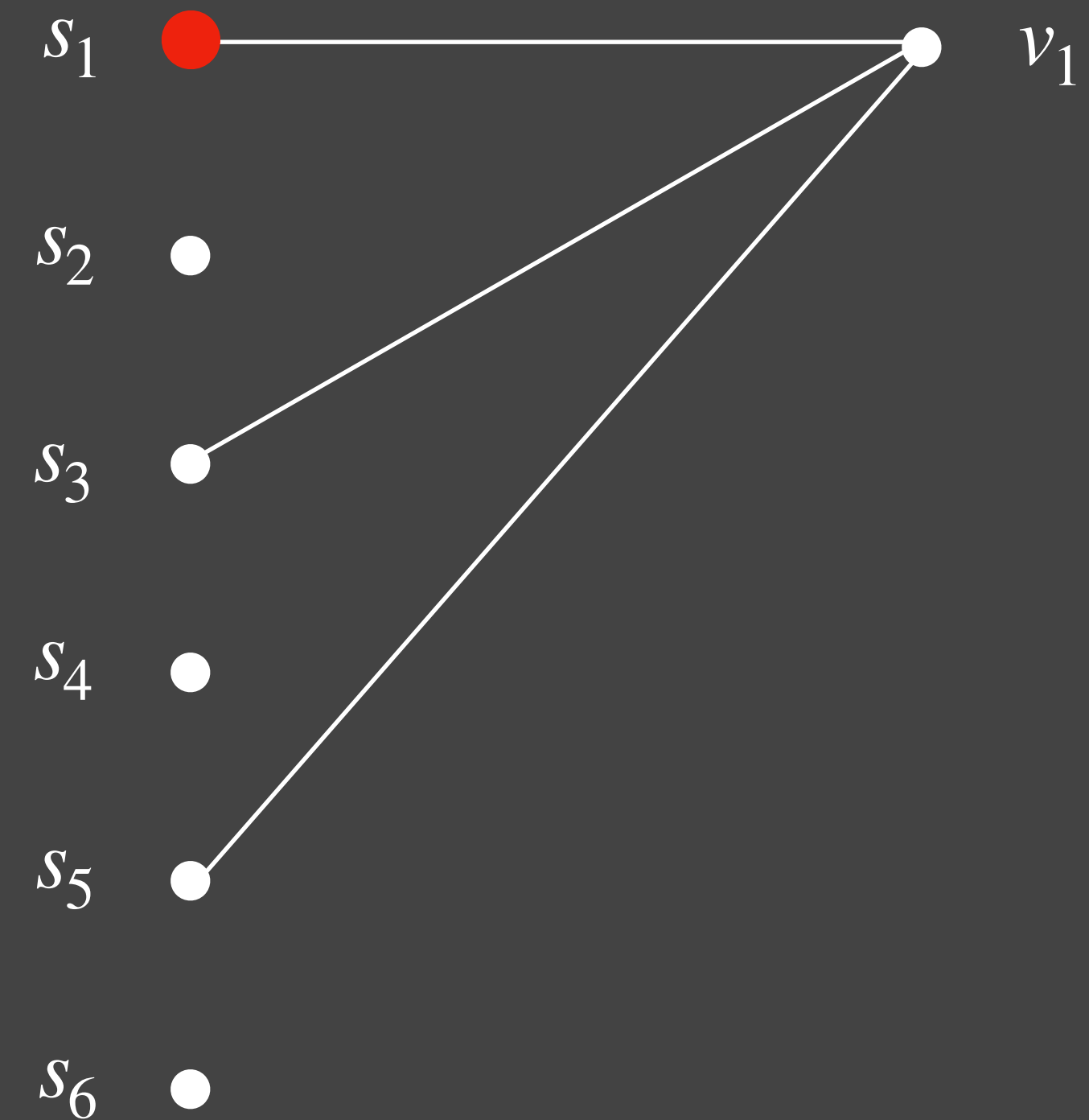
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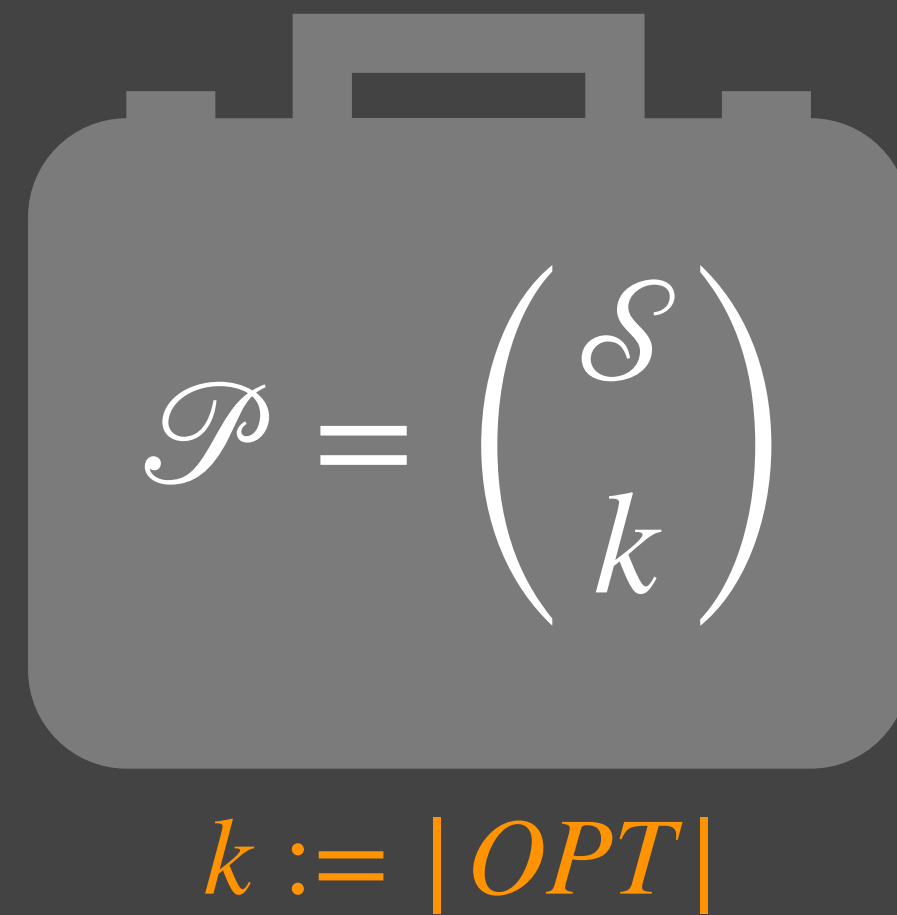
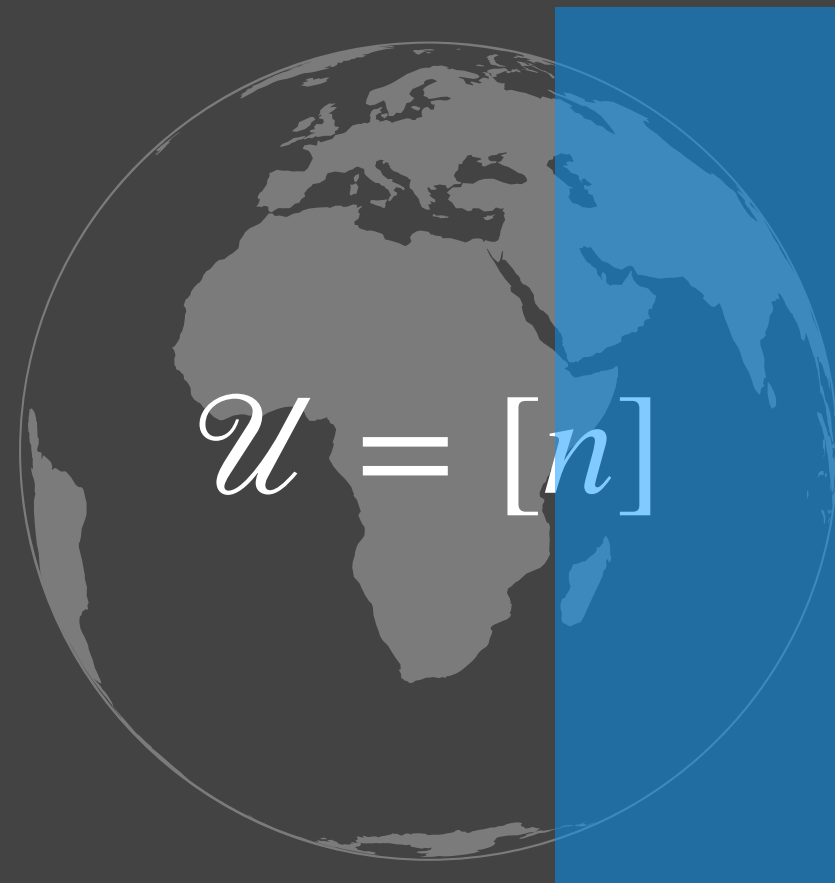
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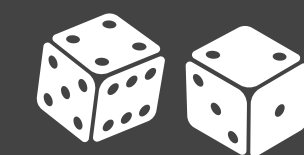
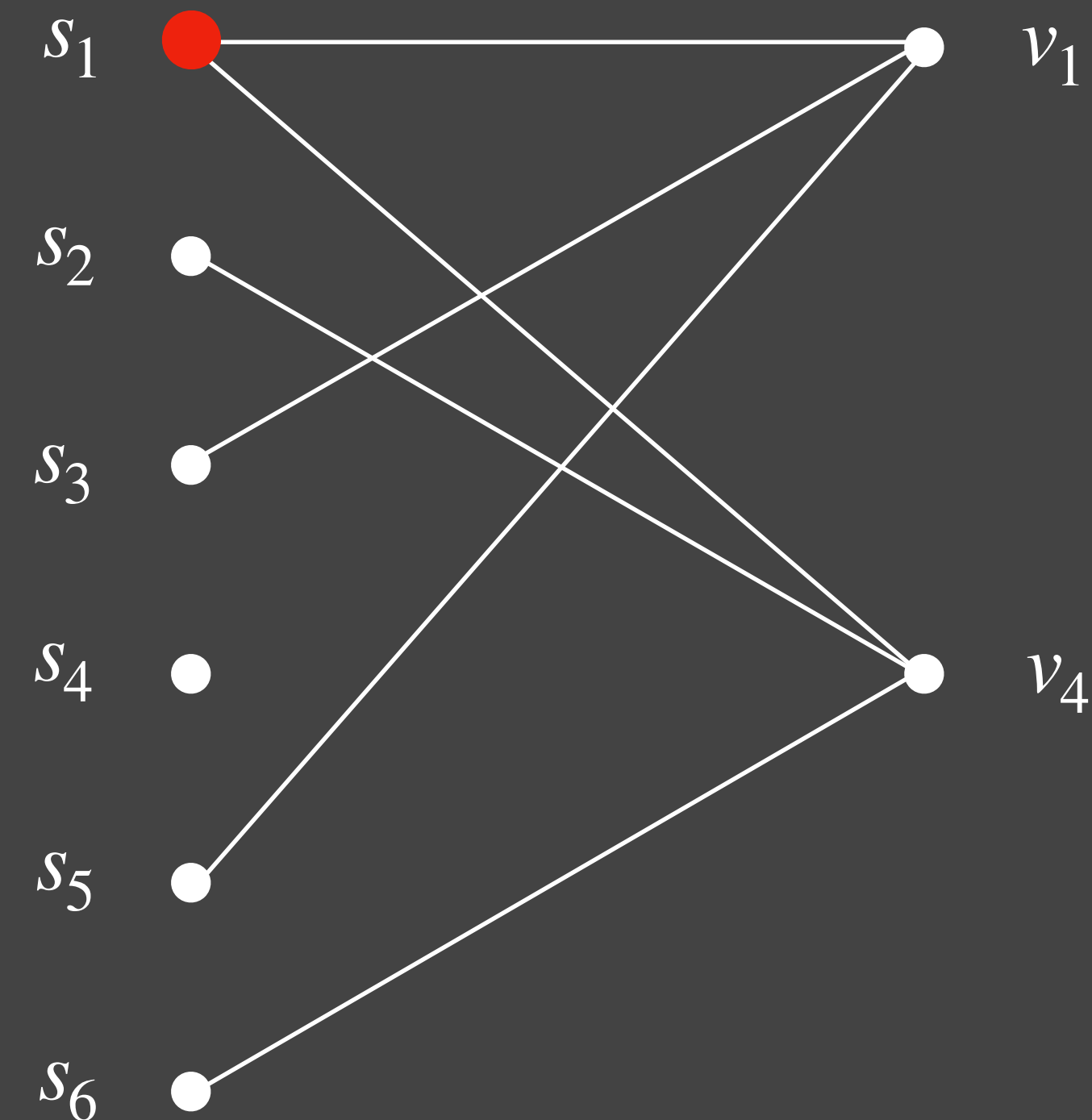
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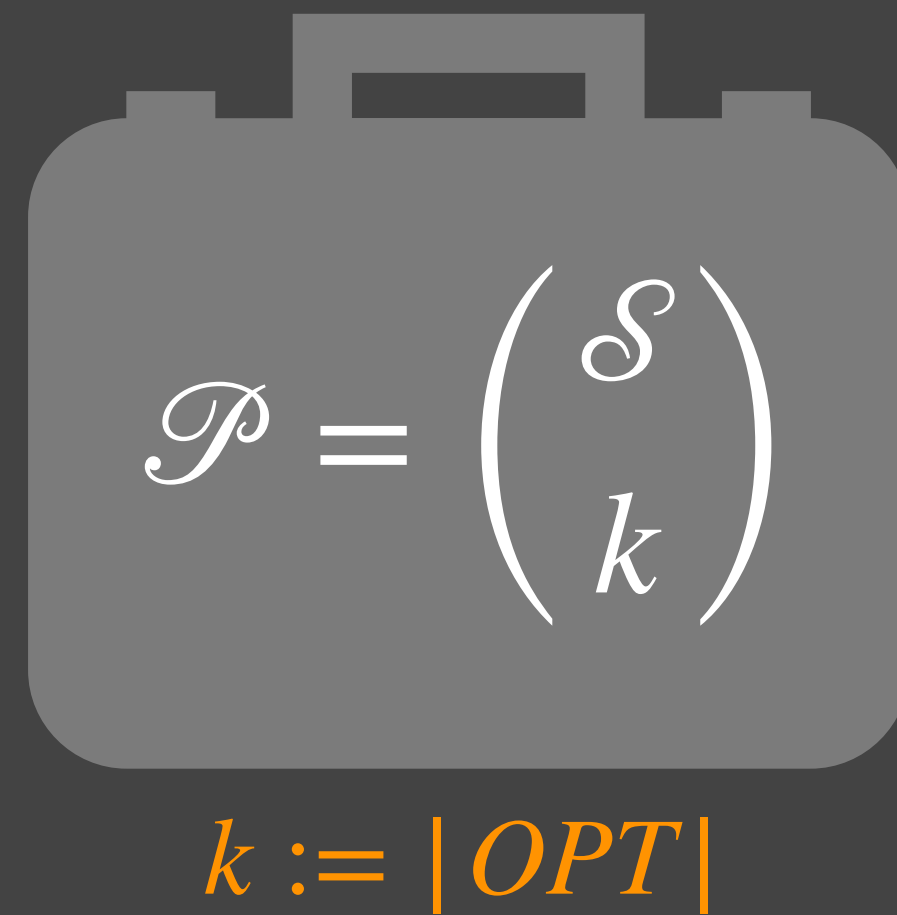
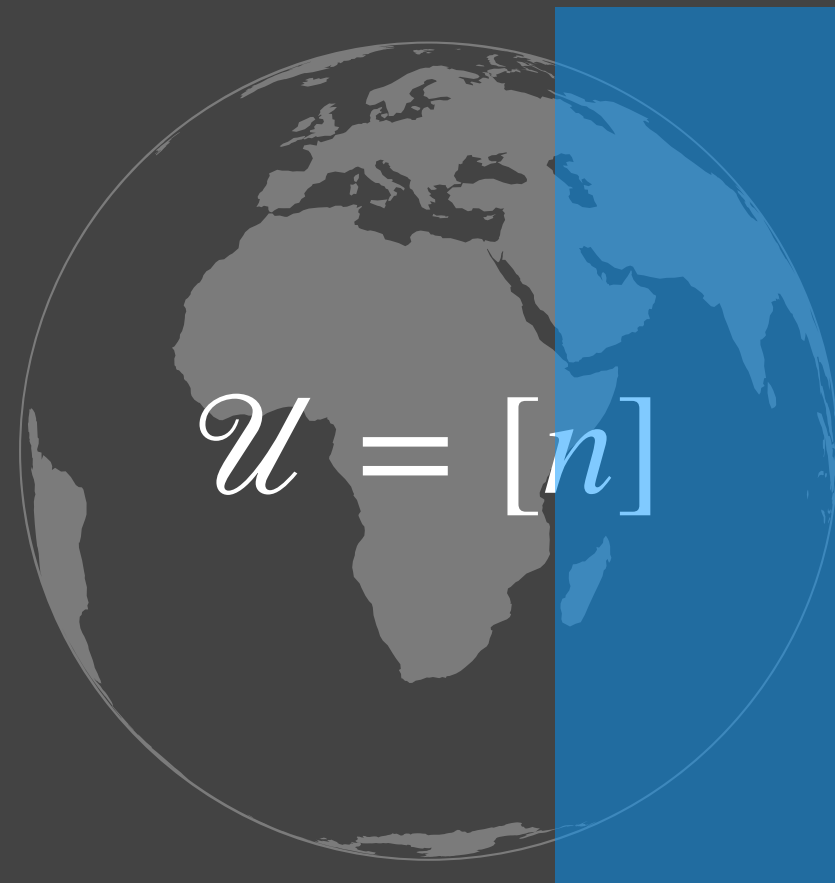
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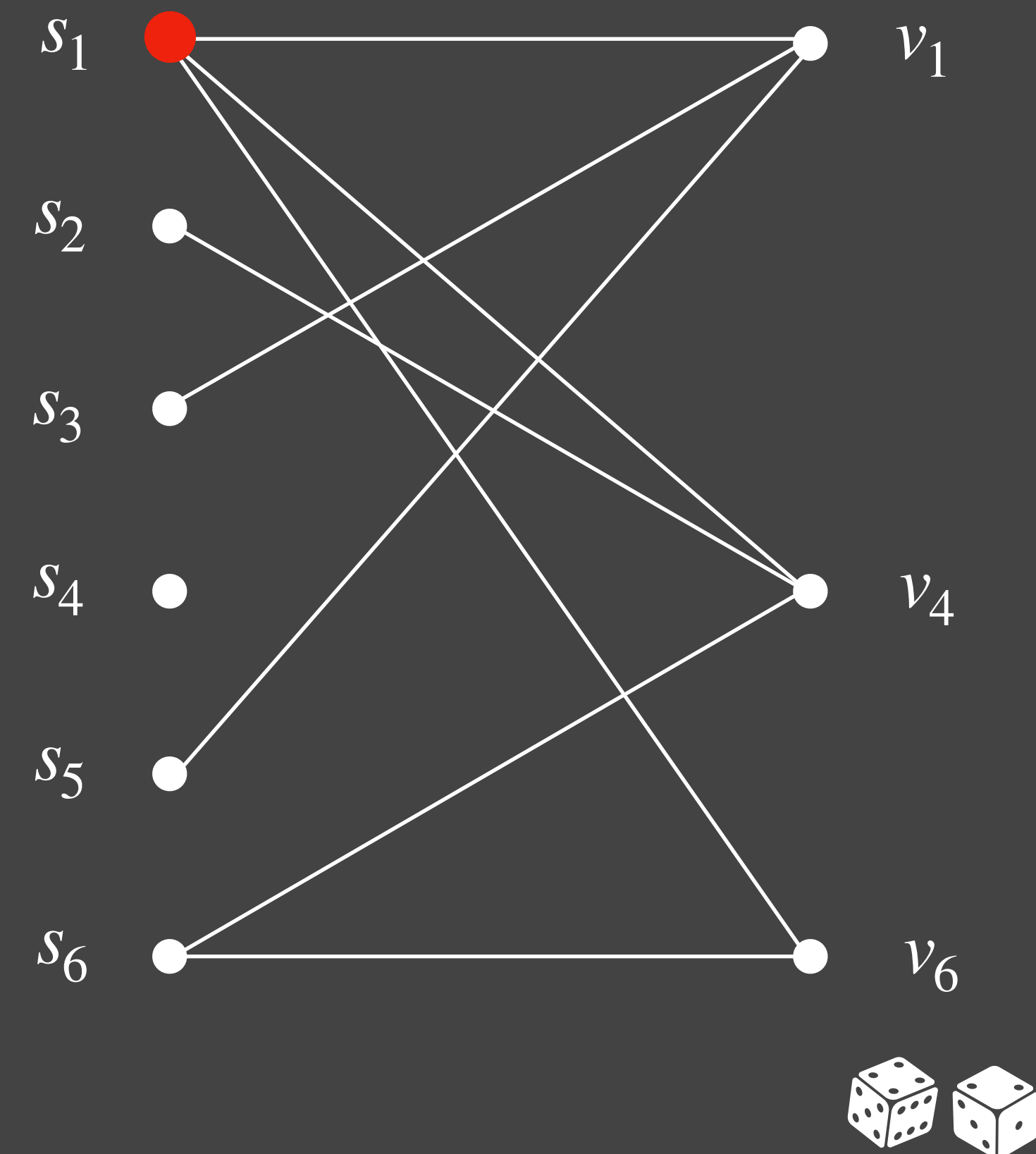
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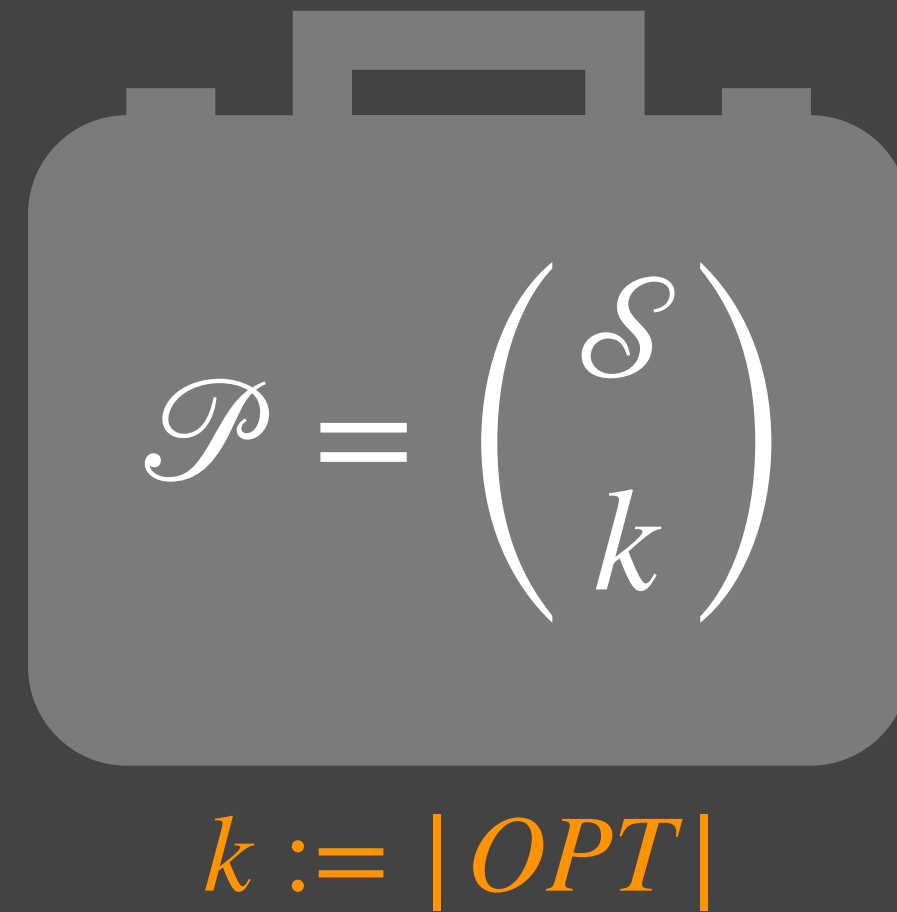
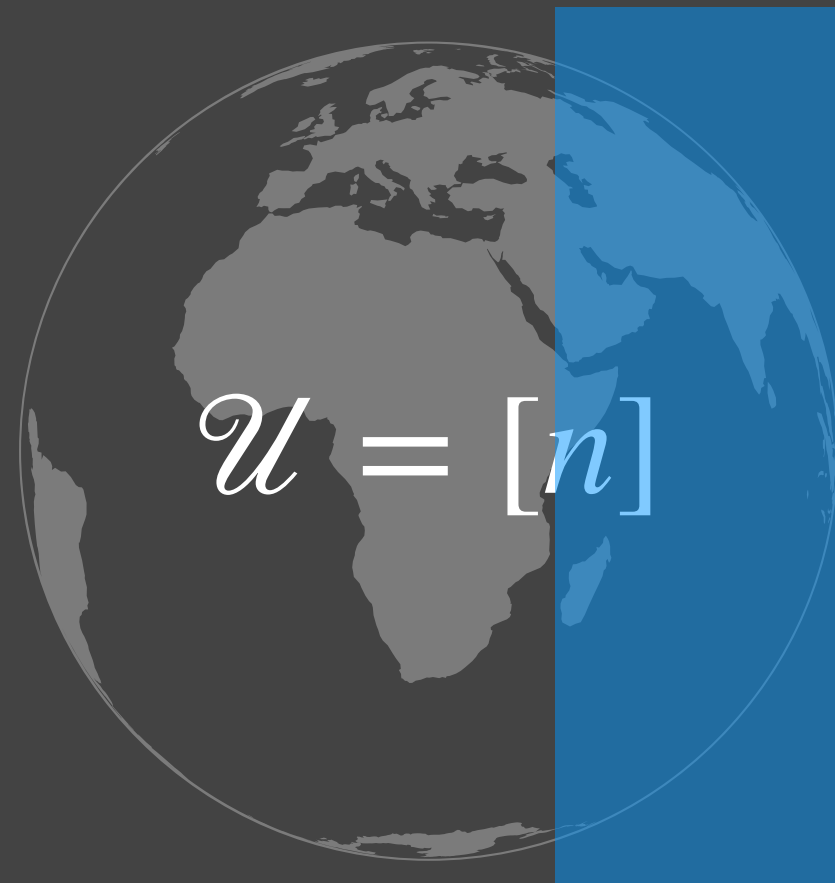
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LearnOrCover

Proof idea: progress **learning** or **covering**.



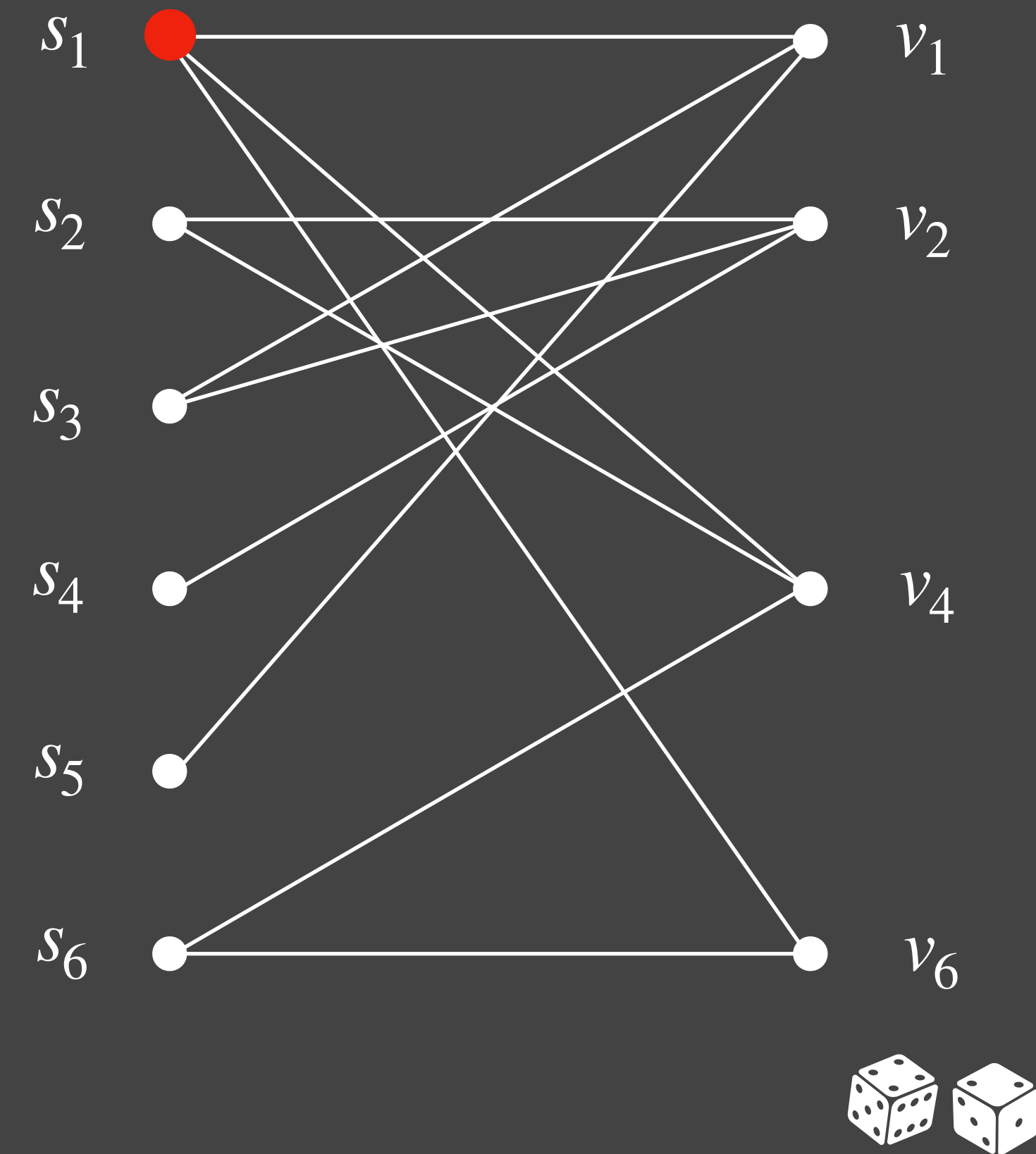
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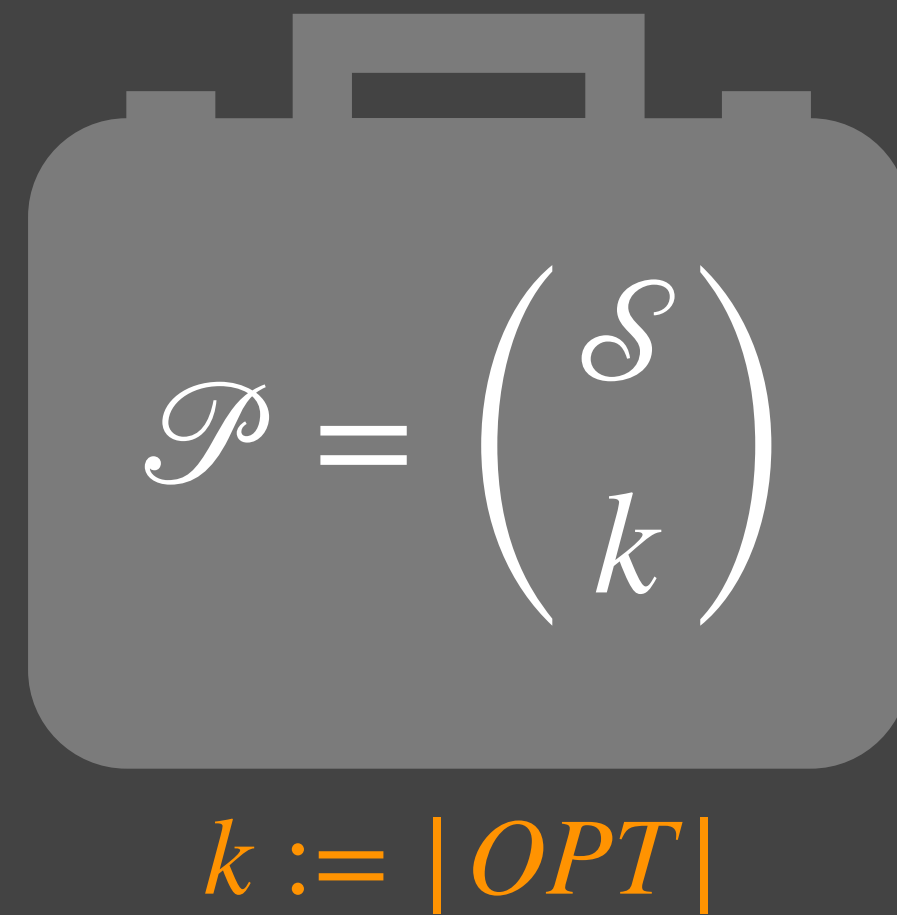
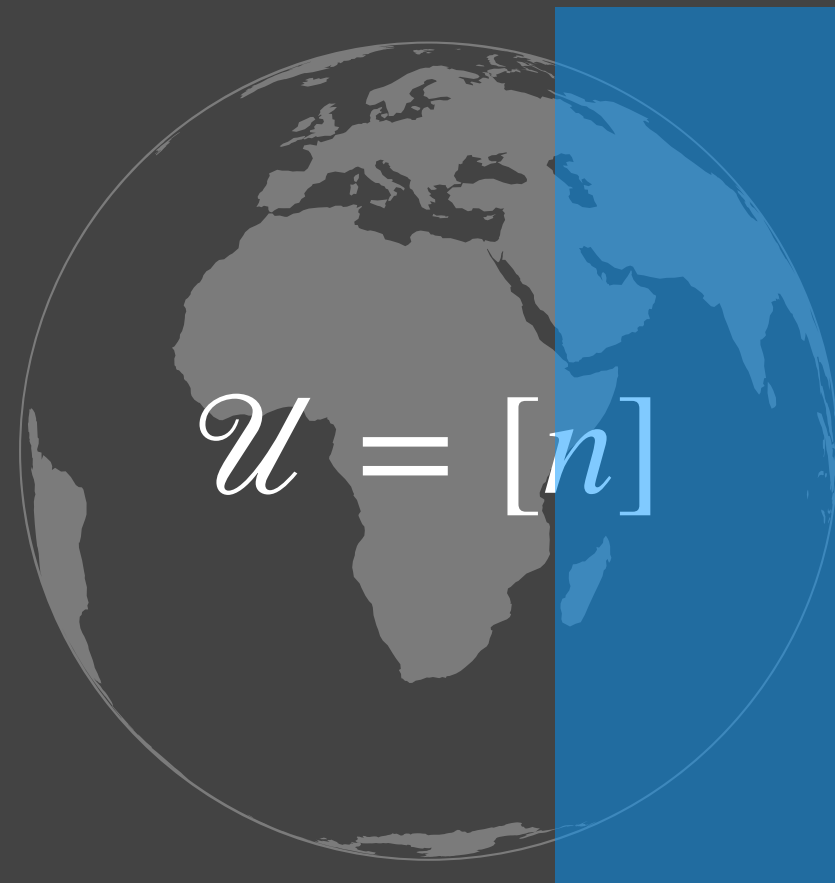
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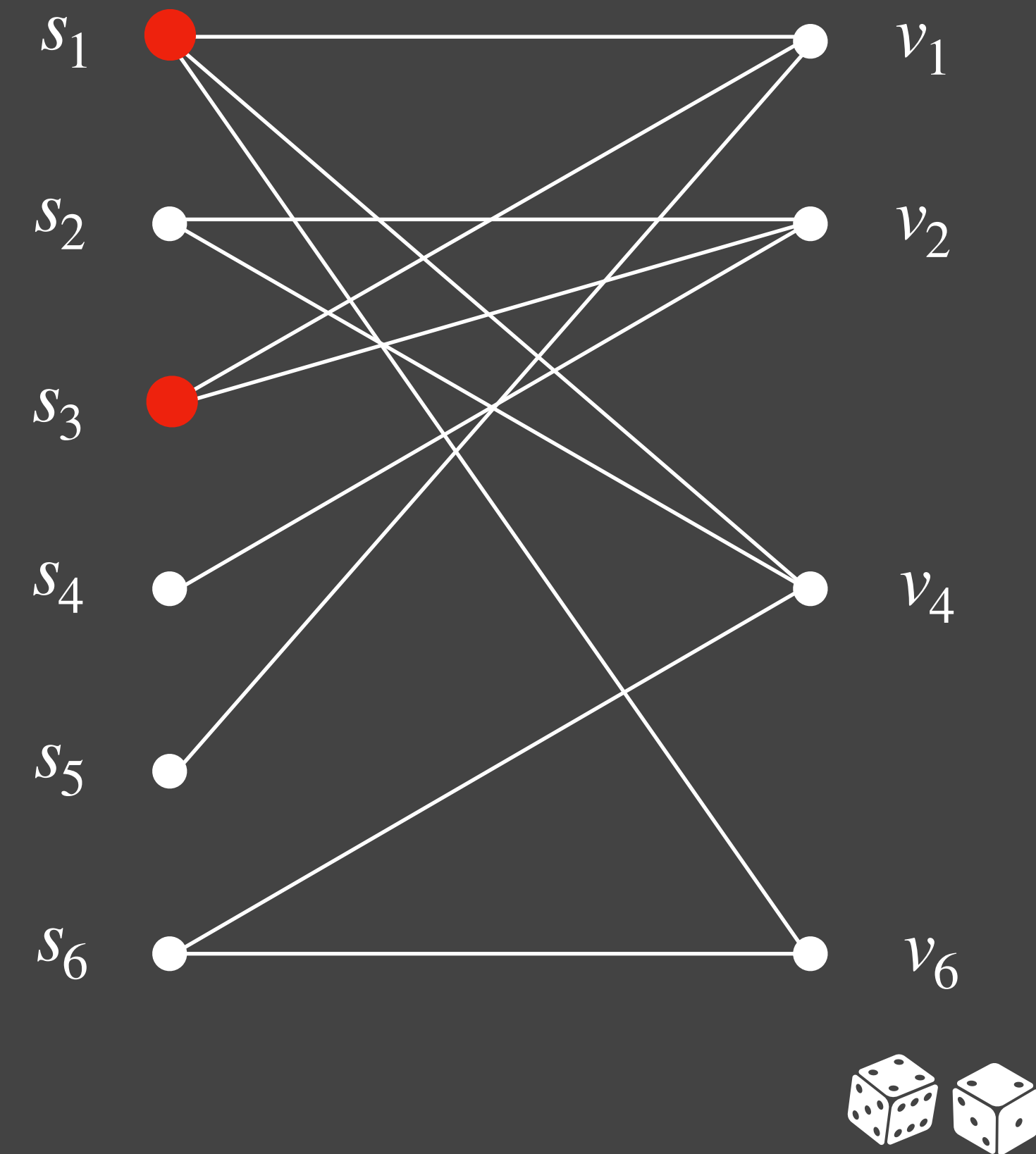
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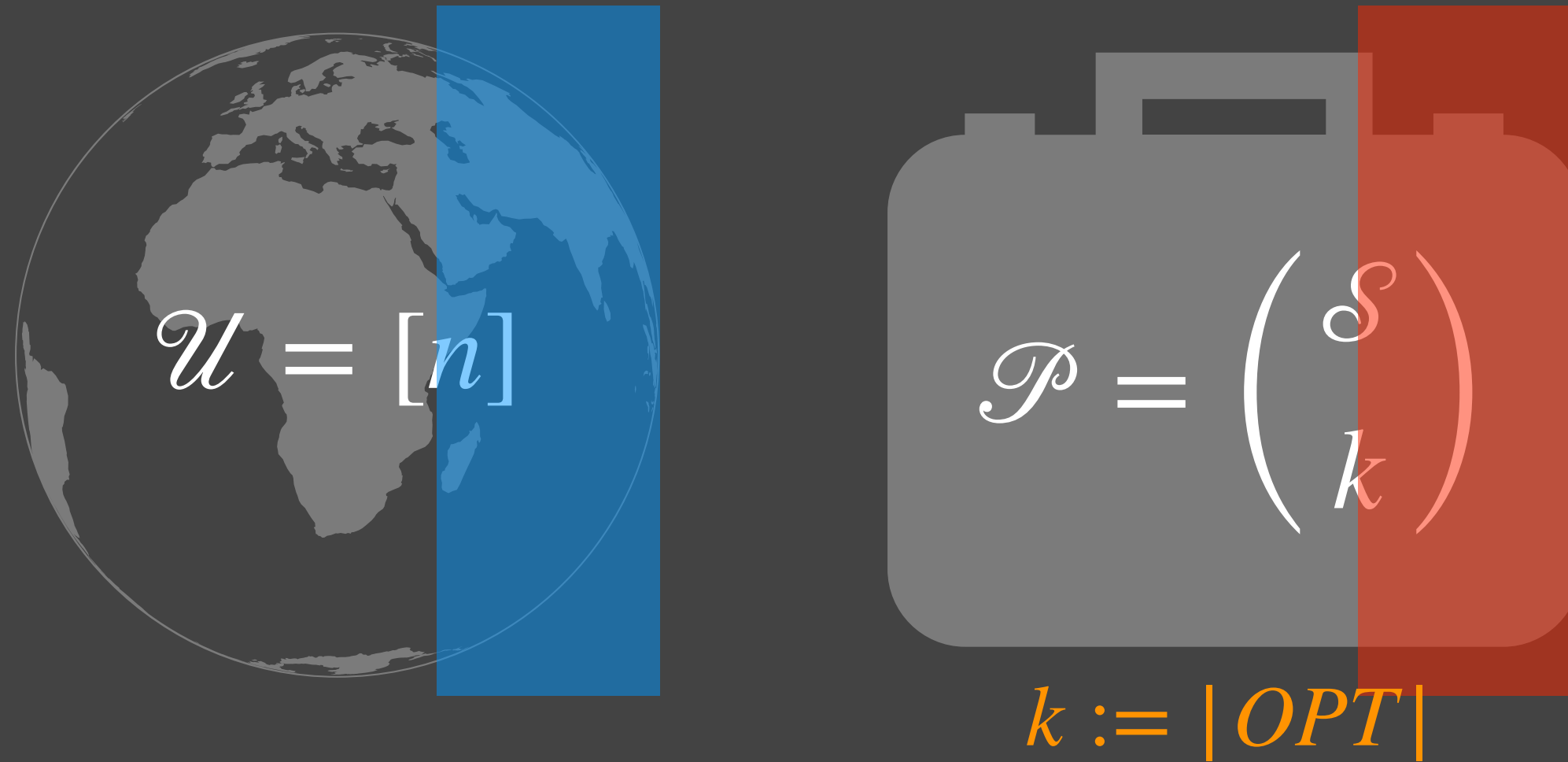
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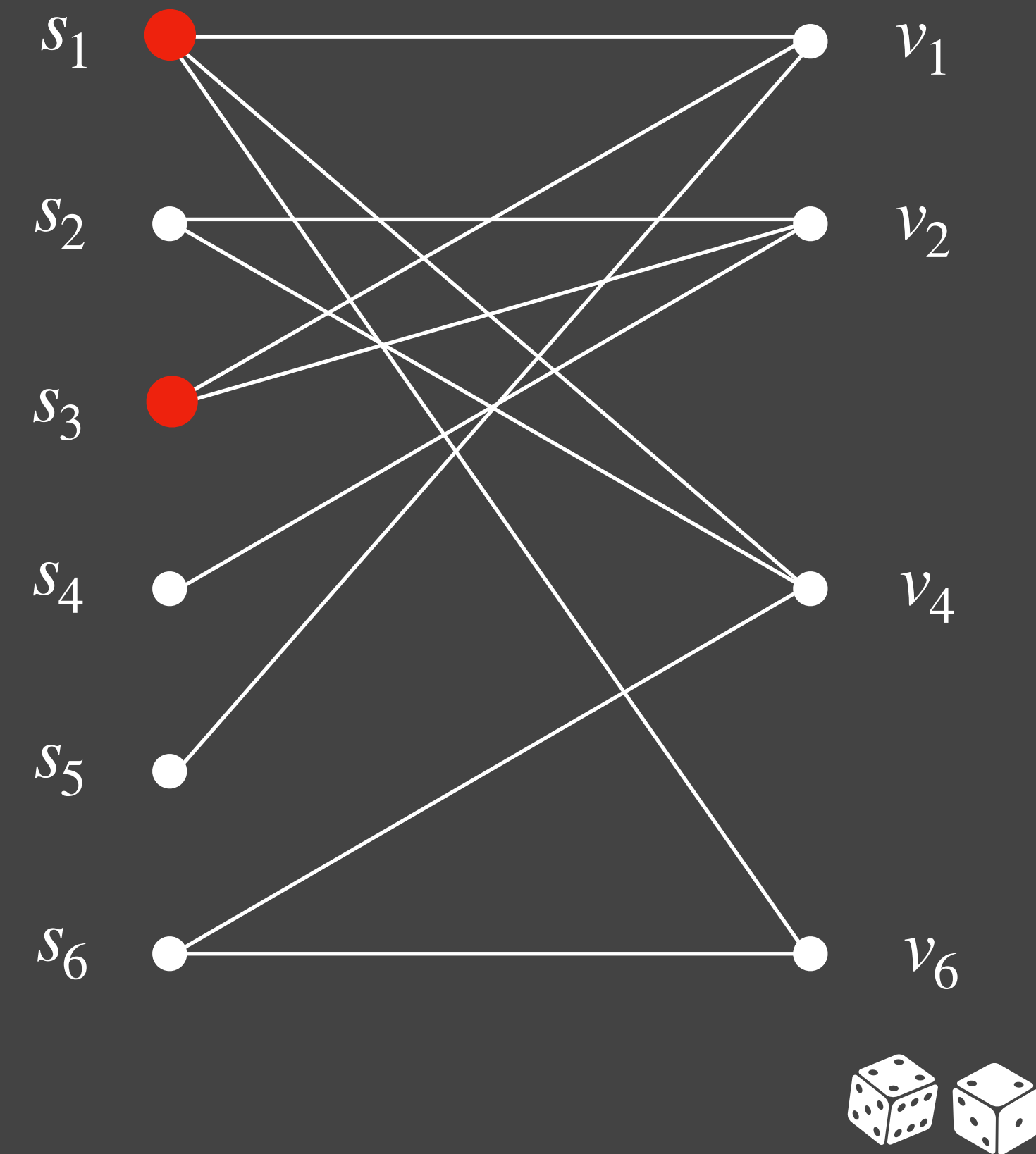
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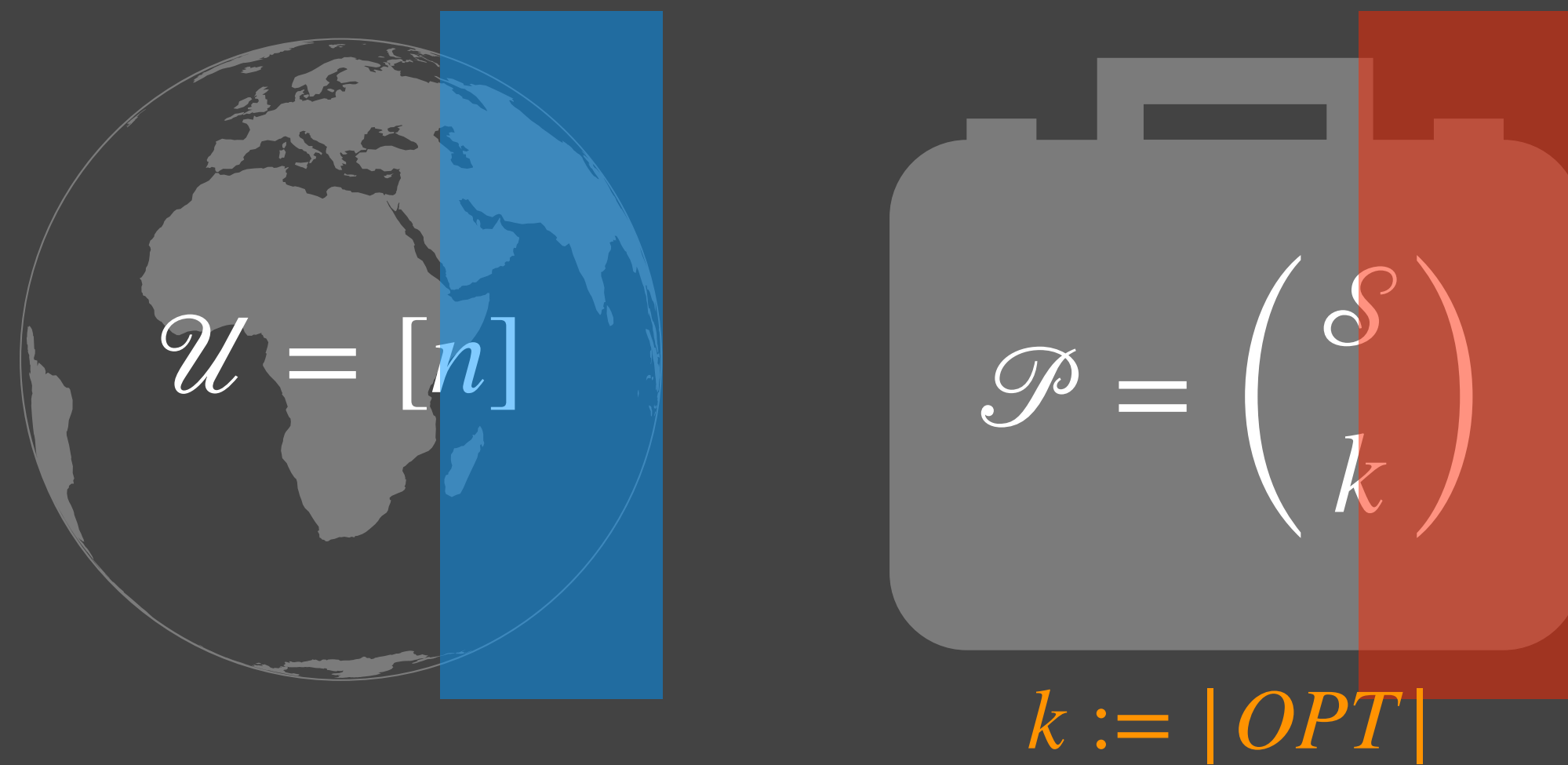
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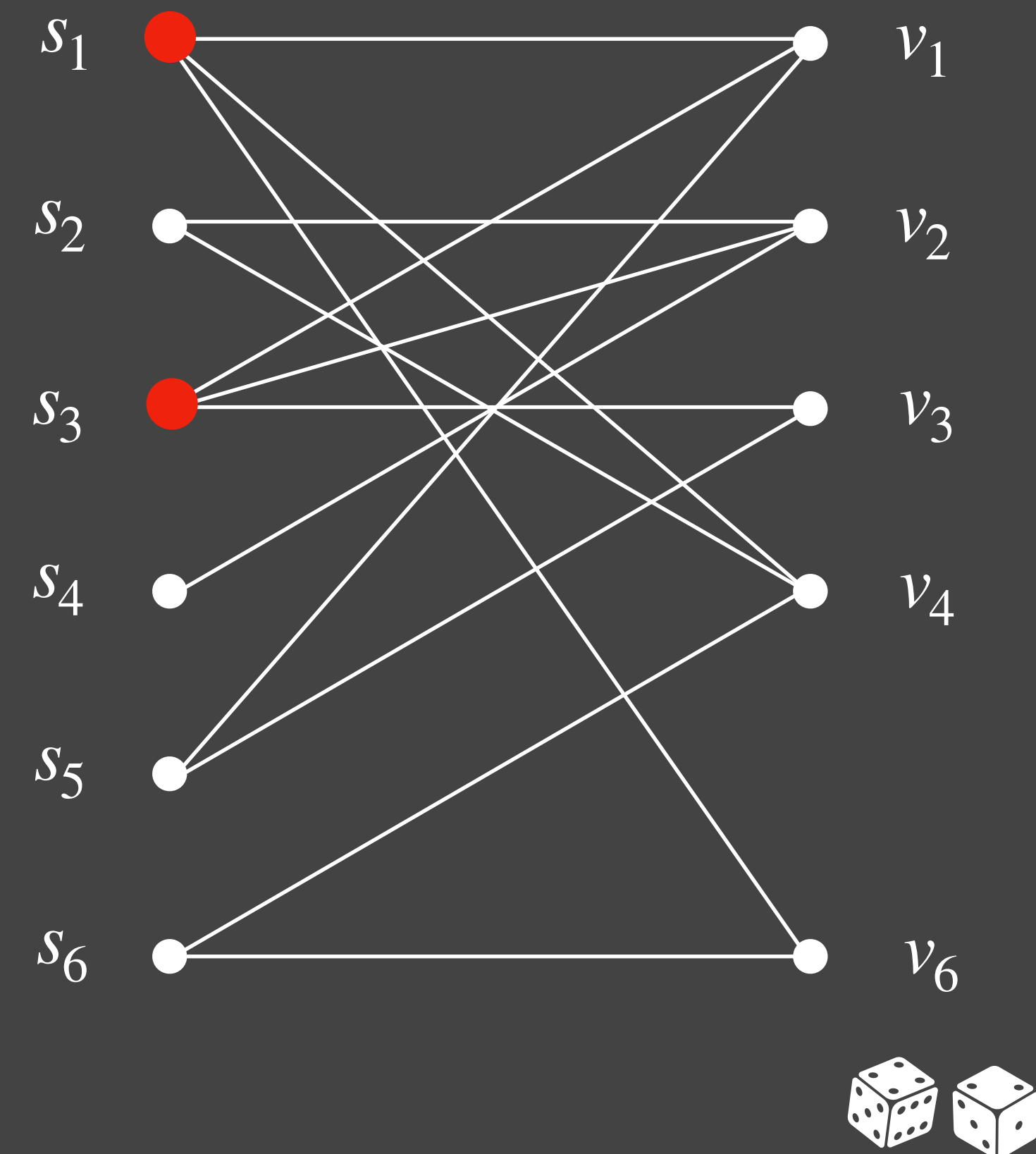
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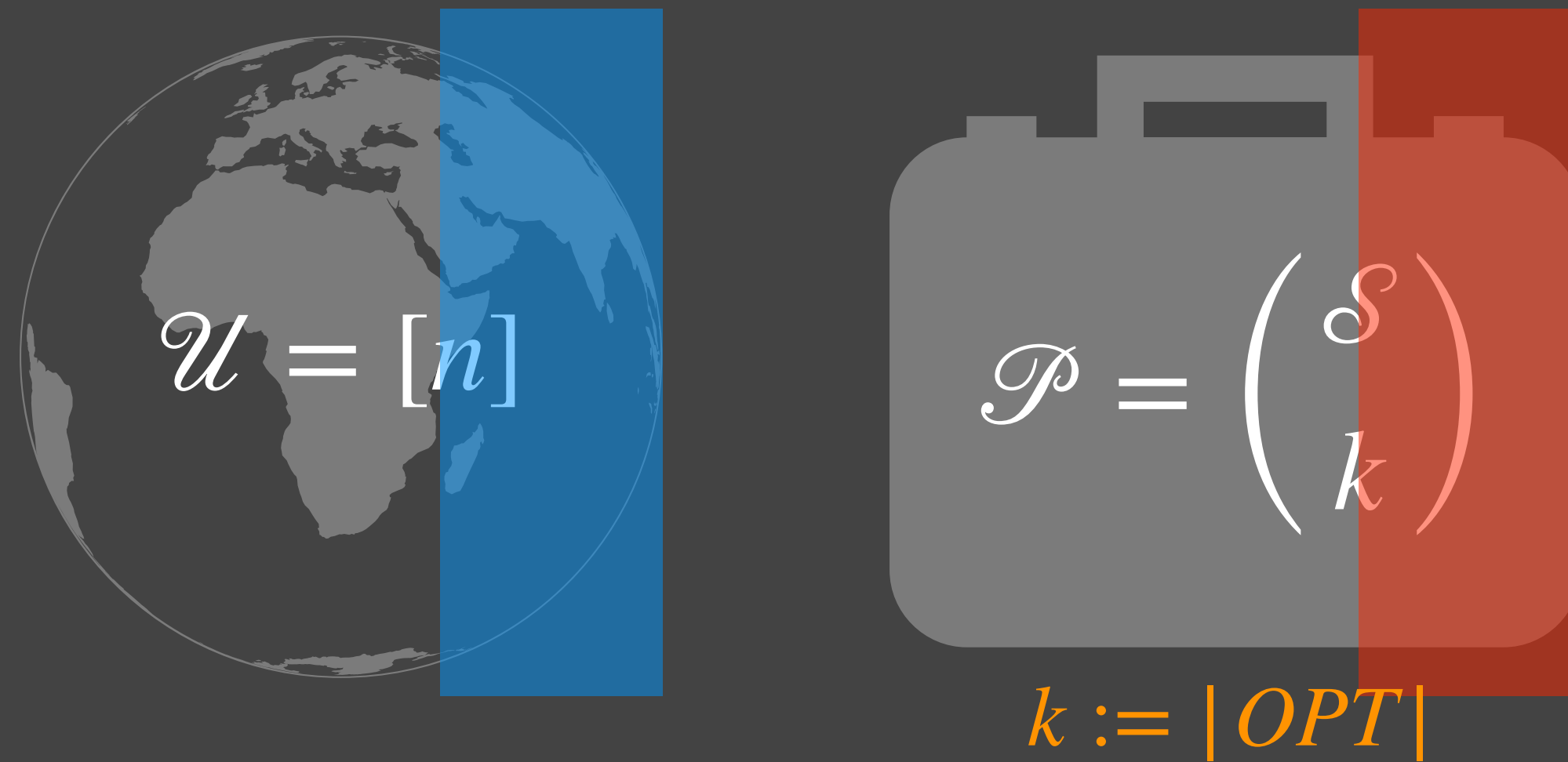
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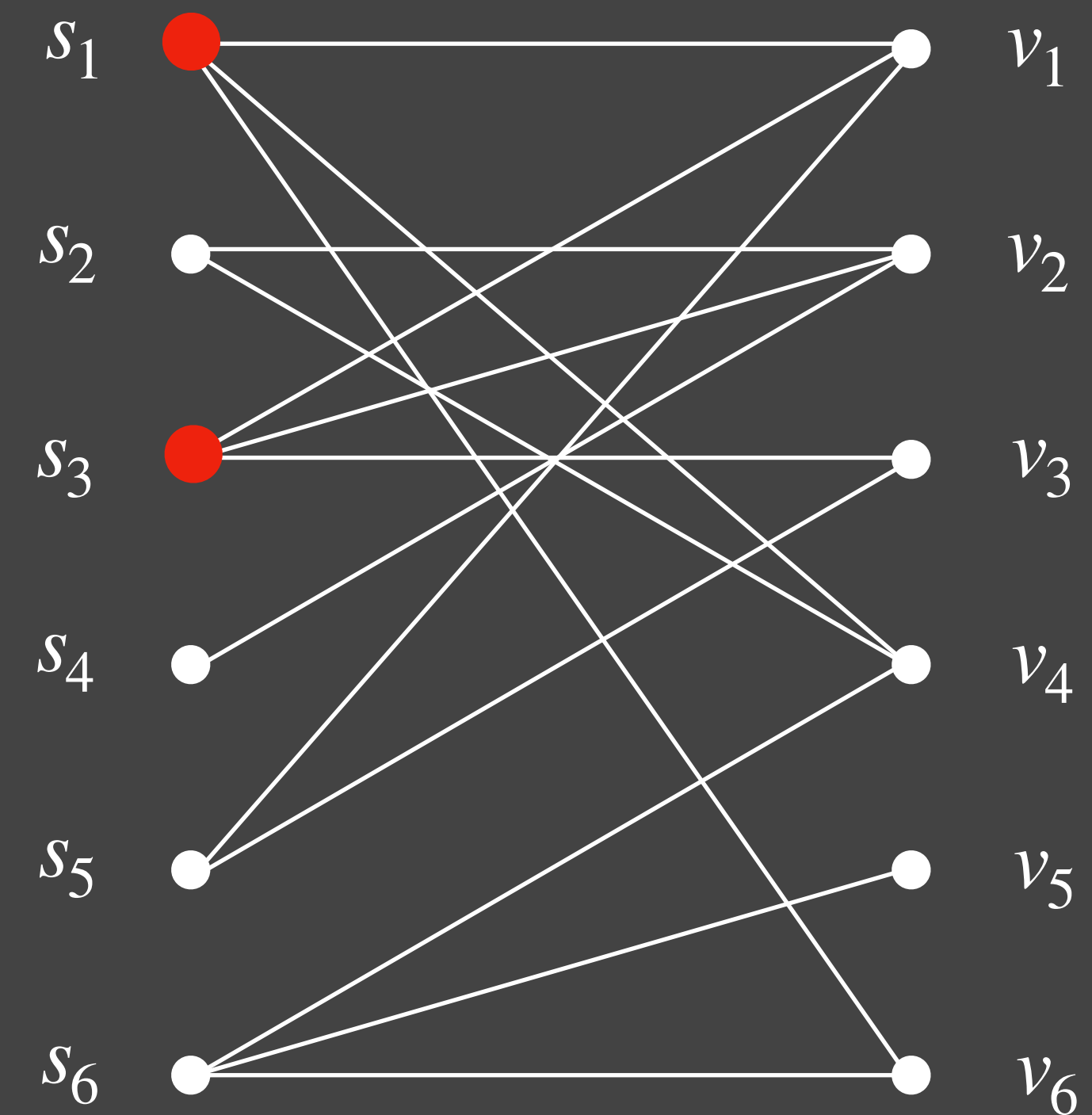
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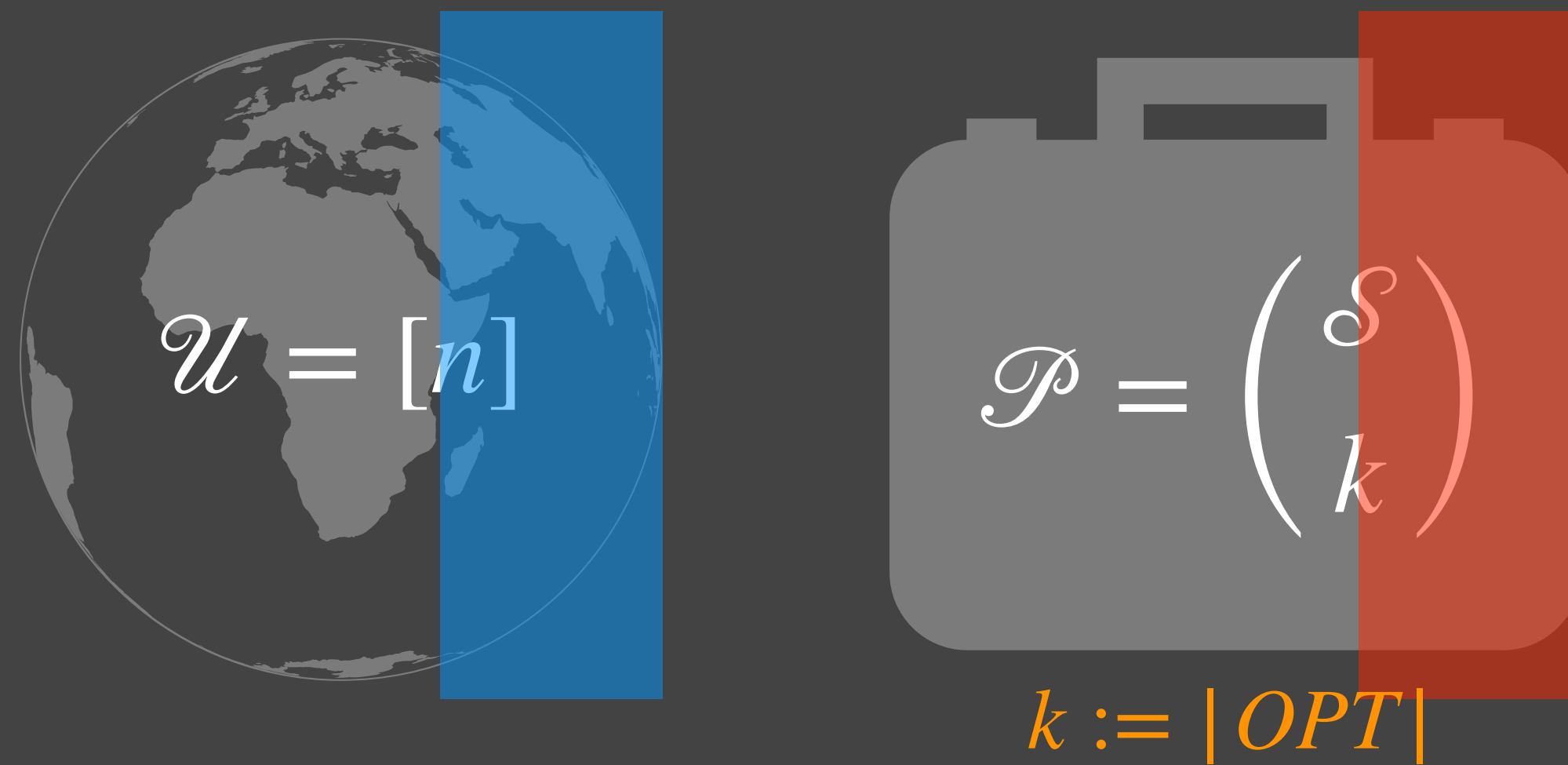
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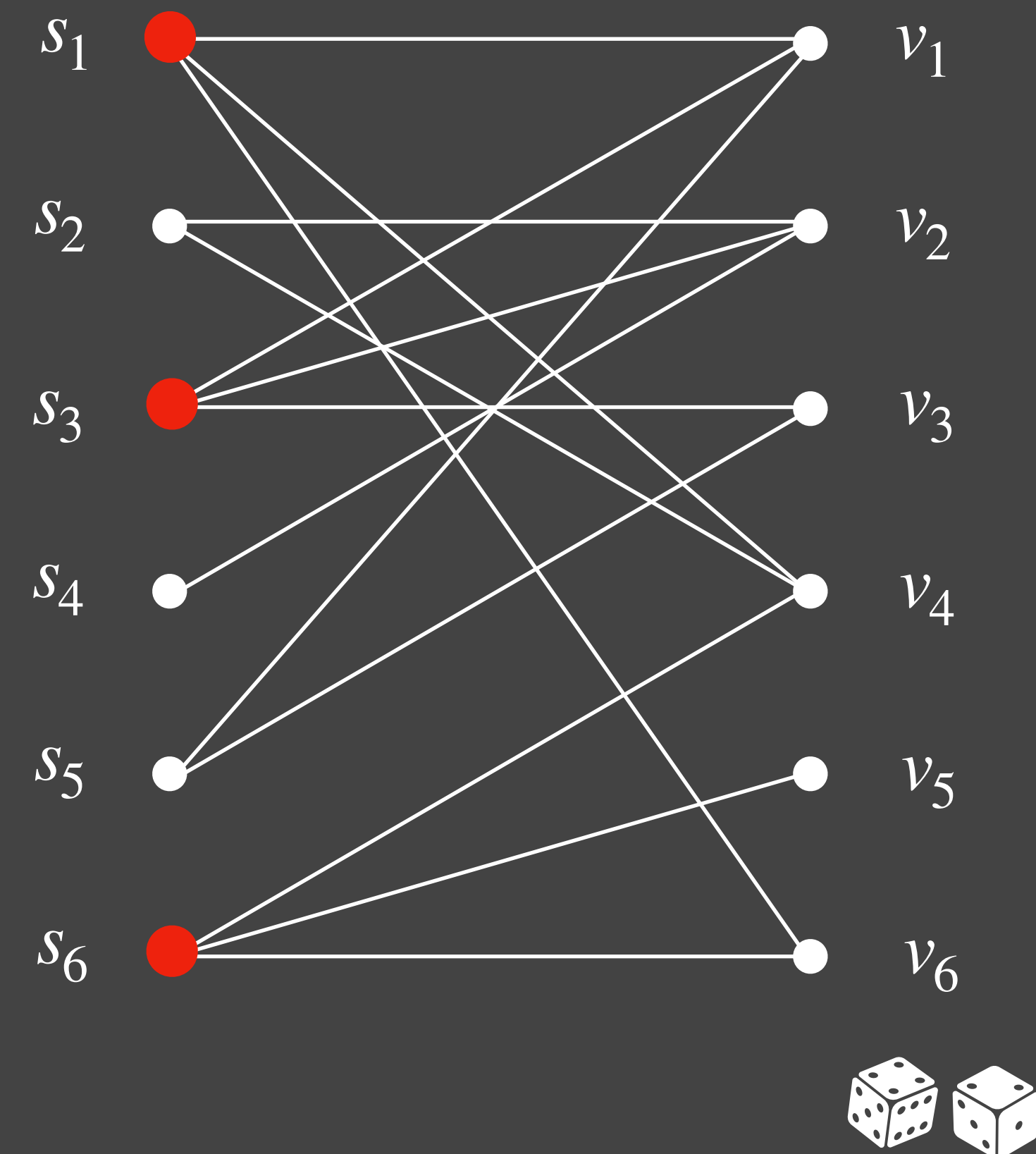
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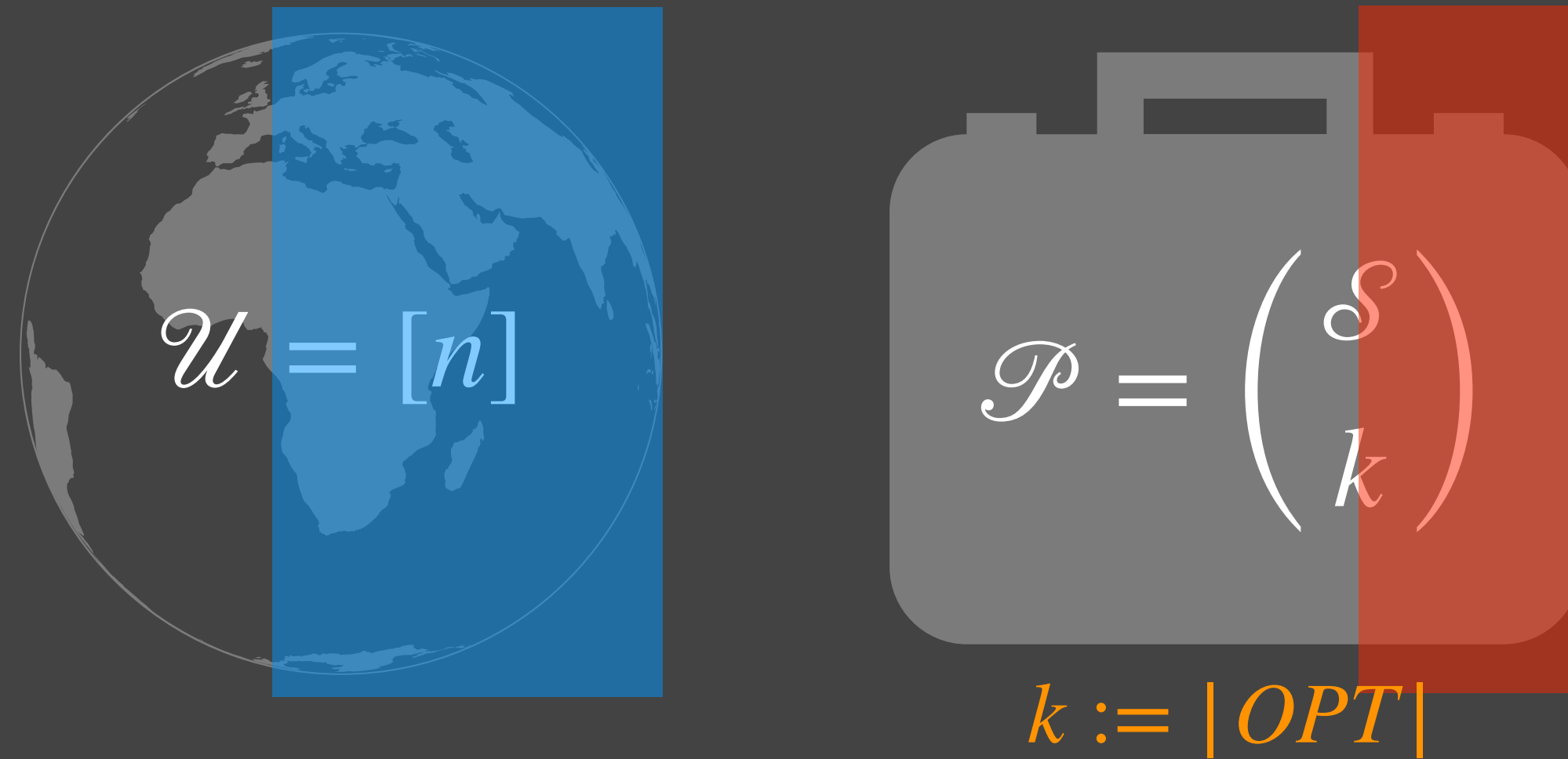
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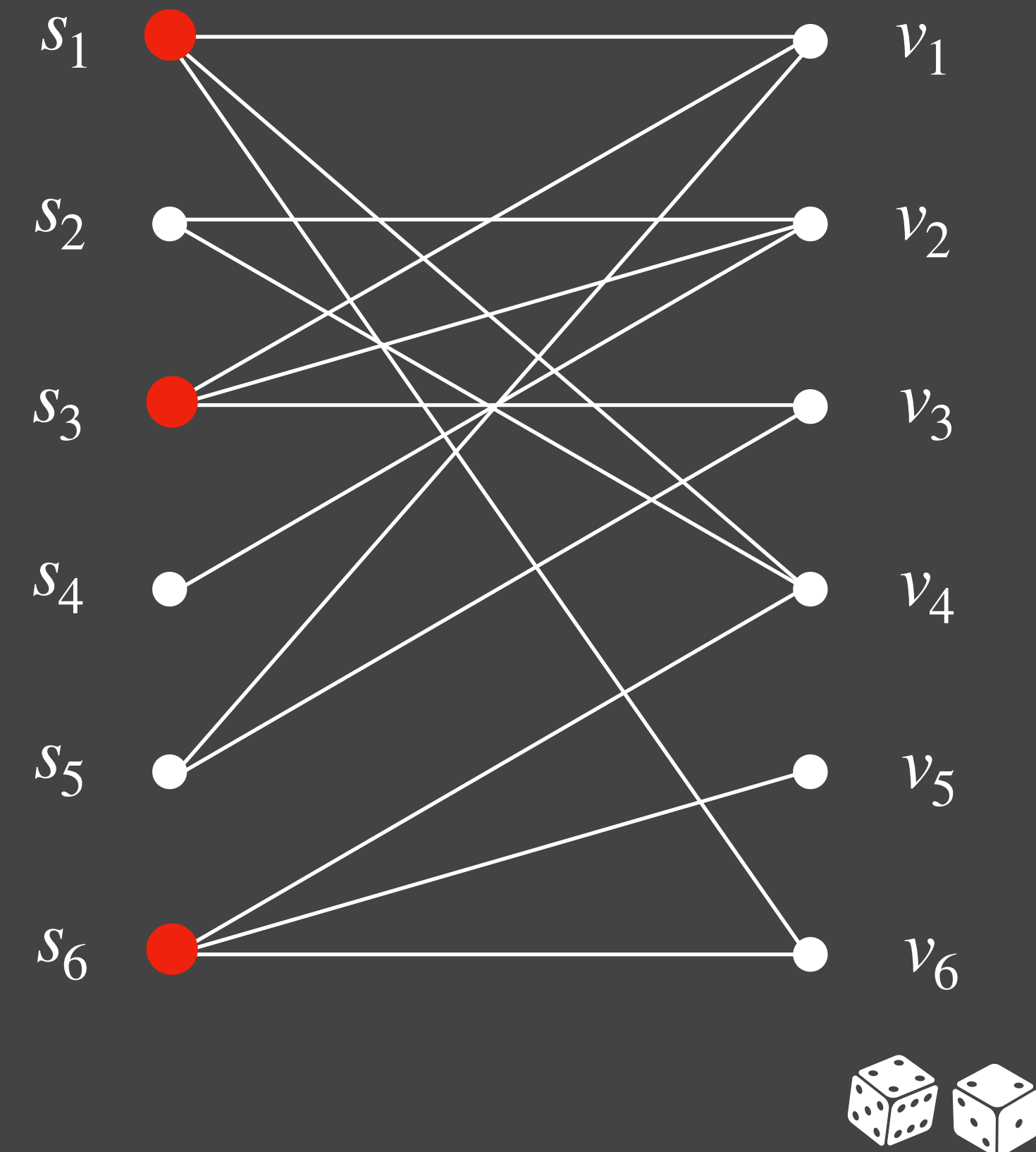
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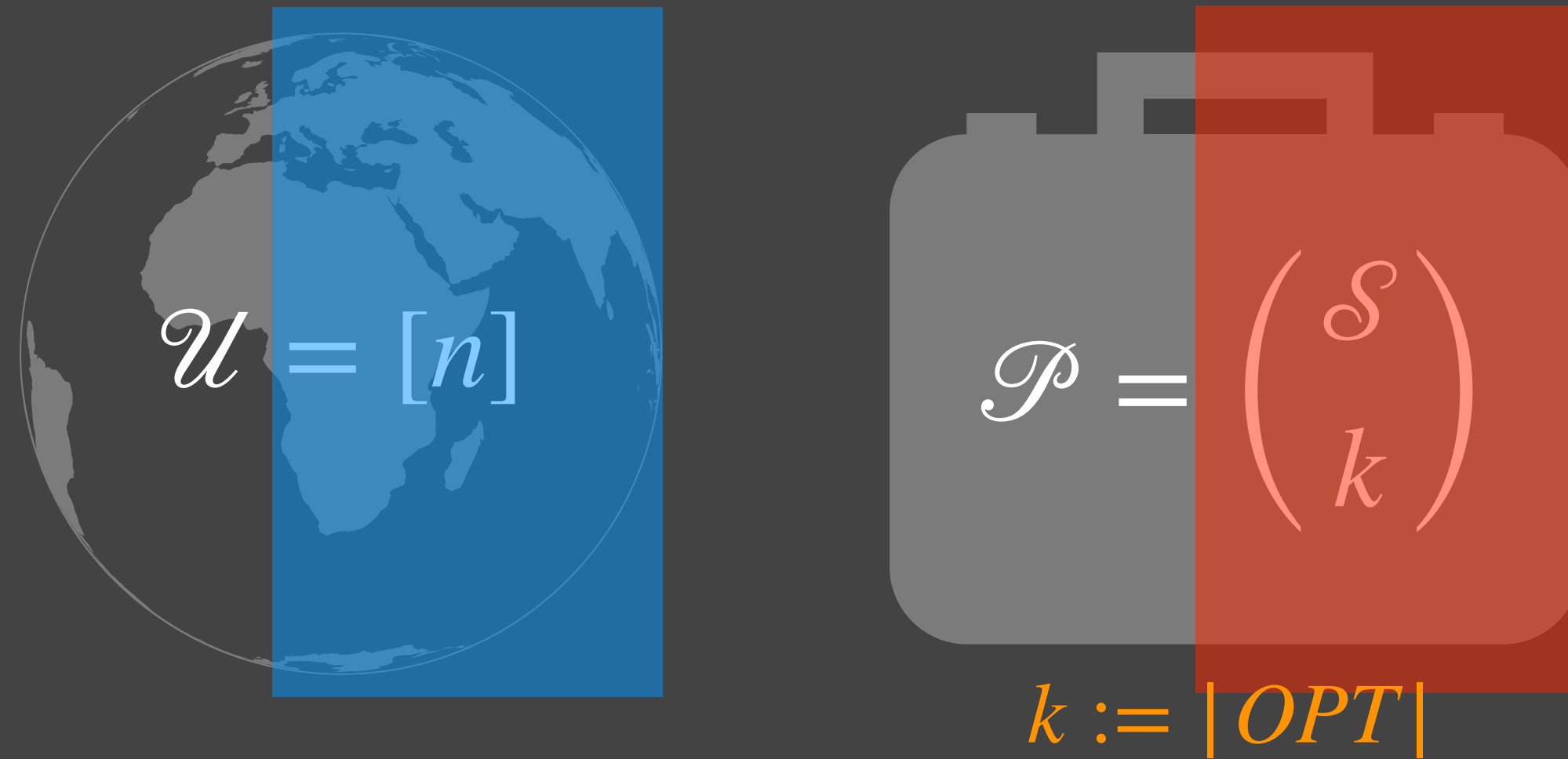
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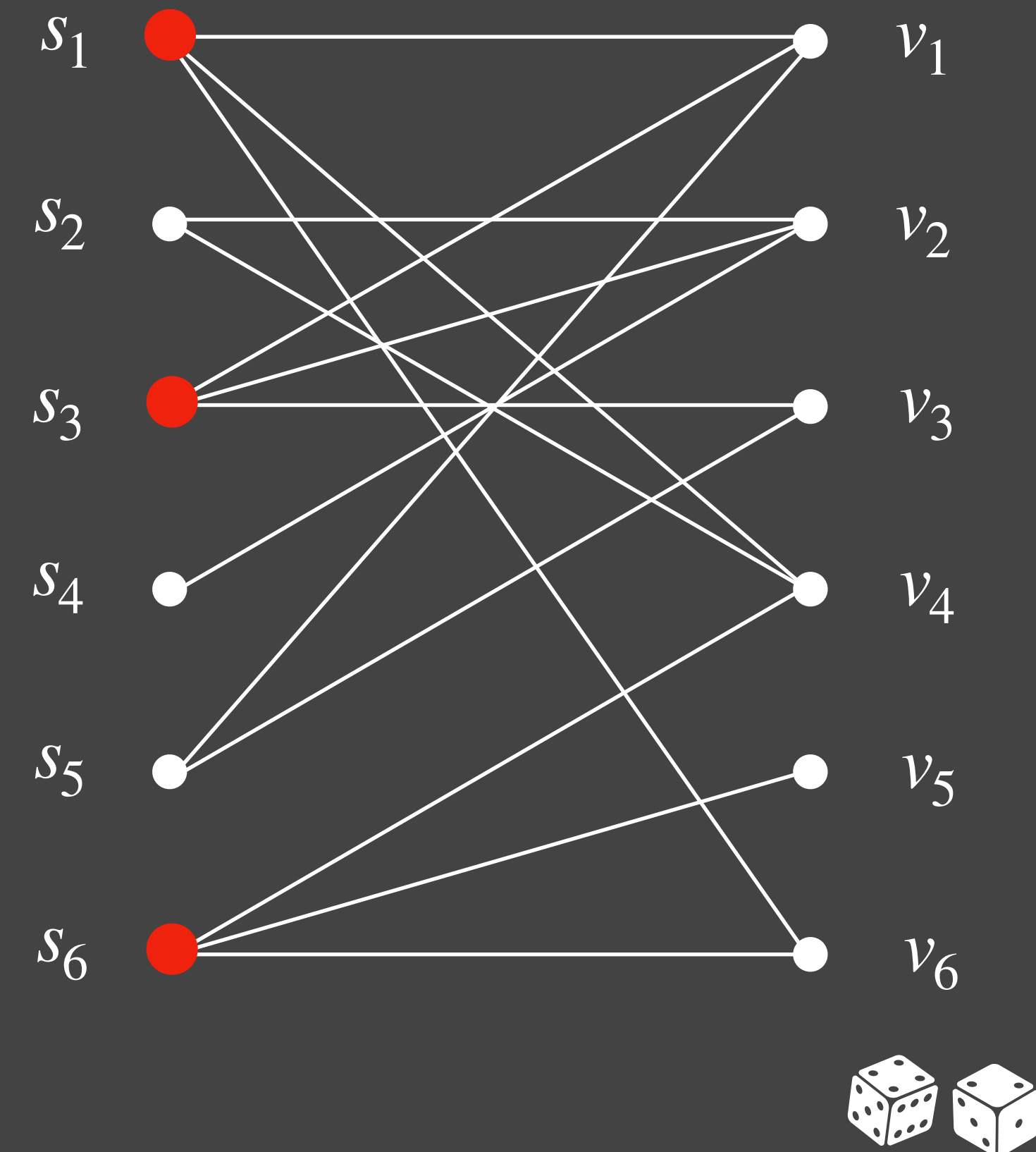
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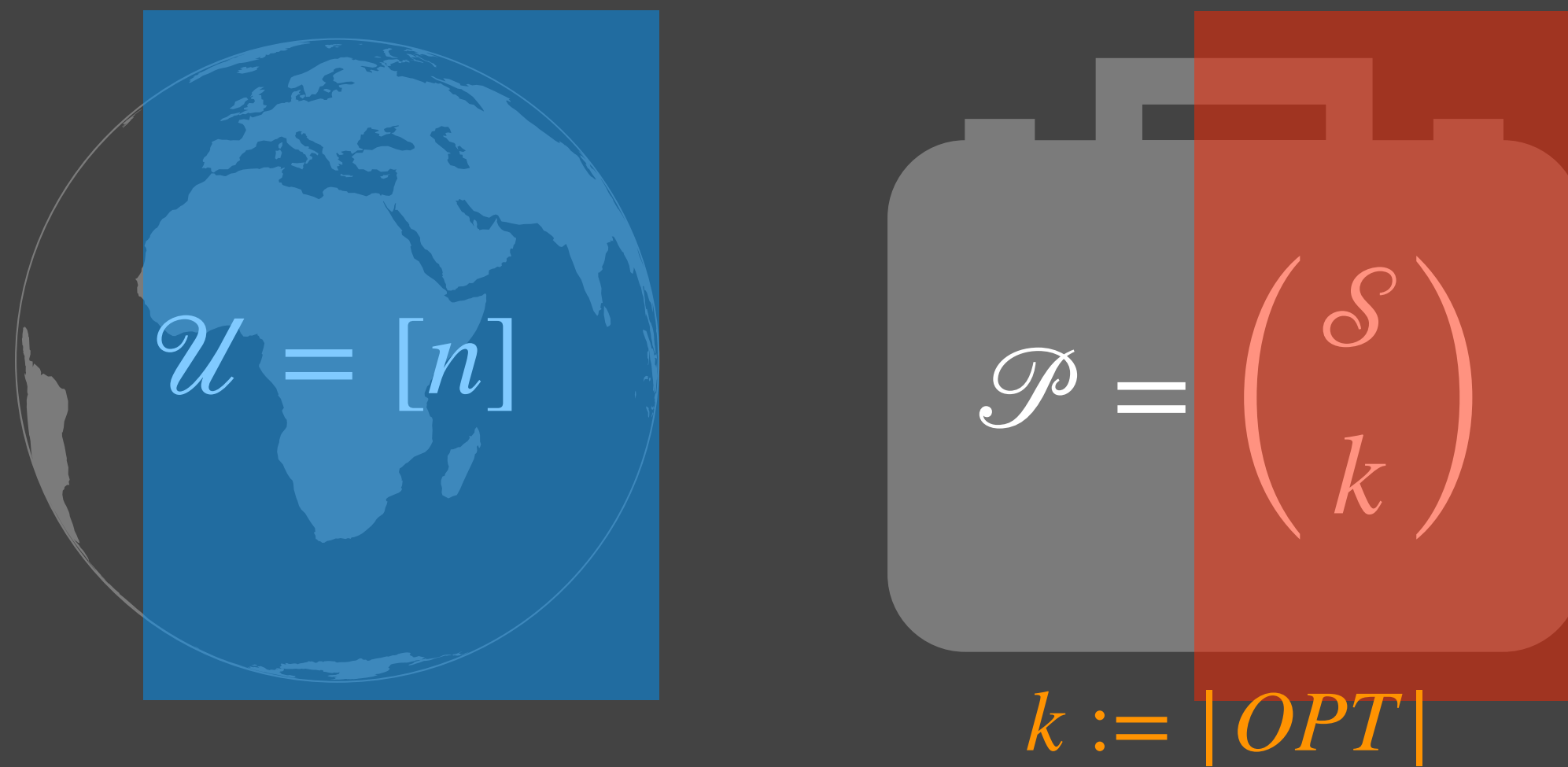
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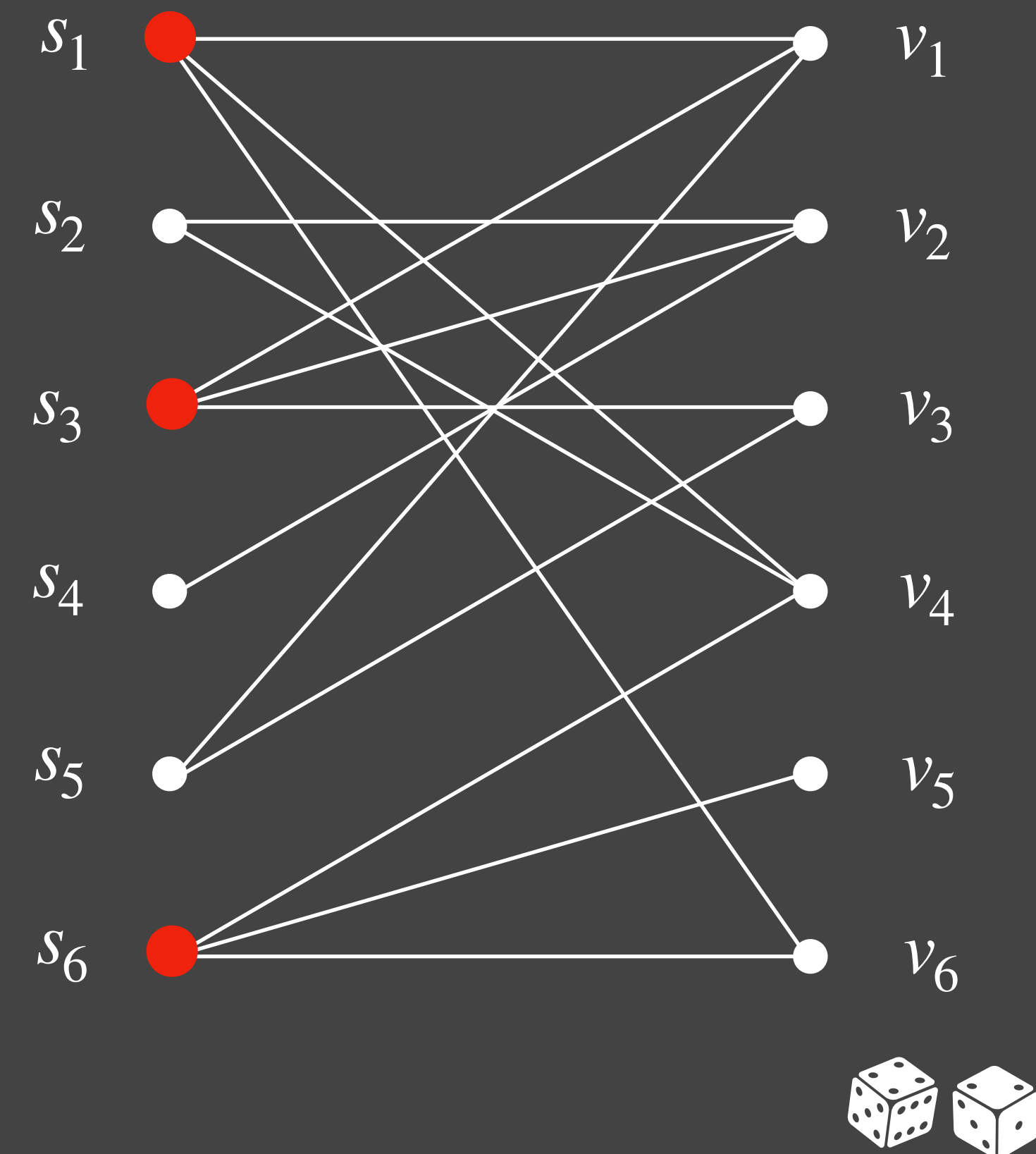
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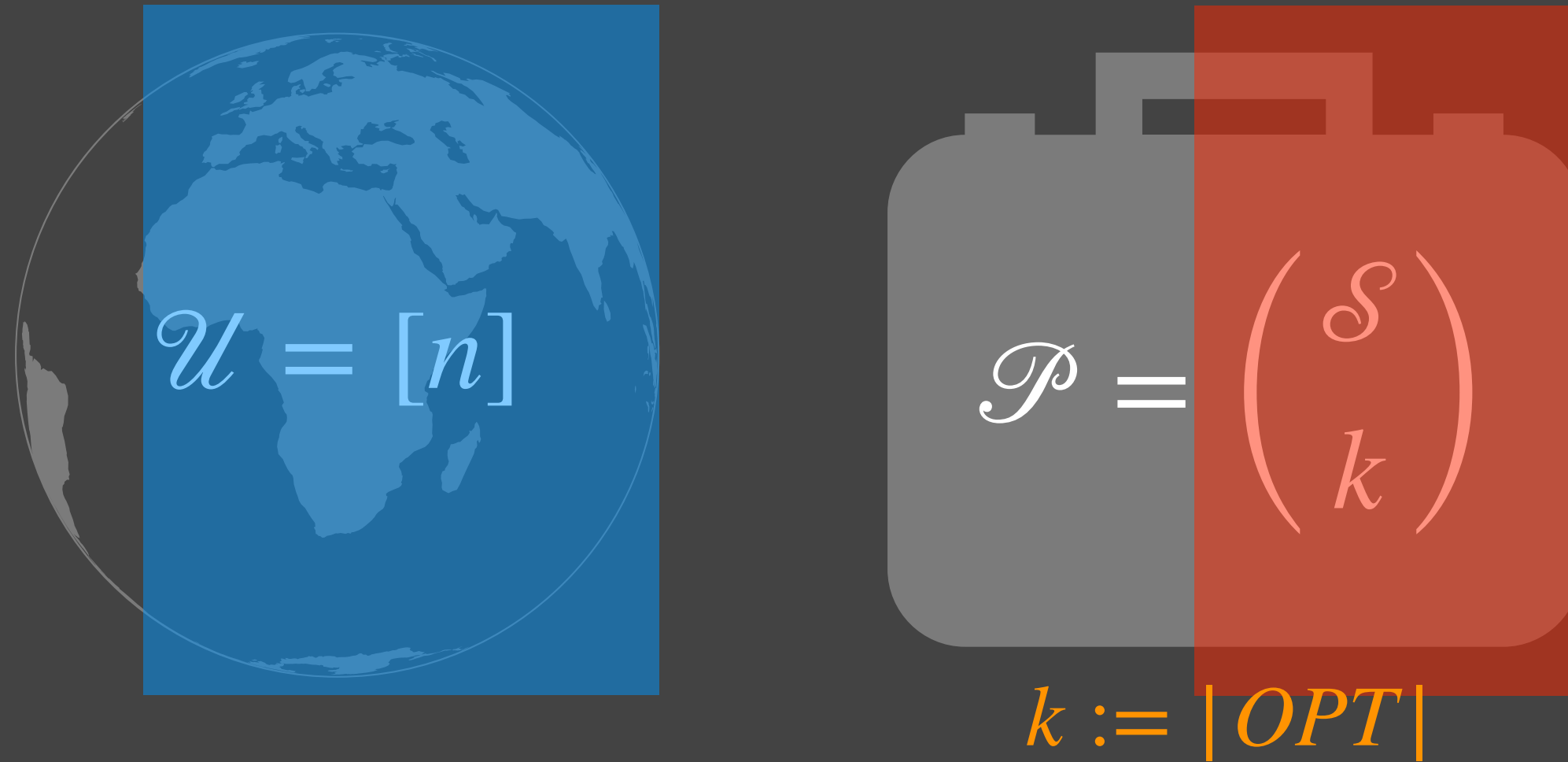
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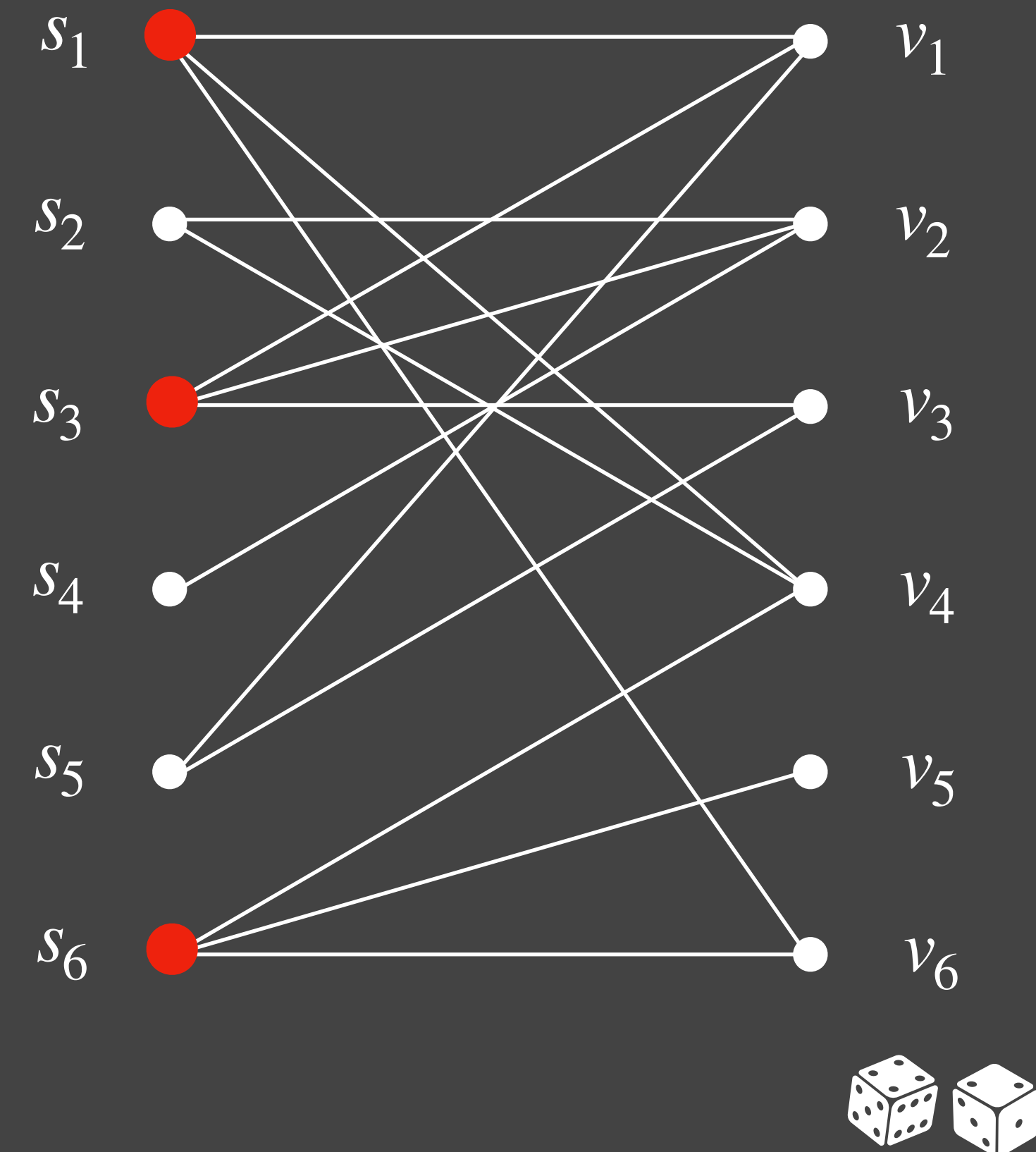


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After $O(\log n) \cdot \text{OPT}$ steps,

$|\mathcal{U}| = 0$ or $|\mathcal{P}| = 1$.

LearnOrCover [GKL. 21] enters the canon

In syllabus of Algorithmic Foundations course @ EPFL

Algorithmic Toolbox --- How to Solve Set Cover in x Ways

Credits 2
Lecturer [Ola Svensson](#)
Office hours Wednesdays 14:00 - 16:00 in INJ 112
Schedule Mondays 14-16 in INM201.



Short description

The goal of this PhD course is to give PhD students a toolbox of algorithmic techniques in order to successfully address their favorite problems. The course emphasizes the illustration of the main ideas of these techniques. We prefer simplicity over details and we illustrate the algorithmic techniques in the simple and clean setting of the set cover problem. The algorithmic techniques that we plan to cover include

- Greedy algorithms
- Local search algorithms
- Linear programming
 - Randomized rounding (independent, threshold, exponential clocks)
 - Duality (primal-dual algorithms, dual fitting, and the use of complementarity slackness)
- Multiplicative weight update
- Online algorithms in adversarial and random order streams (primal-dual, potential function, and projection based)

In addition, to attending the lectures, students are required to submit a project report where they apply one of the algorithmic techniques in a more complex setting.

Schedule and references

- **Lecture 1 (Monday February 27):** Introduction. Greedy and Local Search Algorithms. References: [Greedy algorithm](#), [Local Search Algorithm \(Section 2.1\)](#).
- **Lecture 2 (Monday March 6):** Linear programming, Threshold and Randomized rounding. References: [LPs and Threshold Rounding](#), [Independent Randomized Rounding](#), see also [for a very nice analysis](#).
- **Lecture 3 (Monday March 13):** Exponential clocks, TU matrices, VC-dimension. References: [Appendix A for exponential clocks](#), [TU matrices](#) and [consecutive ones property](#), for VC-dimension see [here](#) and [here](#).
- **Lecture 4 (Monday March 20):** TU matrices, VC-dimension. References: [Ola's notes](#)

Take Away III

[Gupta Kehne L. FOCS 21]

[Gupta Kehne L. In Submission]

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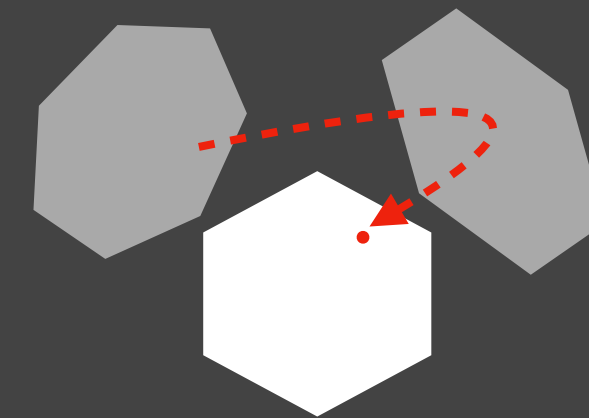
A2: Random instance is as easy as offline.

Outline

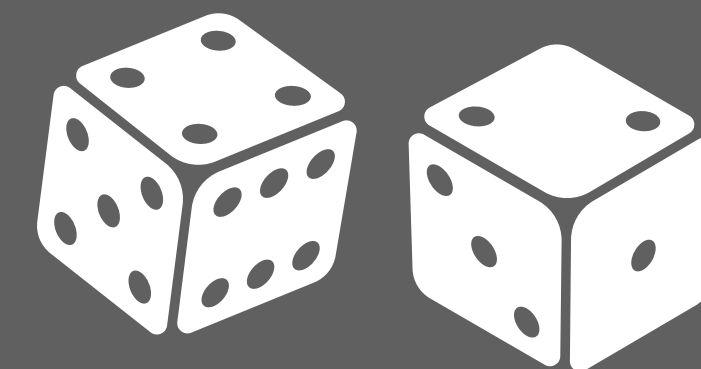
Theme I — Submodular Optimization

$$f(\text{🍕} \mid \text{🥕}) \geq f(\text{🍕} \mid \text{🥕}, \text{🍩})$$

Theme II — Stable Algorithms



Theme III — Beyond Worst-Case Analysis



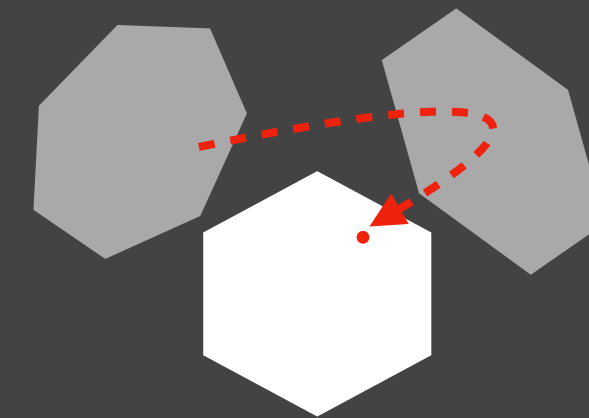
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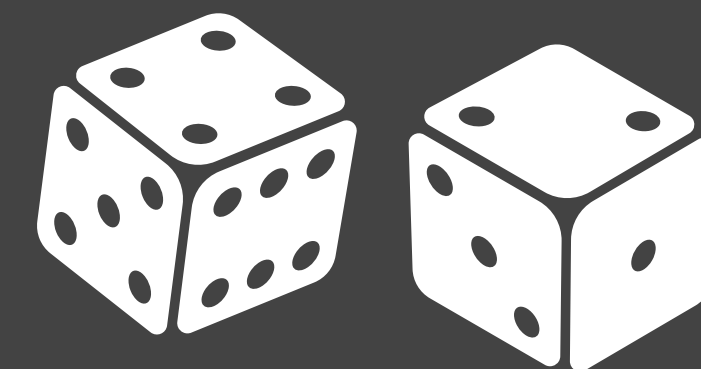
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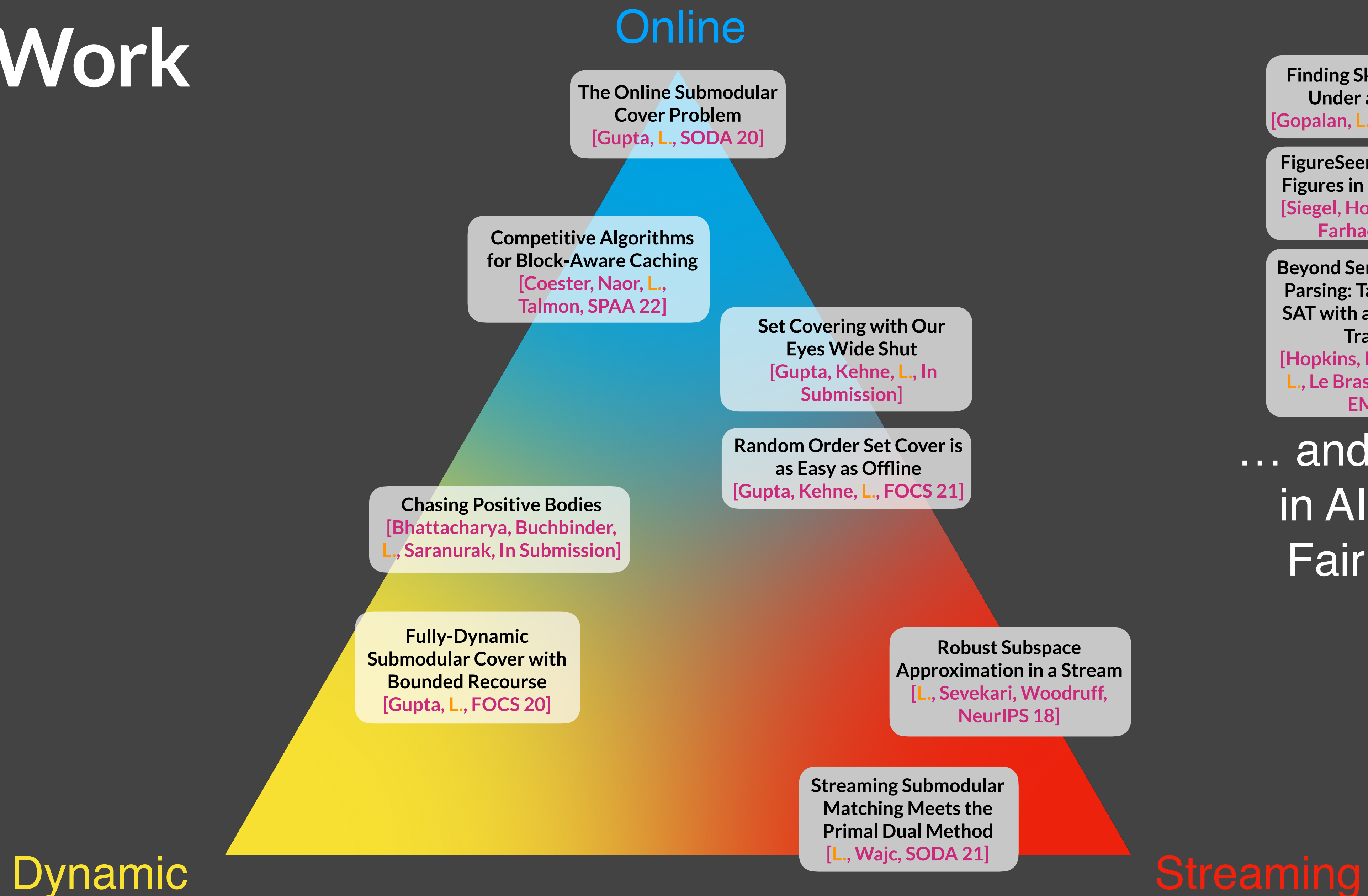
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My Work



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3. **Apply Theory in Practice**: Does my work inform useful heuristics? New collaborations on real world applications?

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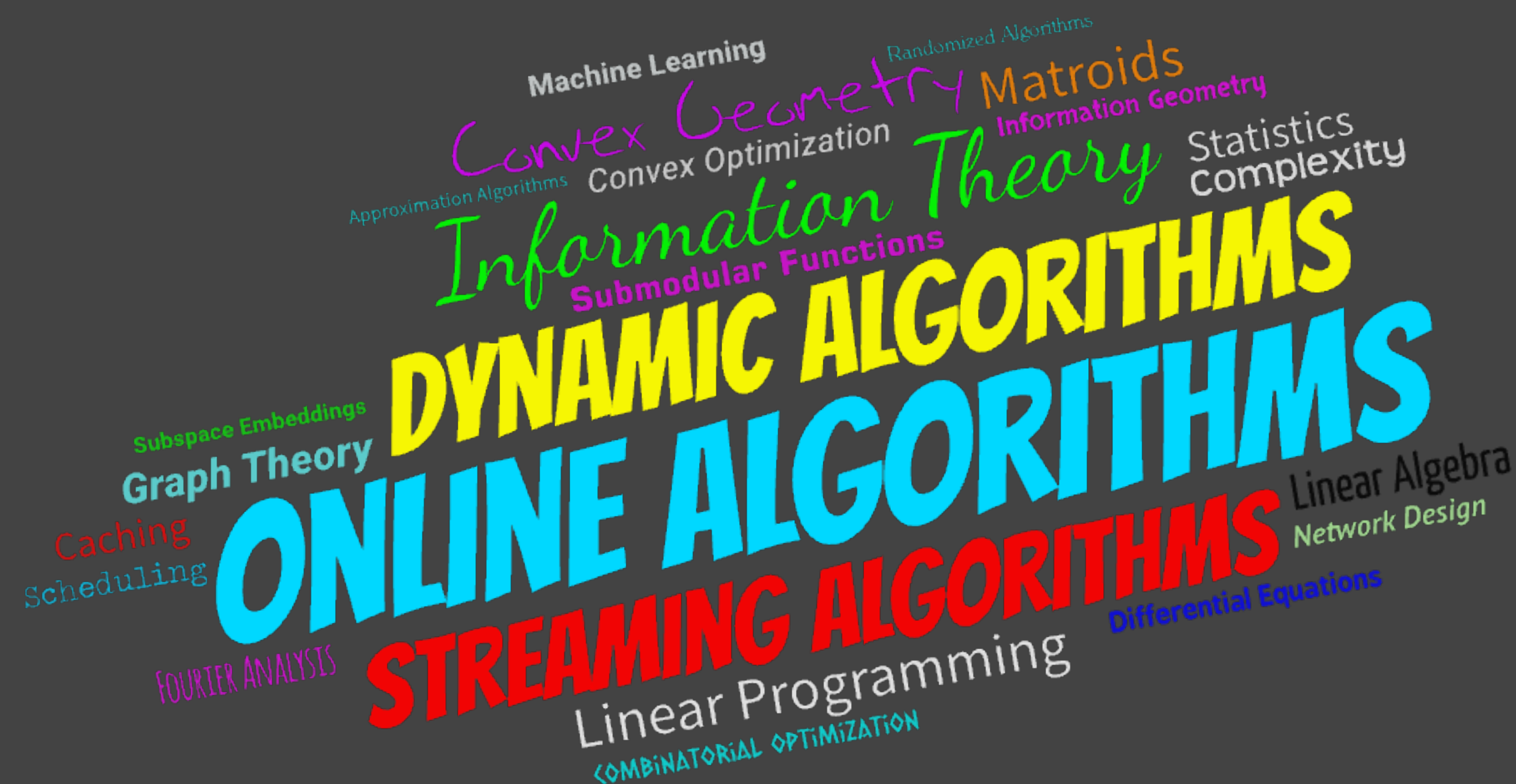
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Algorithms & Uncertainty

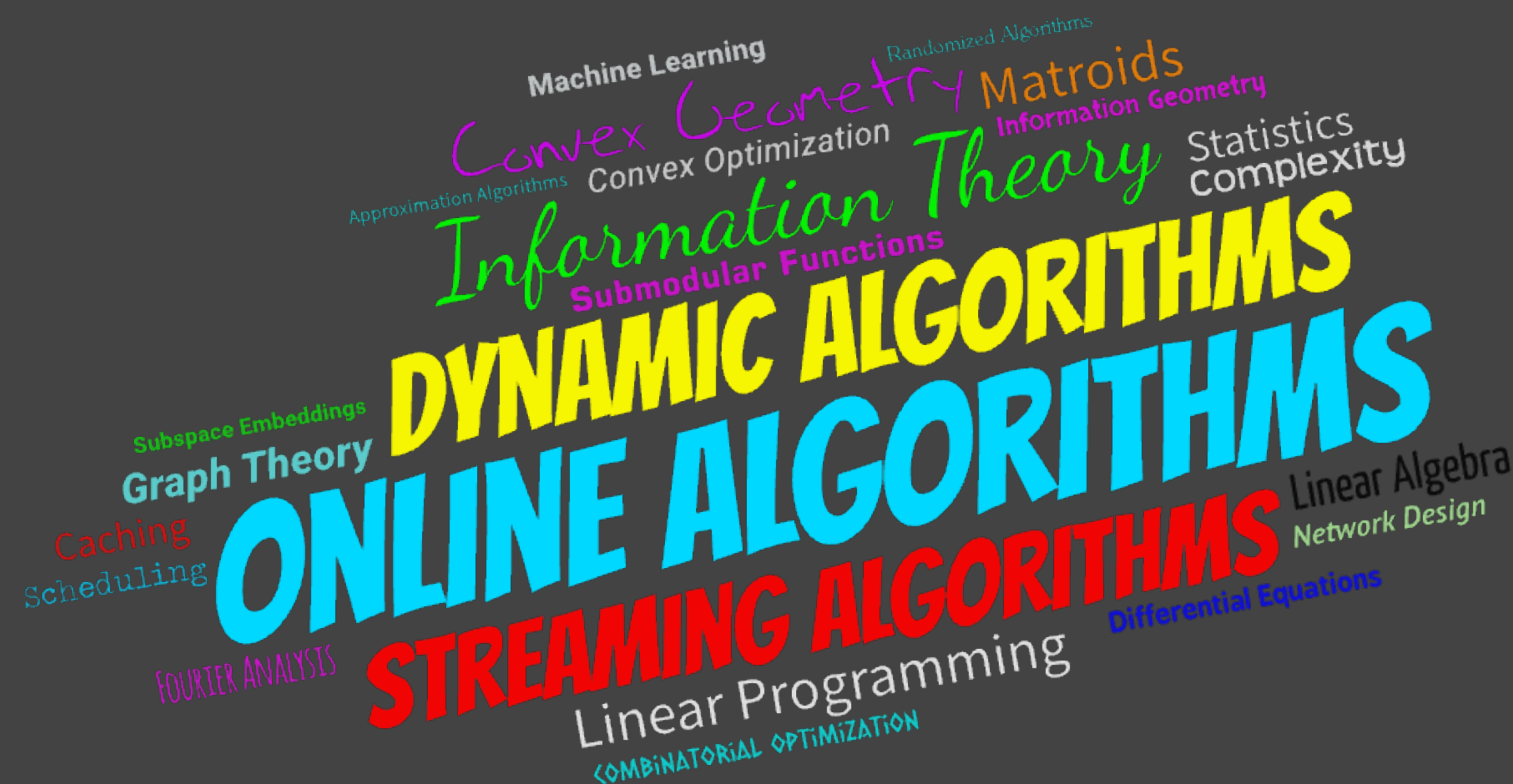
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Recent/Current Collaborators

- **Carnegie Mellon University**: Anupam Gupta, Anish Sevekari, David Woodruff
- **Harvard**: Gregory Kehne
- **U Michigan**: Thatchaphol Saranurak
- **Duke**: Debmalya Panigrahi
- **Tel Aviv University**: Niv Buchbinder, Haim Kaplan, Yaniv Sadeh
- **Technion**: Seffi Naor, Ohad Talmon, David Naori
- **Oxford**: Christian Coester
- **University of Warwick**: Sayan Bhattacharya
- **London School of Economics**: Neil Olver, Franziska Eberle
- **University of Bremen**: Nicole Megow
- **Google Research**: Ravi Kumar, Rajesh Jayaram, David Wajc
- **Apple**: Parikshit Gopalan
- **VMWare**: Udi Wieder

Thanks!

